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DEPARTMENT BE (E)

SUBJECT DSP

Q1 (a) Determine the reason - - - - - - - - - - -

To the input $x(n) = 4^n u(n)$

ANS:- We have already determined the solution to the homogenous difference equation for this system.

$$y_h(n) = c_1(-1)^n + c_2(4)^n \quad -(2)$$

Normally we could assume a solution of the form

$$y_p(n) = K(4)^n u(n)$$

However we observe that $y_p(n)$ is already contained in the homogenous solution.

we have already treated multiple roots in the characteristic equation this we assume that

$$y_p(n) = Kn(u)^n u(n)$$

$$Kn(u)^n u(n) - 3K(n-1)(u)^{n-1} u(n-1) - 4K$$
$$(n-2)(u)^{n-2} u(n-2)$$
$$= (u)^n u(n) + 2(u)^{n-1} u(n-1)$$

satesfy to accomplished this we return to

① times which we obtained

$$y(0) = 3y(-1) + 4y(-2) + 1$$
$$y(1) = 3y(0) + 4y(-1) + 6$$
$$= 13y(-1) + 12y(-2) + 9$$

On the other hand (5) equalent at $n=0$ and $n=1$ yeilds

$$y(0) = c_1 + c_2$$
$$y(1) = -c_1 + 4c_2 + 2\ 4/5$$

(3)

Since we have already solved for the zero input response we can simply

$= y(-2) = 0$, then we have

$$c_1 + c_2 = 1$$

$$-c_1 + 4c_2 + \frac{24}{5} = 9$$

Here $c_1 = -1/25$ and $c_2 = \frac{26}{25}$.

So $y(n)$ is

$$\boxed{y_{zs}(n) = \frac{-1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6n}{5}(4)^n \quad n \geq 0}$$

Q1(b) Determine the impulse response

- - - - - - - - - - - - - -

$$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$

Solution:- The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$\lambda = 0.2, 0.4$ Hence

$$y(\lambda) = c_1 \frac{1^n}{5} + c_2 \frac{2^n}{5}$$

$x(n) = \delta(n)$ the initial condition are

$$y(0) = 1$$

$$y(1) - 0.6y(0) = 0 \Rightarrow y(1) = 0.6$$

Hence $c_1 + c_2 = 1$ and

$$\frac{1}{5}c_1 + \frac{2}{5} = 0.6 \Rightarrow c_1 = -1, \; c_2 = 3$$

Therefore $h(n) = \left[-(1/5)^n + 2(2/5)^n\right]u(n)$

step response is

$$\delta(n) = \sum^{n} h(n-k) \quad n \geq 0$$

$$= \sum_{k=d}^{n} \left[2\left(\tfrac{2}{5}\right)^{n-k} - \left(\tfrac{1}{5}\right)^{n-k} \right]$$

$$= \tfrac{1}{0.12}\left[\left(\tfrac{2}{5}^{n+1} - 1\right) - \tfrac{1}{0.16}\left[c\tfrac{1}{5}^{n+1} - 1\right] u(n)\right.$$

Q2(a) :- Determine the causual
- - - - - -
- - - - fraction method.

Sol:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$A = 4, \quad B = -3, \quad c = 1$$

$$\boxed{\text{Hence } x(n) = \left[4(2)^n - 3 - n\right] u(n)}$$

Q2(b) Determine the partial fraction - - - - - - -

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-1}}$$

Solution:- ① First we eliminate the negative powers by multiplying both numerator and denomenator by z^2.

$$X(z) = \frac{z^2}{z^2-1.5z+0.5}$$

the poles of $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

multiplying equation with denomenator

$$z = (z-0.5)A_1 + (z-1)A_2$$

Now set $z = P_1 = 1$

$$1 = (1-0.5)A_2 \text{ and } 0.5 = (0.5-1)A_2$$

$A_2 = 1$ result of expression is

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

(7)

