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Paper Differential Equation.
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Q1:-
part a):-

Ans:- Homogeneous differential equation
It involves only derivatives of y & terms involving y, & they are set to 0, as in this example.

$$\frac{d^4y}{dx^4} + x\frac{d^2y}{dx^2} + y^2 = 0.$$

Non homogeneous differential eq:

Same as homogeneous differential equations, except they can have terms involving only x on right side as given.

$$\frac{d^4y}{dx^4} + x\frac{d^2y}{dx^2} + y^2 = 6x+3.$$

Ans

Q1:- b:- Solve the following 2nd order linear homogeneous/ non-homogeneous differential equation?

i):- $4y'' - 6y' + 7y = 0$

Soln:

First finding $\sqrt{\ }$

So,

$$4\lambda^2 - 6\lambda + 7 = 0$$

$$\lambda = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$$

$$\lambda = \frac{6 \pm \sqrt{36-112}}{8} \Rightarrow \lambda = \frac{6 \pm \sqrt{-76}}{8}$$

$$\lambda = \frac{6}{8} + \frac{2\sqrt{19}}{8} i \Rightarrow \lambda \frac{3}{4} \pm \frac{\sqrt{19}}{4} i$$

$$\lambda_1 = \frac{3}{4} + \frac{\sqrt{19}}{4} i, \quad \lambda_2 = \frac{3}{4} \frac{\sqrt{19}}{4}$$

So it has the complex conjugate roots.

$$\phi_1(x) = e^{\lambda_1 x} \cos \lambda_1(x)$$

$$\phi_2(x) = e^{\lambda_2 x} \sin \lambda_2(x)$$

$$y = C_1 e^{\frac{3}{4}x} \cos \frac{2\sqrt{19}}{4}(x) + e^{\frac{3x}{4}} \sin i \frac{\sqrt{19}}{4}(x) \, C_2.$$

Answer.

Q1

b):

ii):- $y'' - 4y' - 12y = 3e^{5x}$

Solution

The characteristics equation & its roots

$y^2 - 4y - 12 = (y-6)(y+2) = 0$

$y_1 = -2$ , $y_2 = 6$.

The complementary solution is then:

$y_c(t) = c_1 e^{-2t} + c_2 e^{6t}$.

Answer

Q2:- Solve the following IVP for the $2^{nd}$ order linear equations.

i):- $16y'' - 40y' + 25y = 0$ $y(0) = 3$ $y'(0) = -9/4$

Solution.

The characteristic equation & root are as given.

$16y^2 - 40y + 25 = (4y^{(2)} - 5)^2 = 0$ $y_1 = 5/4, y_2 = 5/4$

The general solation & its derivative are

$$y(t) = C_1 e^{5t/4} + C_2 e^{5t/4}.$$

$$y'(t) = 5/4\, C_1 e^{5t/4} + C_2 e^{5t/4} + 5/4\, C_2 t e^{5t/4}$$

Now Put it in the initial position.

$$3 = y(0) = C_1$$

$$-9/4 = y'(0) = 5/4\, C_1 + C_2.$$

The solution for IVP is then

$$y^t = 3e^{5t/4} - 6te^{5t/4}.$$

Answer

Q2
ii):- $y'' + 14y' + 49y = 0$. $y(-4) = -1$ $y'(-4) = 5$

<u>Soln</u>

The characteristic equation & its root are

$y^2 + 14y + 49 = (y+7)^2 = 0$ $y_1 = 7, y_2 = -7$

The general soln & its derivative are

$y(t) = C_1 e^{-7t} + C_2 t e^{-7t}$.

$y' t = -7C_1 e^{-7t} + C_2 e^{-7t} - 7C_2 t e^{-7t}$

Putting in the initial condition.

$-1 = y(-4) = C_1 e^{28} - 4C_2 e^{28}$

$5 = y(-4) = -7C_1 e^{28} + C_2 e^{28} + 28 C_2 e^{28}$

$= -7C_1 e^{28} + 29 C_2 e^{28}$.

It gives the following constant by solving-

$C_1 = -9e^{-28}$

$C_2 = -2e^{-28}$.

The soln for IVP is,

$y(t) = -9e^{28} e^{-7t} - 2te^{-28} e^{-7t}$

$y(t) = -9e^{-7(t-4)} - 2te^{-7(t+4)}$.

Answer

Q2:

iii): $y'' - 4y' + 9y = 0 \quad y(0) = 0, \; y'(0) = -8.$

Soln

The characteristic equation for this DE is.

$$y^2 - 4y + 9 = 0.$$

The $\sqrt{\ }$ of equation are.

$$y_1 = 2 \pm \sqrt{5}\, i$$
$$y_2 = 2 \pm \sqrt{5}\, i$$

The general soln to the D.E is

$$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t).$$

Applying initial condition along with derivatives.

$$y(t) = C_2 e^{2t} \sin(\sqrt{5}t).$$
$$y'(t) = 2i\, 2e^{2t} \sin(\sqrt{5}t) + \sqrt{5}\, C_2 e^{2t} \cos(\sqrt{5})$$

$$-8 = y'(0) = \sqrt{5}(2) = C_2 = -8/5$$

Soln is.

