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Section:

B

Depth:

BE(K)

Subject:

Structure Analysis

Submitted to:

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## Question No 012

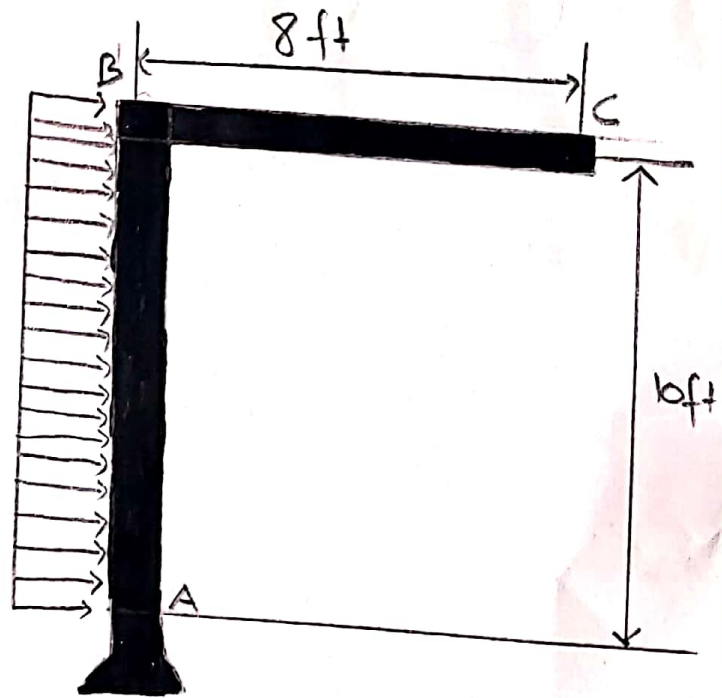
Determine the vertical displacement of free end point C on the frame in figure. Take  $E = 29 (10^3) \text{ ksi}$  and  $I = 600 \text{ in}^4$  for both members. Use method of virtual work.

### Given Data:

$$E = 29 (10^3) \text{ ksi}$$

$$I = 600 \text{ in}^4$$

4 kft



Required:

Vertical displacement = ?

Solution:

Now vertical moment

①

For reaction 2

$$\sum M_A = 0$$

$$-4(10)(5) + C_y(8) = 0$$

$$C_y = 25 \text{ kips}$$

$$\sum F_y = 0 \uparrow +$$

$$25 + A_y = 0$$

$$A_y = -25 \text{ kip}$$

$$\sum F_x = 0 \rightarrow +$$

$$40 - A_x = 0$$

$$A_x = +40$$

Real moment;

