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Section	B
exam	mid term

(P1)

Q1 Solve the following objective type Question.

part (i)

$A_{m\times p} B_{p\times n} AB = m\times n$ Ans

Part (ii)

No of non zero Row in echolen from is called Rank.

Part (iii)

$A = 8$

Part (iv)

3

Part (v)

Scaler matric

Part (vi)

$\log y = x - x^2 + c$

Part (vii)

order 1, degree = 3

Part (viii)

order and degree is not def as its not polynomial.

(ix) part

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5$$

(x) $bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$

Q 2 part (A)

Express the determination.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

<u>Solution.</u>

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a\,(b^2c^3 - b^3c^2) - b\,(a^2c^3 - a^3c^2) + c\,(a^2b^3 - a^3b^2)$$

$$ab^2c^3 - ab^3c^2 - ba^2c^3 + a^3bc^2 + ca^2b^3 - ca^3b^2$$

Q 2 part (B)

Find the Eigen value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Solution

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equation $\Rightarrow |A - \lambda I| = 0 \rightarrow A$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

expand by R1

$$2-\lambda \begin{vmatrix} 3-\lambda & 1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \Rightarrow$$

P5

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow B$$

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ expand by R1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big)+1\big((-1)(2-\lambda)-(-1)(-1)\big)$$
$$-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \rightarrow (a)$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

by C1

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

expand by C1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

put A, b and c in B

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$\Rightarrow -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda -$
$\lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda -$
$8\lambda + 6\lambda + 6\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

<u>By</u> synthatic division we get

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

<u>By</u> factorization Method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)\cancel{Or}(\lambda-4)$

$\lambda = 4 \quad \lambda = 4$

$\lambda_1 = 0 \quad , \lambda_2 = 2 \quad , \lambda_3 = 4 \quad , \lambda_4 = 4$

Q 3

$(x^2+3y^2)dx - 2xydy = 0$

Find the general solution at $x=2$ ∴ $y=6$

Solution

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y^2)\,dx = 2xy\,dy$$

$$\Rightarrow \frac{x^2+3y^2}{2xy} = \frac{dy}{dx}$$

$$\Rightarrow \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2xy} = \frac{dy}{dx}$$

$$\frac{x}{2y} + \frac{3y}{2x} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (1)$$

Comparing eq (1) with $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

eq (i) is a homogenous eq of degree (1) so we put $y/x = v$

$\Rightarrow y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

put in equation (i)

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

$$\Rightarrow 2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{v^2+1}{v}$$

$$\frac{2v}{v^2+1}\,dv = \frac{1}{2}\,x\,dx$$

$$= \int\frac{2v}{v^2+1}\,dv = \frac{1}{2}\int\frac{1}{x}\,dx$$

$$\ln(v^2+1) = \ln x + \ln c$$

$$\frac{v^2+1}{x} = c \qquad \Rightarrow v^2+1 = xc$$

$$\left(\frac{y}{x}\right)^2 + 1 = xc$$

$$\frac{y^2+x^2}{x^2} = xc$$

$$y^2 + x^2 = x^3c$$

Put $x = 2$ and $y = 6$

$$y^2 + x^2 = x^3c$$

$$(6)^2 + (2)^2 = (2)^3c$$

$$36 + 4 = 8c$$

$$40 = 8c$$

$$40/8 = c$$

$$c = 5$$

$$y^2 + x^2 = 5x^3$$

$$\sqrt{y^2} = \sqrt{5x^3 - x^2}$$

$$y = \pm\sqrt{5x^3 - x^2}$$

$$y = \pm\sqrt{x^2(5x-1)}$$

$$\boxed{y = \pm x\sqrt{5x-1}}$$