So we have pole position

$$\frac{(z-A_K)x(z)}{z} = \frac{(z-P_K)A_1}{z-P_1} + \cdots A_{K} + \cdots$$

$$\frac{(z-P_K)A_n}{z-P_N}$$

Consequatly with $z = P_K$ yields the Kh coefficiant as

$$A_K = \left.\frac{(z-P_K)x(z)}{z}\right|_{z=P_K} \quad K = 1, 2, \cdots N$$

Q3(a) A two pole low pass filter has the system response

$$H(z) = \frac{b_0}{(1-Pz^{-1})^2}$$

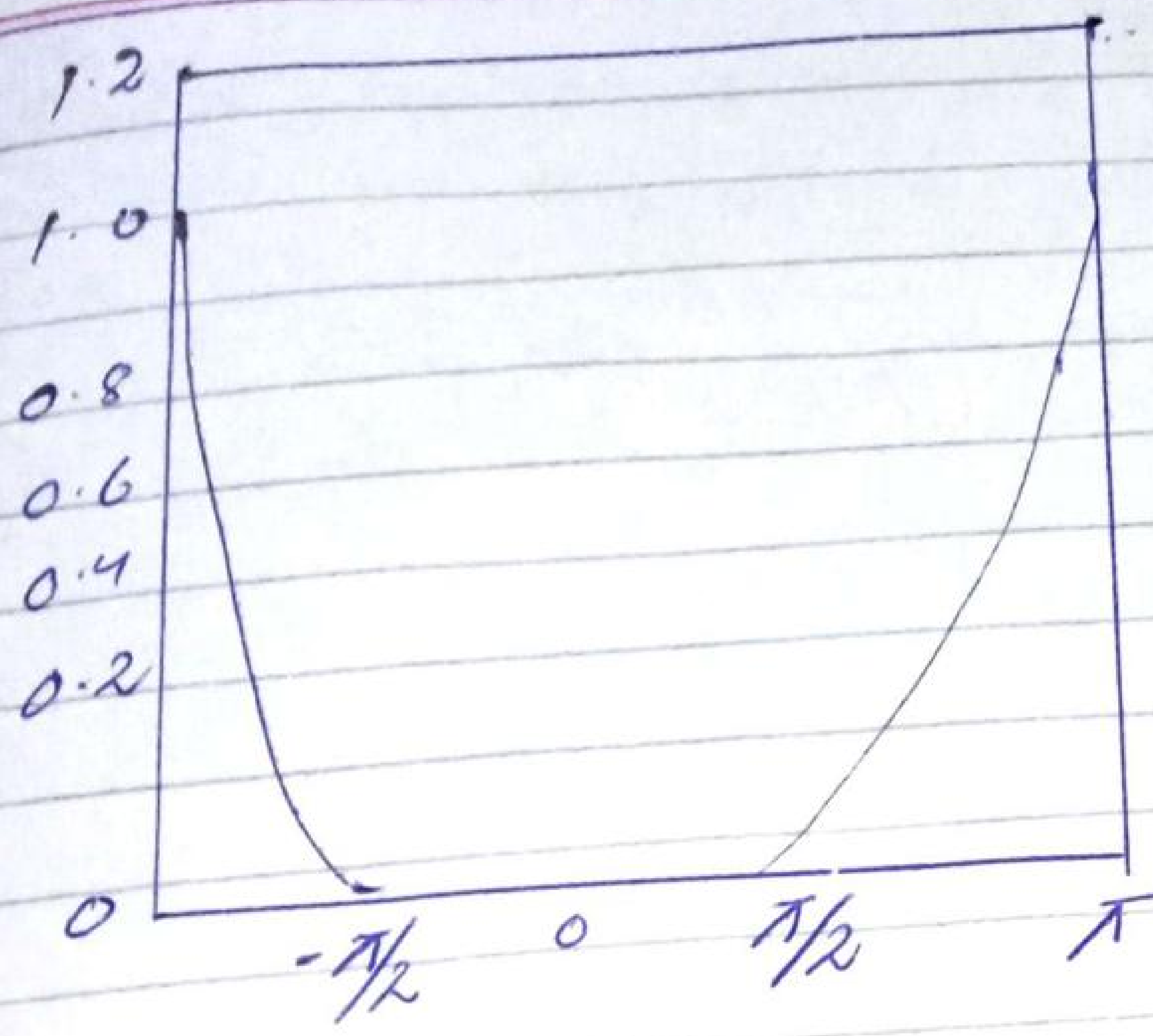
- - - -

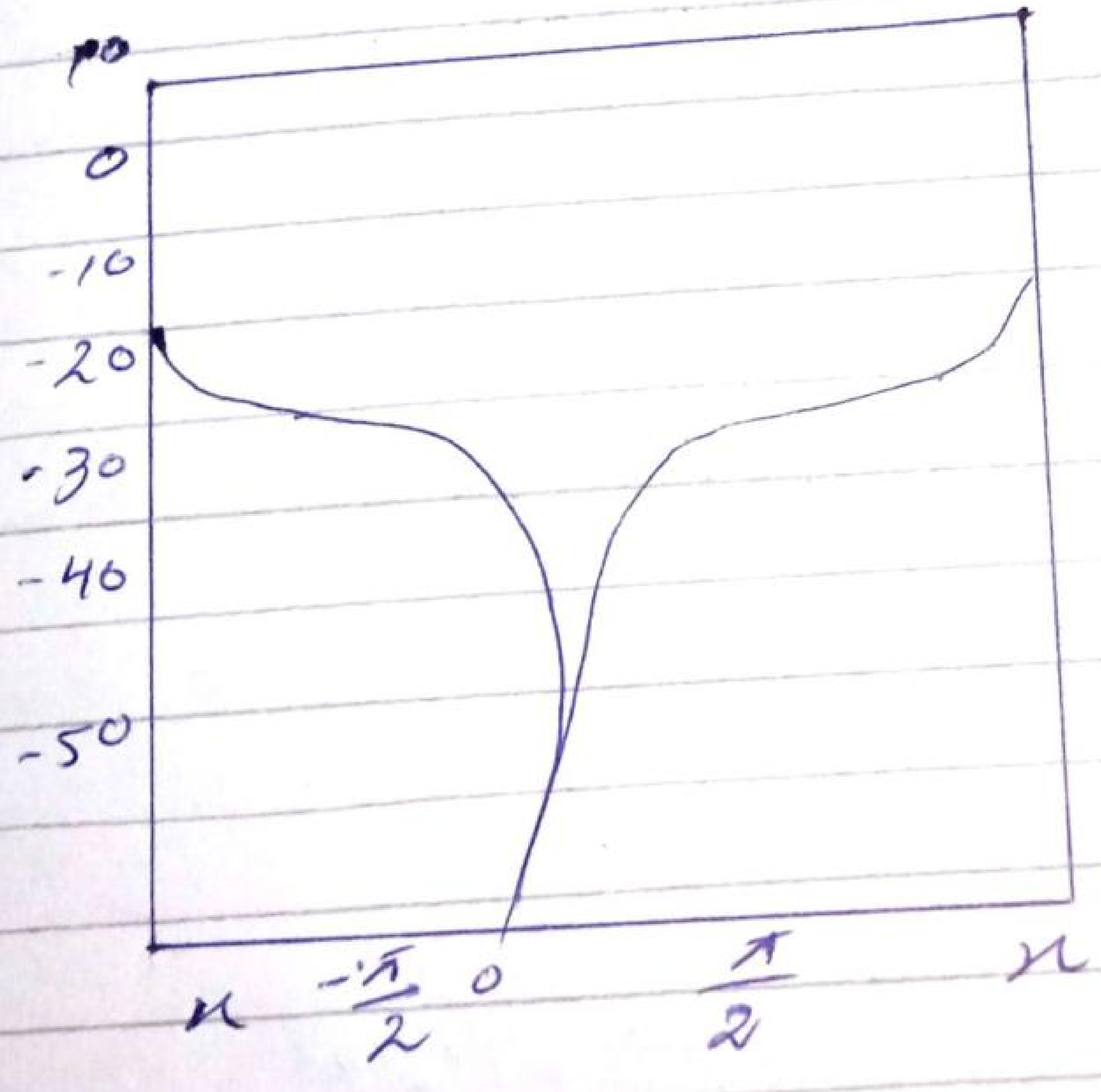