$$\sum M_1 = 0$$

$$-40(x_1) + 4x_1\left(\frac{x_1}{2}\right) + M_1 = 0$$

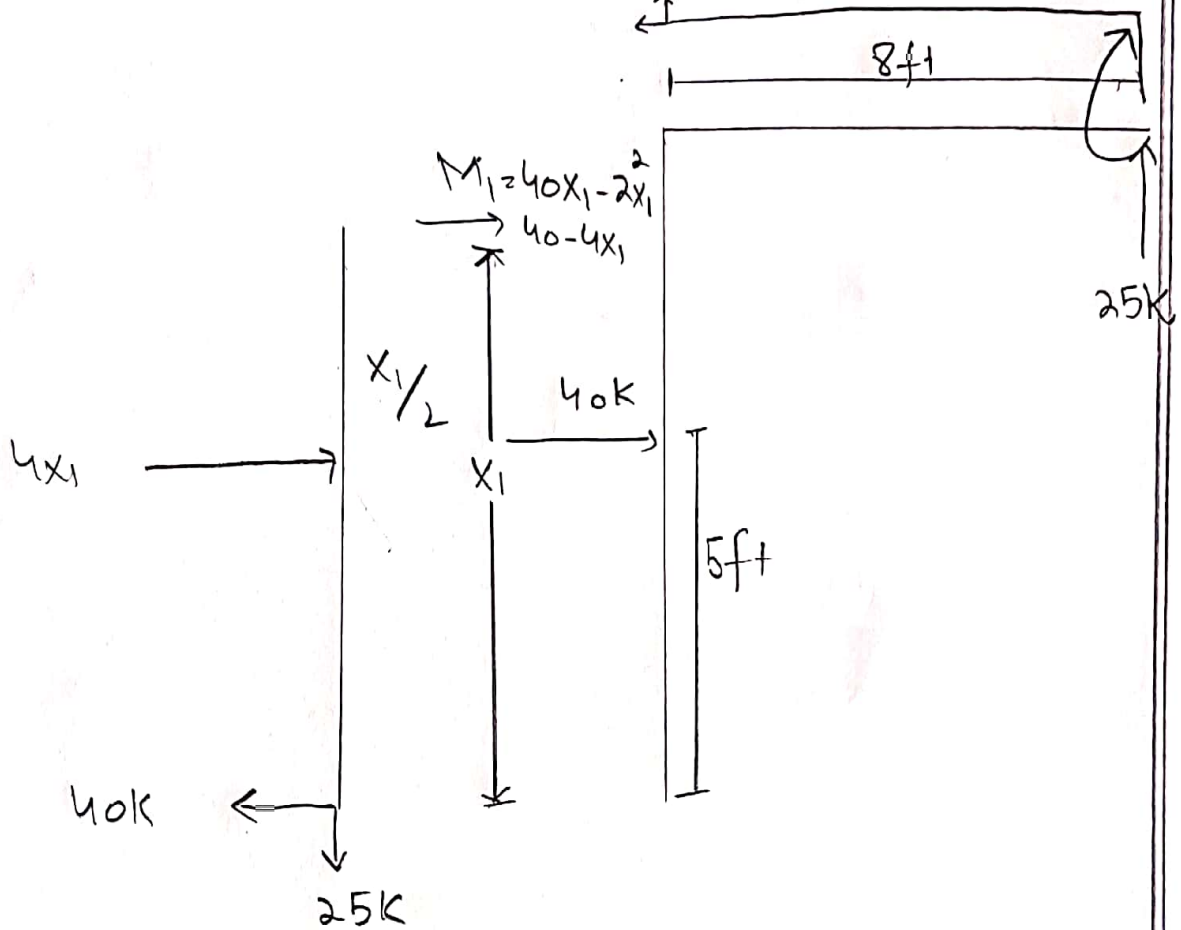
$$M_1 = 40x_1 - 2x_1^2$$

2

$$-25x_2 + M_2 = 0$$

$$M_2 = 25x_2, V_2 = -25$$

$$M_2 = 25x_2$$



Virtual Moments:

