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7966

SEC - B

CIVIL (4th SEM)

MAAM SHUMAILA

DIFFERENTIAL EQUATION

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Q No: 1 ①

The wave Equation:

The wave equation is an important second order linear partial differential equation for the descaiption of wave as they occur in classical Physic - such as mechanial wave (e.g water Waves sound wave & semic wave.

⇒ We generally visit beach & if we stand on an ocean share & take a snapshots

②

the waves the picture

show as - - - - -

-- - - - - -

- - - -- equation.

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

i) $w = \sin(x+ct) + \cos(2x+2ct)$

ii) $w = \tan(2x+ct)$

i $w = \sin(x+ct) + \cos(2x+2ct)$.

Sol:-

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

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$$\Rightarrow \quad \frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

$$c^2 \cdot \frac{\partial^2 w}{\partial x^2}$$

Hence

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

ii:- $W = \tan(2x + ct)$.

Sol:-

$W = \tan(2x + ct)$

A we know that

$W = \tan(2x + ct)$

Taking derivate w.r.t x

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t} \tan(2x + ct) \cdot c$$

$$\Rightarrow \frac{\partial w}{\partial t} = \sec^2(2x + ct) \cdot c$$

$$\frac{\partial w}{\partial t} = c \sec^2(2x + ct)$$

Agian Taking Derivate.

$$\frac{\partial^2 w}{\partial t^2} = c \frac{\partial}{\partial t} \sec^2(2x + ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c \cdot 2 \sec(2x+ct) \cdot \sec(2x+ct) \times \tan(2x+ct)\, c$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2 \sec^2(2x+ct) \cdot \tan(2x+ct)$$

Now

$$w = \tan(2x+ct)$$

Taking derivate w.r.t x

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x} \tan 2x + ct.$$

$$\frac{\partial w}{\partial x} = \sec^2(2x+ct) \cdot 2$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct).$$

$\Rightarrow$ Again taking derivate

$$\frac{\partial^2 w}{\partial x^2} = 2\frac{\partial}{\partial x} \sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2 \cdot 2 \sec(2x+ct)\sec(2x+ct) \tan(2x+ct) \cdot 2$$

$$\Rightarrow \frac{\partial^2 w}{\partial x^2} = 6\sec^2(2x+ct)\tan(2x+ct)$$

As we know the wave Equation

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

Put values in wave equation.

$$1 \neq 3$$

Hence

L.H.S $\neq$ R.H.S.

not proof wave equation.

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Q NO: 2

Expand the following function in a fourier Series.

Sol:

$$F(x) = x, \quad -\pi < x < 0$$

$$= 2x, \quad 0 < x < \pi$$

$a_0 = ?$ $a_n = ?$, $b_n = ?$

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\Big|_{-\pi}^{0}\right] + \frac{2}{\pi}\left[\frac{x^2}{2}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}} - 0}{\cancel{2}}\right]$$

$$a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2} \longrightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= -\frac{1}{\pi}\left[x\int_{-\pi}^{0} x\cos nx\,dx\right] + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= -\frac{1}{\pi}\left[x\left(\frac{\sin nx}{x}\right) - \left(-\frac{\cos nx}{n^2}\right)\Big|_{-\pi}^{0}\right] +$$

$$\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\Big|_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2} + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \begin{cases} \frac{-2}{\pi n^2}, & \text{if } n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx$$

$$+ \frac{2}{\pi}\int_{0}^{\pi} x \sin nx.$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\Big|_{-\pi}^{0}\right]$$

$$+ \frac{2}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\Big|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right].$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

Hence.

$$F(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)}{(2n-1)^2} + 3$$

$$\sum_{n=1}^{\infty}\frac{\sin nx}{n.}$$ Required Fourier Series

Q No: 3

Solve the initial value problem

$$y'' - 4y' + 13y = 8\sin 3x$$

$$y(0) = 1 \quad , \quad y'(0) = 2$$

Sol:-

$$y'' - 4y' + 13y = 8\sin 3x \quad \rightarrow (1)$$

Associated Homogenous Eq (1) is

$$y'' - 4y' + 13y = 0 \quad \rightarrow (2)$$

Into Auxiliary Equation

put $y = m$ in $\rightarrow (2)$

$$m^2 - 4m + 13 = 0$$

$\Rightarrow$ Use Quadratic Formula.

$$a = 1 \quad , \quad b = -4 \quad , \quad c = 13.$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2}$$

$$m = \frac{4 \pm \sqrt{16-52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

$$m = \frac{4 \pm \sqrt{36i}}{2} = \frac{4+6i}{2}$$

$$m = 2 \pm 3i$$

$$m_1 = 2+3i \quad , m_2 = 2-3i$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow A$$

Let $y_p = A\cos 3x + B\sin 3x \rightarrow A_{II}$

Differenate w.r.t "x"

$$y_p = -3A\sin 3x + 3B\cos 3x.$$

Again Differentiate w.r.t x.

$$y_p'' = -9A\cos 3x + 3B\cos 3x$$

put in Eq (1)

$$(-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3D\cos 3x) + 13(A\cos 3x + B\sin 3x) - 8\sin 3x.$$

$$(-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$$

$$(4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$$

Comparing Coefficient

$$\sin 3x \Rightarrow 4B + 12A = 8 \dashrightarrow a$$

$$\cos 3x \Rightarrow 4A - 12B = 0$$

$$4A = 12B \Rightarrow \boxed{A = 3B} \rightarrow *$$

Put in * Eq a

$$4B + 12(3B) = 8$$

$$4B + 36B = 8$$

$$40B \Rightarrow B = \frac{1}{5} \rightarrow c$$

Put Equation c in eq (*).

$A = \frac{3}{5} \rightarrow (d)$

Put c & d in $(*)$

$$y_p = \frac{B}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$$

The General Solution is

$$y = y_c + y_p.$$

$$y = e^{3x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

Now we have to find the values of c_1 & c_2

For this.

Put $x=0$ & $y=1$ in (C)

$$1 = e^{x(2)}(c_1 \cos 3(0) + c_2 \sin 3(0)) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0)$$

$$1 = \left(c_1 (1)' + c_2 (0)\right) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = c_1 + \frac{3}{5}$$

$$c_1 = \frac{2}{5} \longrightarrow (*_1)$$

Differentiate c w.r.t x

$$y' = c_1 \left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) + c_2\left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \rightarrow (D)$$

Put $y' = 2$, $x = 0$ in (D)

$$y' = c_1 \left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) + c_2 \left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x.$$

Put $y' = 2$, $x = 0$

$$2 = c_1\left(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)\right) - \frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0)$$

$$2 = C_1(2) + C_2(3) - 0 + \frac{3}{5}$$

$$2 = 2C_1 + 3C_2 + \frac{3}{5}$$

Put $C_1 = \frac{2}{5}$

$$2 = \frac{4}{5} + 3C_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3C_2$$

$$3C_2 = 2 - \frac{7}{5} \Rightarrow C_2 = \frac{3}{15} \rightarrow \star_2$$

Put ★1 & ★2 in C.

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$$

$$y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$$

Required General Solution.

Q NO : 4

$$(D^2 - DD')Z = \cos x \cos 2y.$$

Sol:-

$$(D^2 - DD')Z = \cos x \cos 2y.$$

In CF is given by.

$$CF = \phi_1(y) + \phi_2(y+x)$$

while its PI is given by.

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Complete solution of the given PDE'S

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y) \text{ Ans.}$$