- - - $H(\pi/4)^2 = 1/2$ - - -

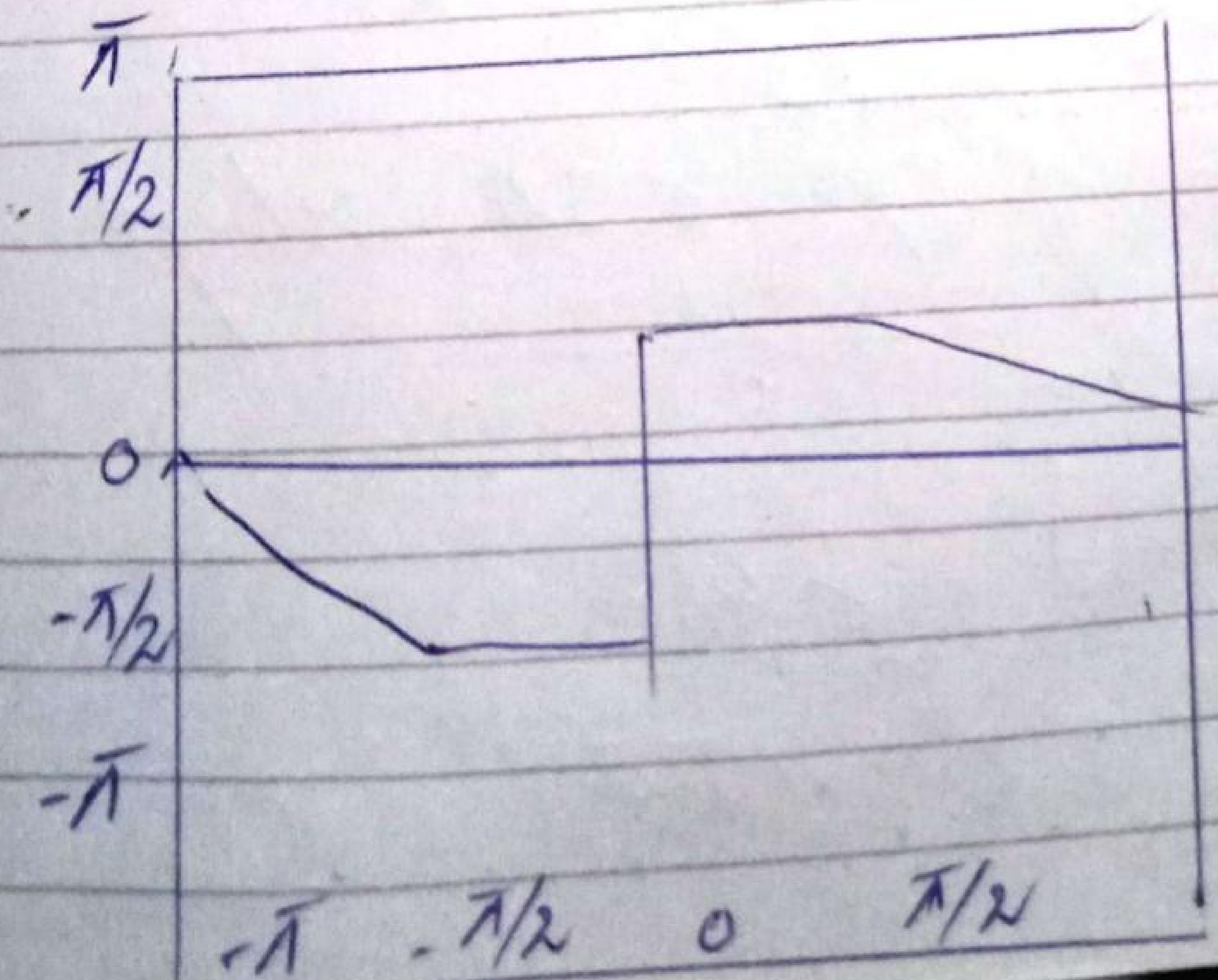
Solution:- at $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

hence $b_0 = (1-P)^2$


1.2
1.0
0.8
0.6
0.4
0.2
0
-π/2
0
π/2
π


0
-10
-20
-30
-40
-50
-π/2
0
π/2


π
π/2
0
-π/2
-π
-π
-π/2
0
π/2

(9)

at $\omega = \pi/4$

$$(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{1-P(\cos(\pi/4)+jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P\sqrt{2}+jP\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^2}{[(1-P/\sqrt{2})^2+P^2/2]^2}$$

$$= 1/2$$

or equally

$$\sqrt{2}(1-P)^2 = 1 + P^2 - \sqrt{2P}$$

the value of $p = 0.32$ satisfy this equation

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

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Q3(b) Design a two-pole band pass filter that has the of its passband at $\omega = \pi/2$ zero in its frequency response characteristic at $\omega = \pi$ and its - - - - $\omega = 4\pi/9$.