$$\sum m_1 = 0$$

$$-1(x_1) + m_1 = 0$$

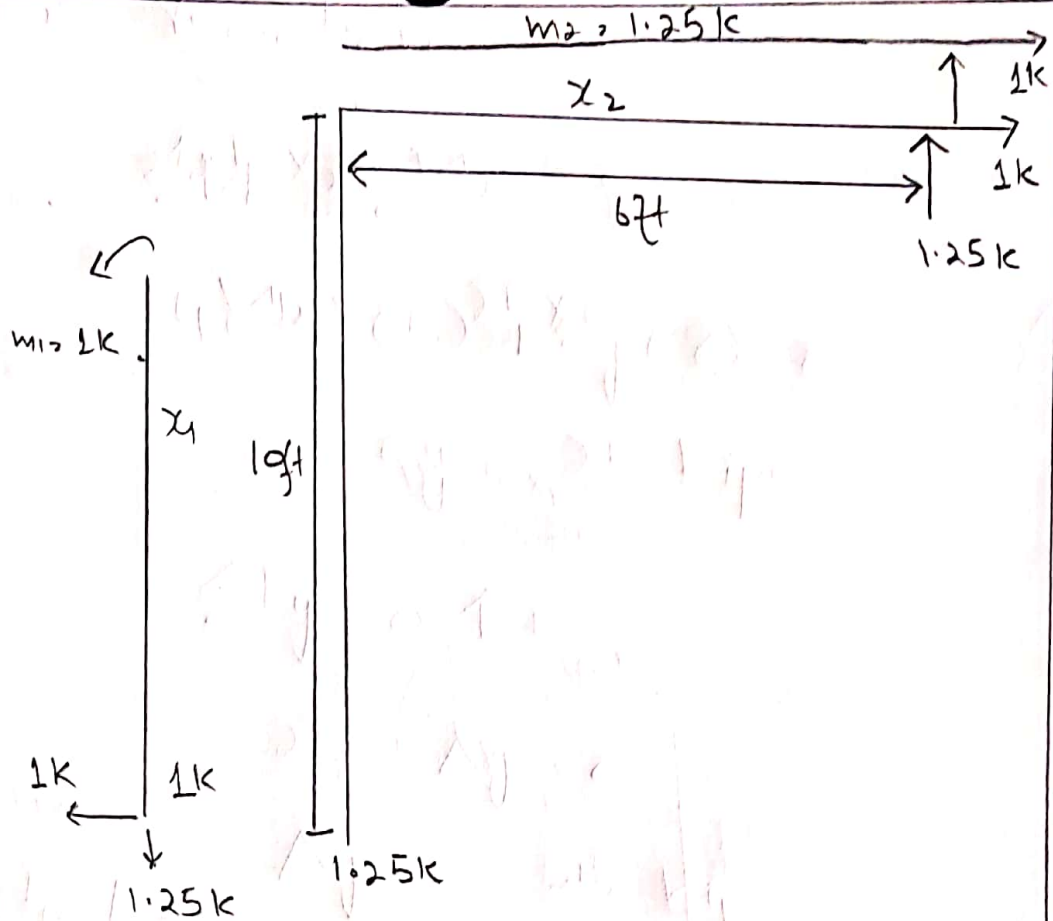
$$m_1 = 1x_1$$

$$-m_2 + 1 \cdot 25x_2 = 0$$

$$m_2 = 1 \cdot 25x_2$$



5



Now from virtual work eqn

$$1k \cdot \Delta_{ch} = \int_0^{10} m \frac{M dx}{EI}$$

$$1k \cdot \Delta_{ch} = \int_0^{10} \frac{(40x_1 - 2x_1^2)(1x_1) dx}{EI}$$

$$\Delta_{ch} = \frac{8333.3}{EI} + \frac{5333.3}{EI}$$

$$\Delta_{ch} = \frac{13666.7}{EI} \text{ k}^3 \cdot \text{ft}^3$$

$$\Delta_{ch} = \frac{13666.7 \text{ k}^2 \cdot \text{ft} (12^3 \cdot \text{in}^3) (1 \text{ ft}^3)}{[29 \times 10^3 \text{ k/in}^2] [600]}$$

Result

$$\Delta_{ch} = 1.357 \text{ in}$$

Q

## Question No 022

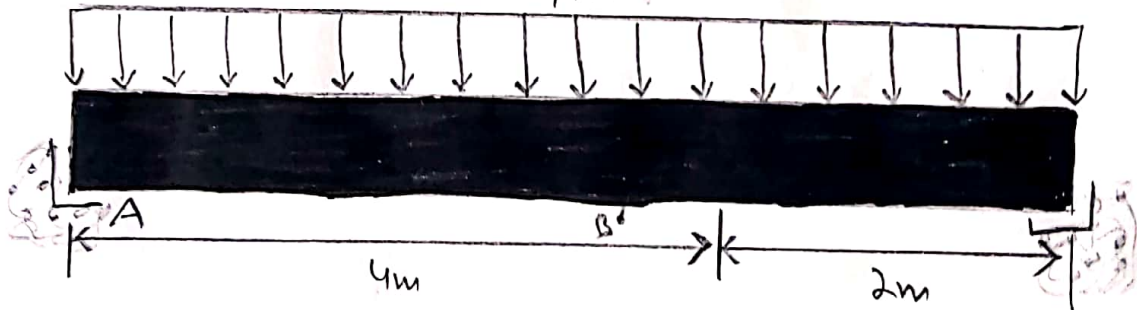
Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take  $E = 200 \text{ GPa}$ ,  $I = 60 (10)^6 \text{ mm}^4$ . Use Castigliano's theorem.