$$y(t) = -8/5\, e^{2t} \sin.$$

Ans

Q2

iv):- $y'' - 8y' + 17y = 0 \quad y(0) = -4, y'(0) = -1$

Soln

The characteristic equation & its roots are

$$y^2 - 8y + 17 = 0.$$
$$y_1 = 4 + i$$
$$y_2 = 4 - i.$$

The general soln as well as derivative is

$$y(t) = C_1 e^{4t} \cos(t) + C_2 e^{4t} \sin(t).$$
$$y'(t) = 4C_1 e^{4t} \cos(t) - C_1 e^{4t} \cos(t)$$

$$y'(t) = 4C_1 e^{4t} \cos(t) - C_1 e^{4t} \cos(t) + 4C_2 C^{4t} \sin(t) + C_2 e^{4t} \cos(t)$$

- By applying the initial condition gives us,

$$-4 = y(0) = C_1$$

$$-1 = y(0) = 4C_1 + C_2.$$

So, the soln is

$$y(t) = -4e^{4t} \cos(t) + 15e^{4t} \sin(t).$$

Ans

Q3:

a):- Laplace

Ans Laplace transform is integral transform that converts a function of real variable (t) to the function of complex var ($x$).

i.e:

The laplace transform $\mathcal{L}$ of a function $f(t)$ for $t>0$ is defined by the following

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt.$$

general eu:
$$f(s) = \int_0^\infty f(t) \cdot e^{-st}\,dt.$$

Q3 Define laplace transform along with example? find the laplace if if transform of the given function is.

i):- $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9.$

Soln:

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

Answer

ii):- $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t).$

Soln

$$g(s) = \frac{4s}{s^2+(4)^2} - \frac{9\cdot 4}{s^2+(4)^2} + \frac{2s}{s^2+(10)^2}$$

$$\frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}.$$

Answer

iii):- $H(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t).$

Soln

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}.$$

$$\frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

Answer

Q4:- Solve the following IVP using laplace form:

1):- $y'' - 10y' + 9y = 5t, \quad y(0) = -1, \quad y'(0) = 2.$

Soln

First taking transform of every term.

$\mathcal{L}\{y''\} - 10\mathcal{L}\{y'\} + 9\mathcal{L}(y) = 5\mathcal{L}\{t\}$

By formula.

$$s^2 Y_{(s)} - s y(0) - y'(0) - 10(sY_{(s)}) - y(0) + 9Y_{(s)} = \frac{5}{s^2}$$

Now, put in initial condition.

$(s^2 - 10s + 9) Y(s) + s - 12 = 5/s^2.$

Solve for $y(s)$

$$Y(s) = \frac{5}{s^2(s+9)(s+1)} + \frac{19-s}{(s-9)(s-1)}$$

$$Y(s) = \frac{5 + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

The partial fraction of transform will be

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$5 + 12s^2 - s^3 = A \cdot s(s-9)(s-1) + B(s-9)(s-1)$
$+ Cs^2(s-1) + Ds^2(s-9).$

Solve for constants.

| | | | |
|---|---|---|---|
| $s=0$ | $5 = 9B$ | $\Rightarrow$ | $B = 5/9$ |
| $s=1$ | $16 = -8D$ | $\Rightarrow$ | $D = -2$ |
| $s=9$ | $248 = 648C$ | $\Rightarrow$ | $C = 31/81$ |
| $s=2$ | $45 = -14A + \frac{4345}{81}$ | $\Rightarrow$ | $A = 50/81$ |

Plugging in the constant gives.

$$Y(s) = \frac{50}{81} + \frac{5/9}{s^2} + \frac{31/81}{s-9} - \frac{2}{s-1}$$

By taking the inverse transform the solution is

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{2t} - 2e^{t}.$$

Answer

Q4:-

ii):- $y'' - 6y' + 15y = 2\sin(3t)$ $\quad y(0)=1,\ y'(0)=-4$

**Soln**

Taking the laplace transform of every Eq & plug it in initial condition.

$$s^2Y(s) - sy(0) - y'(0) - 6(sY(s) - y(0)) + 15Y(s) = 2\frac{3}{s^2+9}$$

$$(s^2-6s+15)Y(s) + s - 2 = \frac{6}{s^2+9}$$

$$Y(s) = \frac{-s^3+2s^2-9s+24}{(s^2+9)(s^2-6s+15)}$$

Lets get the partial fraction decomposition

$$Y(s) = \frac{As+B}{s^2+9} + \frac{(s+D)}{s^2-6s+15}.$$

Now setting numerator equal gets.

$$-s^3+2s^2-9s+24 = (As+B)(s^2-6s+15) + (s+D)(s+9) = (A+C)s^3 + (-6A+B+D)s^2 + (15A + 9C)s + 15B + 9D.$$

Solve for constant.

$$\left.\begin{array}{ll} s^3: & A+C=-1 \\ s^2: & -6A+B+D=2 \\ s^1: & 15A-6B+9C=-9 \\ s^0: & 15B+9D=24 \end{array}\right\} \quad A=\frac{1}{10} \quad B=\frac{1}{10} \quad C=-\frac{11}{10} \quad D=\frac{5}{2}$$

Plugging in the constant gives.

$$Y(s)=\frac{1}{10}\left(\frac{s+1}{s^2+9}+\frac{-11s+25}{s^2-6s+15}\right).$$

$$=\frac{1}{10}\left(\frac{3}{s^2+9}+\frac{1\frac{3}{3}}{3^2+9}-\frac{11(s-3)}{(s-3)^2+6}\right.$$

finally take the inverse transform & solution will be

$$y(t)=\frac{1}{10}\left(\cos(3t)+\frac{1}{3}\sin(3t)-11\cos\sqrt{6}t-\frac{8}{\sqrt{6}}e^{3t}\sin\sqrt{6}t\right).$$

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