Solution:- clearly the filter must have at $P_{12} = re^{\pm j\pi/2}$ and zeroes at $z = 1$ and $z = -1$ consiquantly the system function is

$$H(z) = \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

the value of r is determine by evaluating $|H(\omega)|$ at $\omega = 4\pi/9$

Thus we have

(11)

$$\left|H\left(\frac{4\pi}{2}\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/2)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= \frac{1}{2}$$

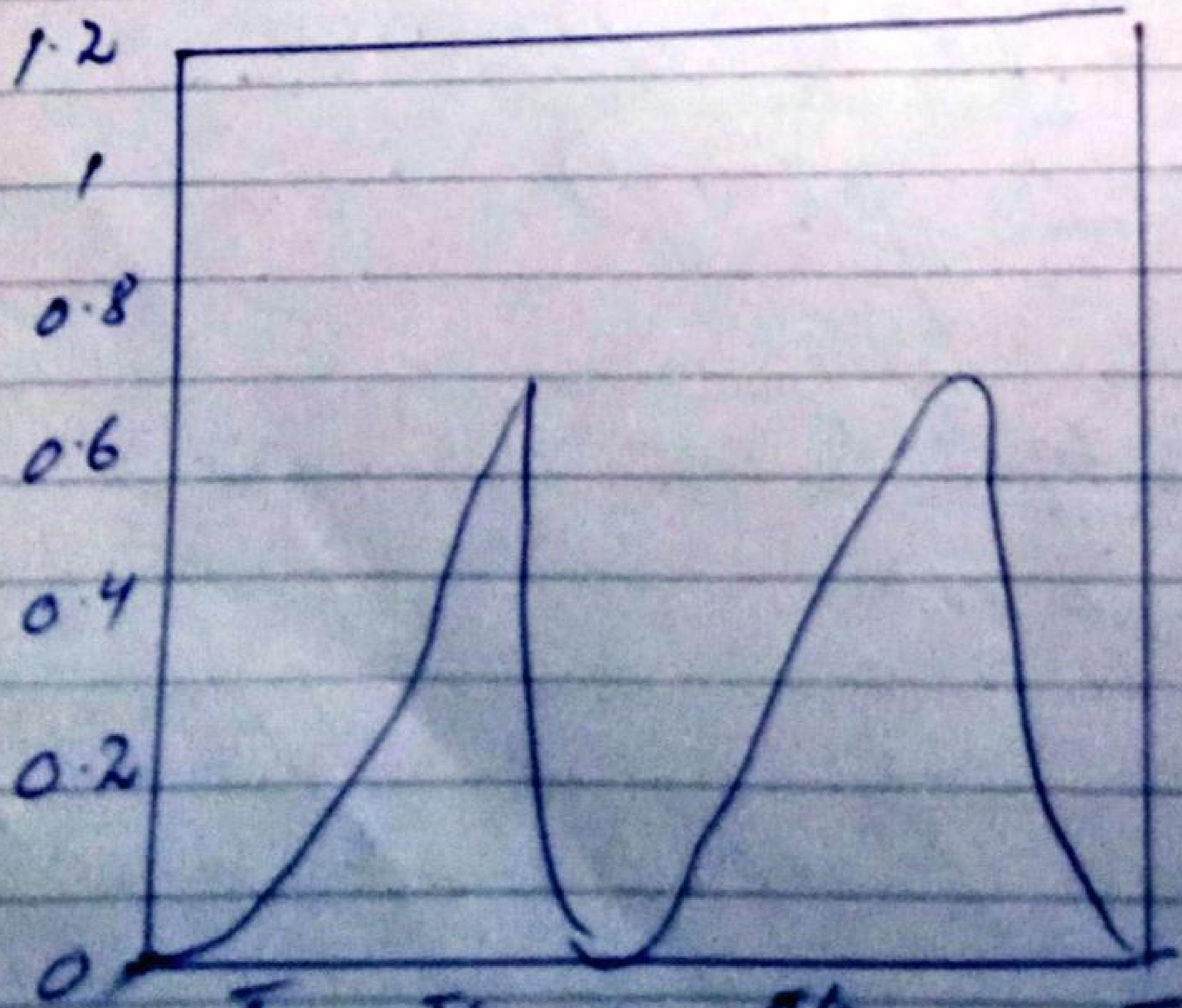
or consequently

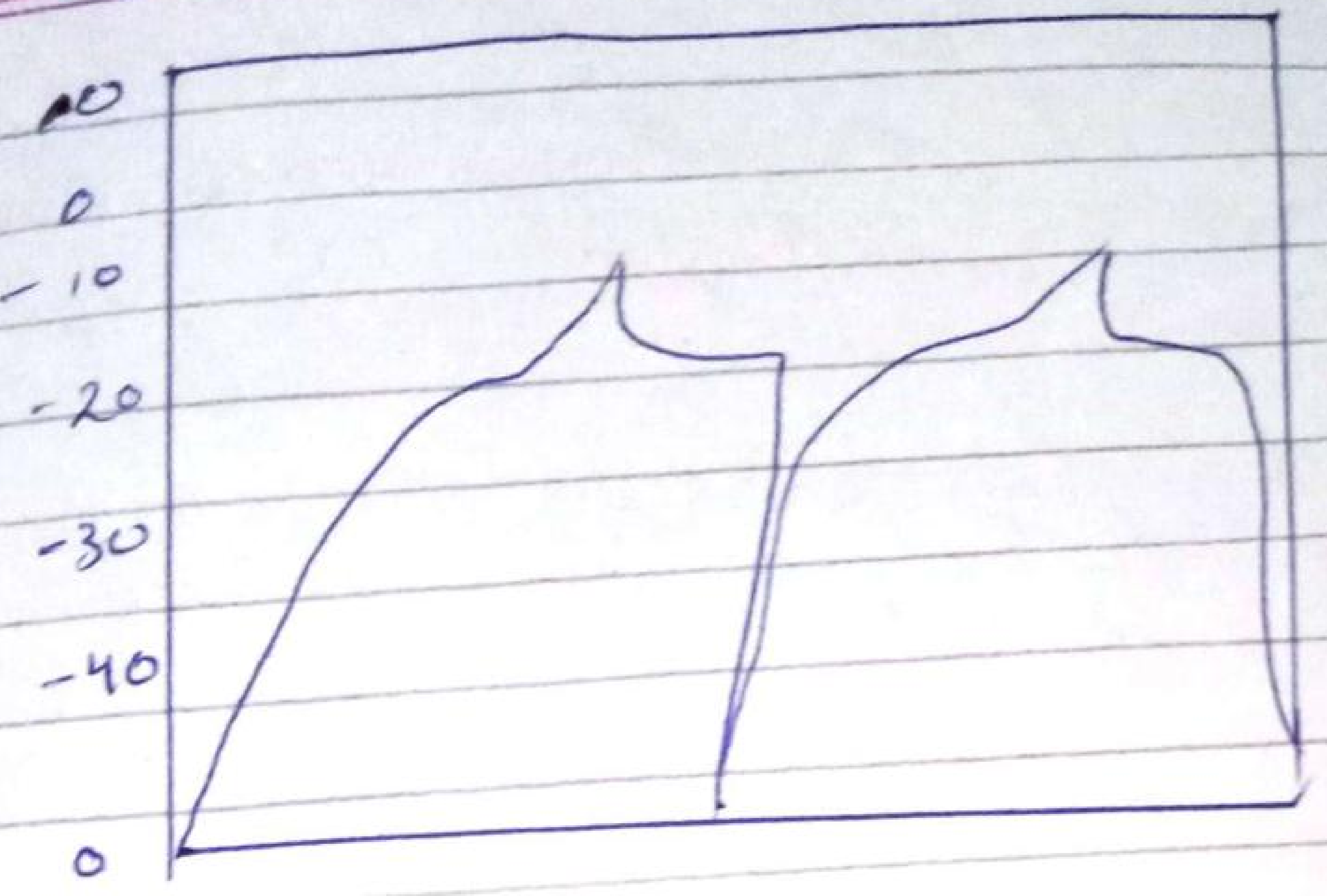
$$1.94*(1-r^2)2 = 1-1.88r^2+r4$$

the value of $r^2 = 0.7$ satisfy the equation

So the system function for the desired filter is

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$





Q4(a):- A finite duration

" " " " for $N \geq L$

Solution:- $X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega L}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega L - n/2}$$