### Given Data

$$E = 200 \text{ GPa}$$

$$I = 60 (10)^6 \text{ mm}^4$$

$$6 \text{ kN/m}$$

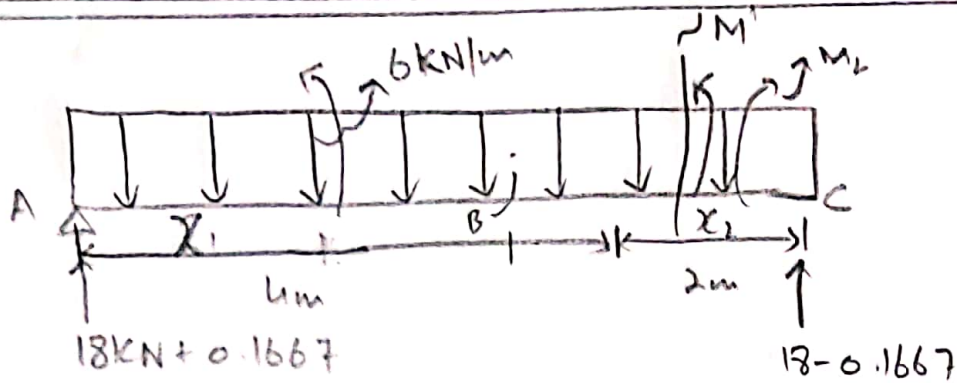


### Required

Slope and displacement at B?

### Solution

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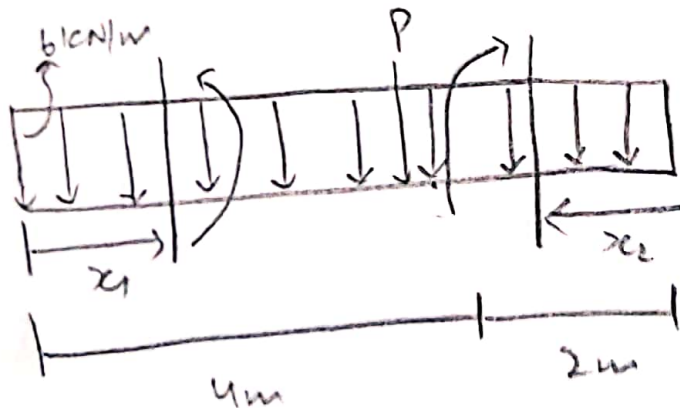
$$R_1 + R_2 = 0$$