the magnitude of $x(\omega)$ and phase illustrated in fig $L=10$

the N DFT of $x(n)$ is simply

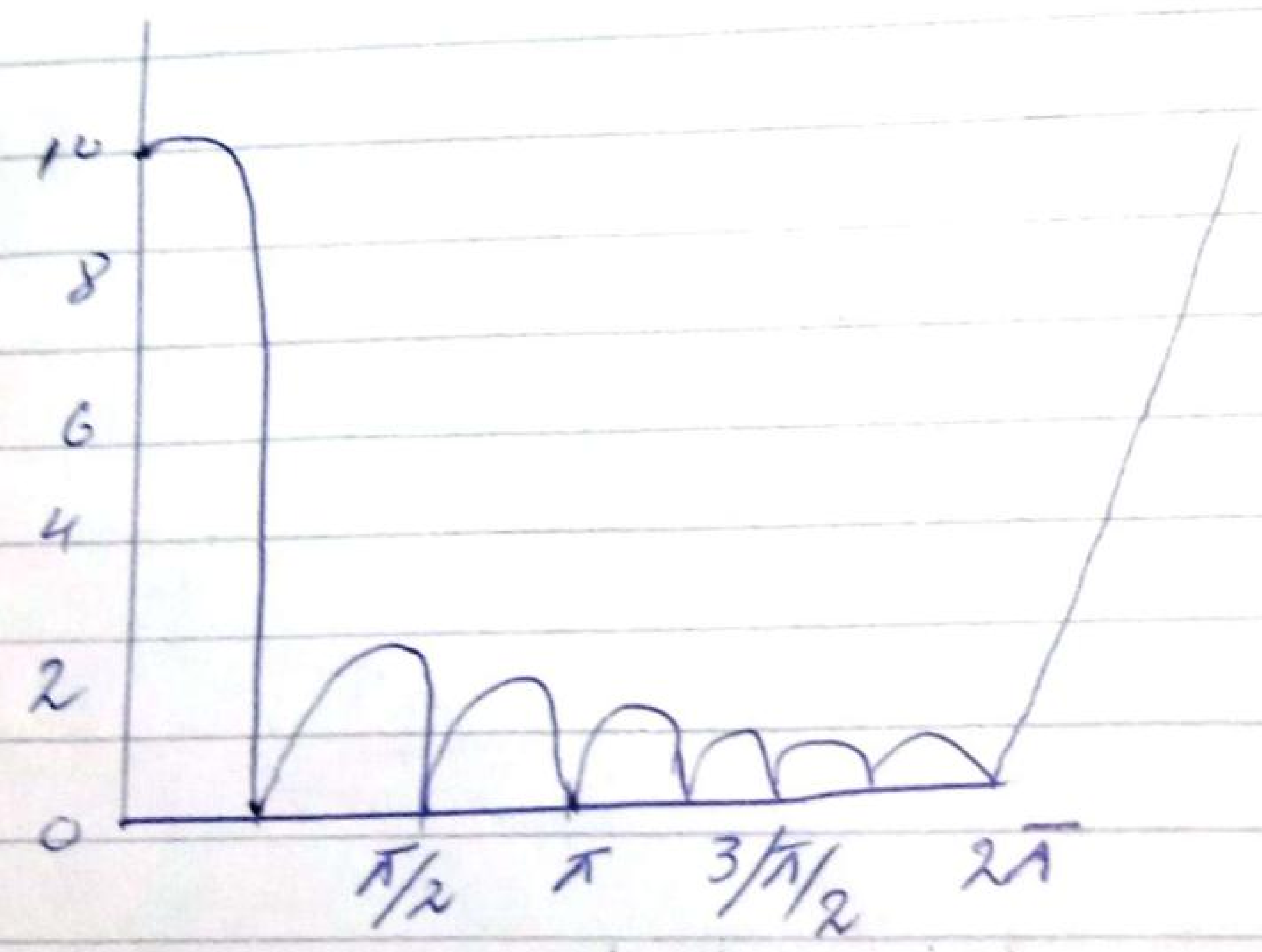
(13)