$$\sum M_A = 0 \quad \curvearrowright$$

$$1 + R_2(6) = 0$$

$$R_1 + (-0.1667) = 0$$

$$R_1 = 0.16667 \text{ KN}$$



$$R_1 + R_2 = 1$$

$$\curvearrowright + \sum M_A = 0$$

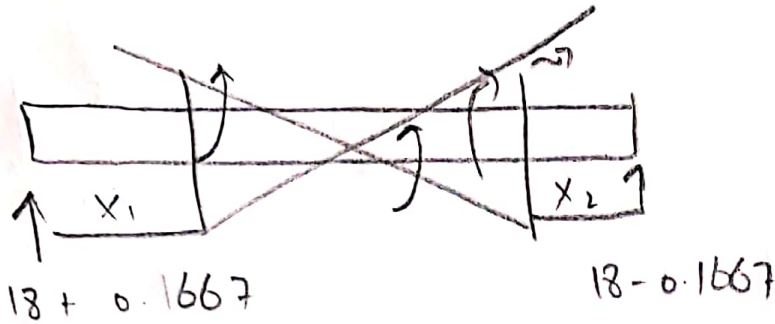
$$-(1)(4) + R_2(6) = 0$$

$$R_1 = 0.16667 \text{ KN}$$

$$R_2 = 1 - 0.16667 \text{ KN}$$

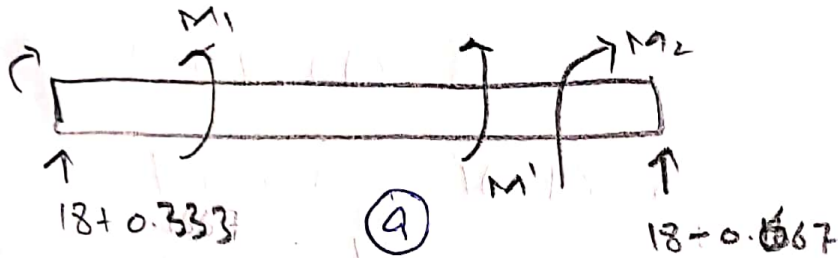
(6)

$$R_2 = 0.333 \text{ kN}$$



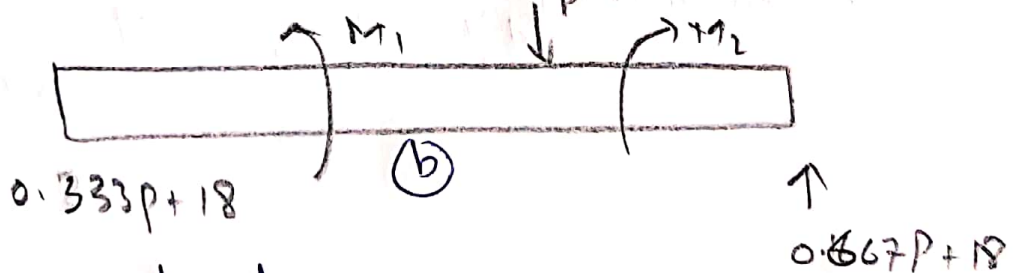
$$M_1 = (18 + 0.1667 M') x_1 - 2x_1^2$$

$$M_2 = (18 - 0.1667 M') x_2 - 2x_2^2$$



$$M_1 = (0.333p + 18) x_1 - 2x_1^2$$

$$M_2 = (0.1667p + 18) x_2 - 2x_2^2$$



The displacement function shown in the fig "a" above

$$\frac{\partial M_1}{\partial M'} = 0.1667x \quad \text{and} \quad \frac{\partial M_2}{\partial M'} = 0.1667x^2$$



⑦

Set  $M' = 0$  then

$$\rightarrow M_1 = (18 + 0.1667 C_0) x_1 - 2x_1^2$$

$$\rightarrow M_1 = (18x_1 - 2x_1^2)$$

$$\rightarrow M_2 = (18x_2 - 2x_2^2)$$

$$C\theta_B = \int_0^2 M \left( \frac{\partial M}{\partial M'} \right) \frac{dx}{EI} = \int_0^1 \frac{(18x_1 - 2x_1^2)(0.1667x_1)}{EI} dx$$

$$+ \int_0^2 \frac{(18x_2 - 2x_2^2)(0.1667x_2)}{EI} dx_2$$

$$C\theta_B = \frac{42.65}{EI} + \frac{6.66}{EI}$$

$$C\theta_B = \frac{49.31}{EI}$$

$$C\theta_B = \frac{49.31}{(200 \times 10^6)(0.00006)}$$

$$C\theta_B = 0.4411 \text{ Rad}$$

for the displacement function are shown in figure 'b'

(8)

$$\frac{\partial M_1}{\partial P} = 0.333x_1 \quad \text{and} \quad \frac{\partial M_2}{\partial P} = 0.667x_2$$

also set  $P=0$

$$\text{Then } M_1 = (18x_1 - 2x_1^2) \text{ KN}\cdot\text{m}$$

$$M_2 = (18x_2 - 2x_2^2) \text{ KN}\cdot\text{m}$$

Thus;

$$\Delta_B = \int_0^2 M \left( \frac{\partial M}{\partial P} \right) \frac{dx}{EI}$$

$$\Delta_B = \int_0^4 \frac{(30x_1 - 2x_1^2)(0.333x_1) dx}{EI}$$

$$\int_0^2 \frac{(30x_2 - 2x_2^2)(0.667x_2) dx}{EI}$$

$$\Delta_B = \frac{218.5}{EI} = \frac{218.5}{(200 \times 10^6)(0.00006)}$$

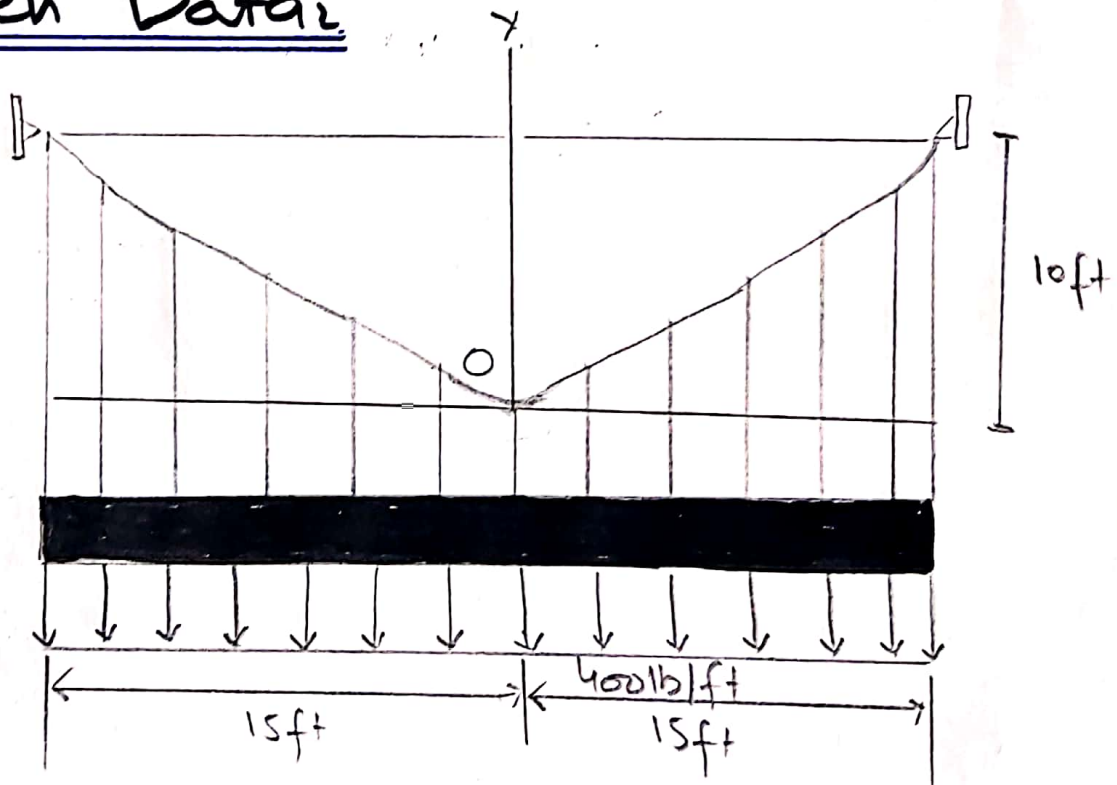
$$\Delta_B = 0.018 \text{ m or } 18 \text{ mm}$$

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## Question No 032

The cable is subjected to the uniform loading if the slope of the cable at point O is zero. Determine the equation of the curve and the force in the cable at O and B.

### Given Data:



Length of origin to top = 10 ft

Length of origin to left = 15 ft

Length of origin to right = 15 ft

Load = 400 lb/ft



(b)

## Required

equation of curve & force?