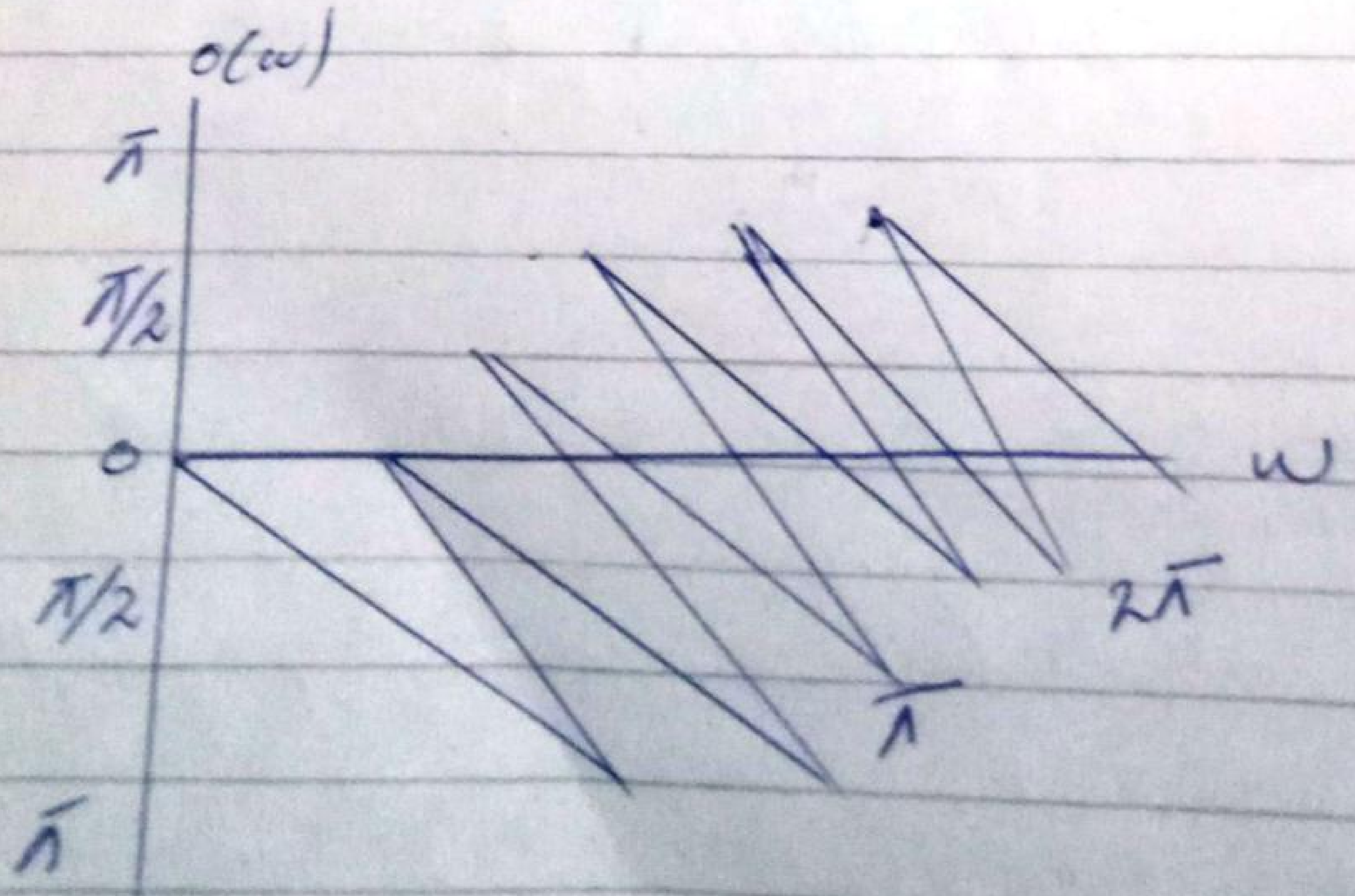
$x(\omega)$ evaluated at the set of N equally spaced frequency $\omega_k = 2\pi k/N$, $k = 0, 1, \cdots N-1$

Hence

$$x(k) = \frac{1-e^{-j2\pi k L/N}}{1-e^{-j2\pi k/N}} \quad k = 0, 1 \cdots N$$

$$= \frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{-j\pi k/L - N/\omega}$$





Q4(b):- Compute the DFT --

$x(n) = (0, 1, 2, 3)$

Solution: $W_N^{K+N/2} = W_N^K$

the matrix W_N may be expressed as

$$W_N = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -5 & -1 & 5 \\ 1 & -1 & 1 & -1 \\ 1 & 5 & -1 & -5 \end{bmatrix}$$

then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ 2+23 \\ -2 \\ 2-29 \end{bmatrix}$$

15

The IDFT of W_4 may be determined by conjugte the element in W_4 to obtained W_4.