## Solution

As we know that

$$y = \frac{h}{L^2} x^2$$

$$= \frac{10}{15^2} x^2$$

$$y = 0.0444 x^2$$

Now,

$$T_0 = F_u = \frac{w_0 L^2}{2h} = \frac{400 (15)^2}{2(10)}$$

$$T_0 = 4500 \text{ lb}$$

by  $\div$  by 1000

$$T_0 = 4.5 \text{ k}$$



We know that

$$T_B > T_{max} = \sqrt{F_U^2 + (W_0 L)^2}$$

$$= \sqrt{(4500)^2 + (400 \times 15)^2}$$

$$= 7500$$

÷ by 1000

$$T_B > T_{max} = 7.5k$$

Now,

$$T_B > T_{max} = W_0 L \sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

$$= (400)(15) \sqrt{1 + \left(\frac{15}{2 \times 10}\right)^2}$$

$$= 7500 \text{ lb}$$

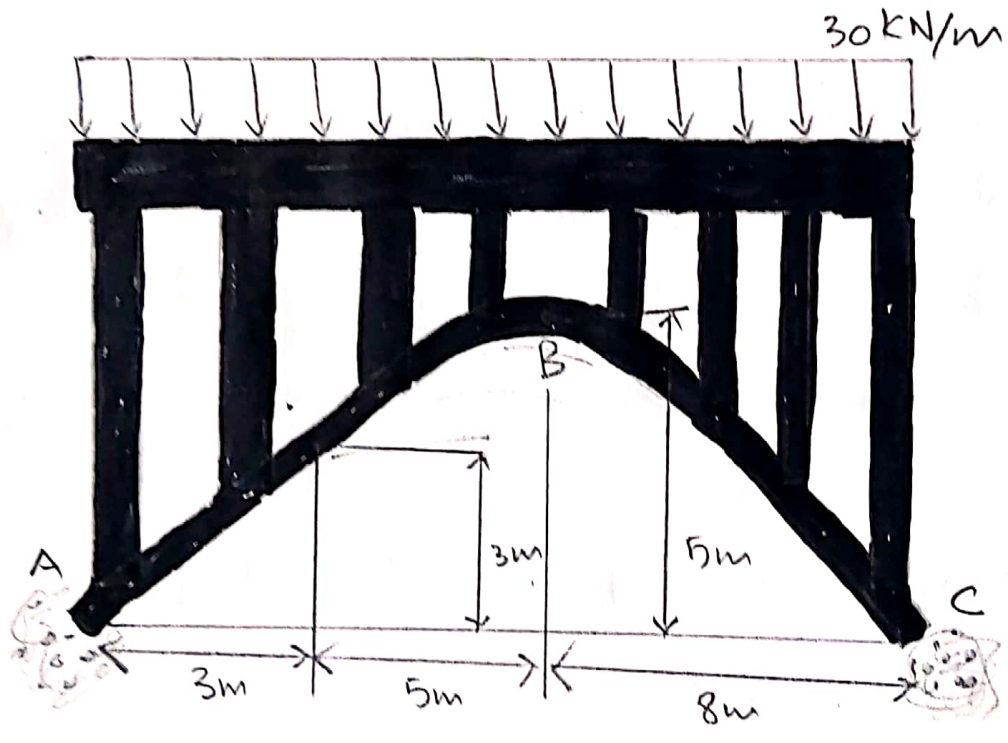
÷ by 1000

$$T_B > T_{max} = 7.500k$$

Question No 04

The Three-hinged Spandrel arch is subject to the uniform load of 30 kN/m. Determine the internal moment in the arch at Point D.

Given Data:



Required:

Internal moment at point D

(13)

## Solution 2

Member AB;

$$\sum M_A = 0$$

$$\Rightarrow B_x(5) + B_y(8) - 240(4) = 0$$

Member BC;

$$\sum M_C = 0$$

$$\Rightarrow -B_x(5) + B_y(8) + 240(4) = 0$$

Solving

$$B_x = 192 \text{ kN}, \quad B_y = 0$$

Segment BD;

$$\sum M_D = 0$$

$$= 192(2) - 150(2.5) - M_D = 0$$

$$M_D = 9 \text{ kN}\cdot\text{m} \quad \text{Ans}$$

(14)

