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SUBJECT "FLUID MECHANICS II"

SUBMITED TO "ENG-ABDUL WAHEED"

"FINAL PAPER"

Que :- 1a.

Define drag . . . .
. . . . .
layer?

Forced on Immersed bodies :-

A body which is wholy immersed in a homogenous fluid may be subjected to 2 kind of forces araising from relative motion b/w body and fluid. These forces are termed as drag and lift. Depending on forces either parallel or rigid angle to motion.

Drag force on submerged body can have 2 components.

* **Pressure drag :-**

It is equal to the Integration of component in direction of motion of all pressure force exerted on surface of body.

$$F_P = C_P \cdot \rho \cdot \frac{V^2}{2} \cdot A$$

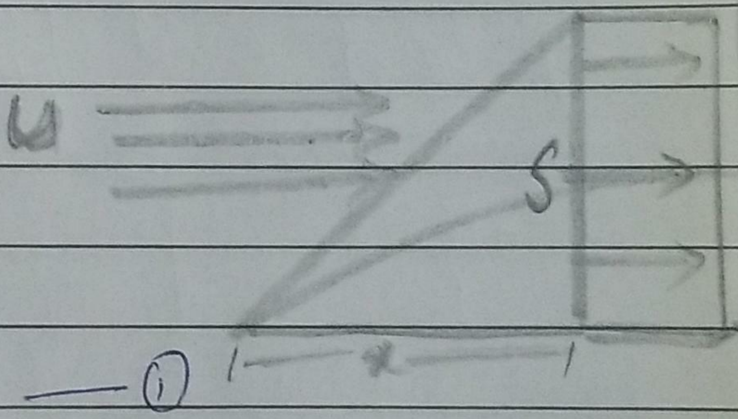
where "C_P" depend on shape.

* Friction Drag:-

It is equal to integration of component at all shear stress ~~only~~ along the Surface in direction of motion.

$Fp = Ct \cdot \int \frac{v^2}{2} (BL)$ "ct" depends on velocity

» Friction drag of bondry layer:-



$\tau_0 = \mu \frac{u\,df(n)}{\delta\, d(n)}$

$\bar{\tau}_0 = \frac{\mu\, UB}{\delta}$ —— (1)

As we know that $\tau_0 = \int U^2 \cancel{x} \frac{ds}{dx}$

compare both

$\int U^2 \cancel{x} \frac{ds}{dx} = \frac{\mu UB}{\delta}$

$\delta d\delta = \frac{\mu B}{\delta U_{\infty}} dx$ ∴ C=0

Integration on both sides

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} dx + C$$

$\therefore C = 0$

$$\delta = \sqrt{\frac{2B}{d}} \sqrt{\frac{\mu x}{\rho u}}$$

$B = 1.62$, $d = 0.135$

$$\delta = \frac{4.91}{\sqrt{R_n}} x$$

where "R_n" is local Reynold number

As we have

$$F_x = \rho B U^2 \delta d$$

where d is a function at boundry layer velocity distribution

Now to find shear stress

$$\tau = \frac{f_x}{A} = \frac{d f_x}{B dx} = \frac{d f_x}{B dx}$$

$$\tau_0 = \rho B U^2 \alpha \frac{d\delta}{B\,dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

$$\tau_0 = \frac{\rho u^2 \alpha\, d\delta}{dx} \longrightarrow \text{general equation}$$

Laminar boundry layer:

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \text{ —— eq (1)}$$

$$\frac{y}{\delta} = \eta \Rightarrow y = \delta\eta$$

$$dy = \delta\, d\eta \longrightarrow \text{eq (2)}$$

$$\frac{u}{U} = f(\eta)$$

$$du = U\,df(\eta) \longrightarrow \text{eq (3)}$$

for Laminar flow

$$\tau_0 = \mu \frac{du}{dy} \text{ —— eq (4)}$$

$$\tau_0 = -\mu . u \frac{df(n)}{\delta\, d(n)}$$

$$\tau_0 = \frac{\mu U B}{\delta} \longrightarrow eq\ (5)$$

As we have $\tau_0 = \rho v^2 \alpha \frac{d\delta}{dx}$

Comparing both

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta d\delta = \frac{\mu B}{\rho U \alpha} dx$$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x + C$$

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$$\delta = \frac{4.91}{\sqrt{R_n}} \longrightarrow eq\ (6)$$

Now eq (6) and (5)

$$\boxed{\tau_0 = 0.332\ \mu U/x \sqrt{R_n}}$$

Now

$$\boxed{F_f = B \cdot \int \tau_0\, dx}$$

where $\tau_0 = 0.332 \dfrac{\mu u}{x} \sqrt{R_n}$

$$R_n = \frac{x\, u f}{\mu}$$

~~then~~ putting the value

$$\boxed{F_f = 0.664 \sqrt{\rho \mu L U^3}}$$

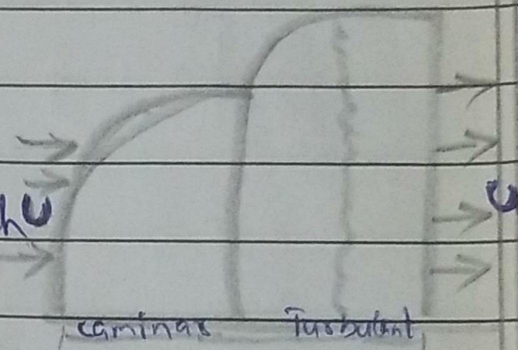
As we have

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

$$C_f = 1.328 \sqrt{\frac{U}{\rho L U}} = \frac{1.328}{R_n}$$

For Laminar $R < 500,000$.

Turbulant Boundry Layer:-



In this fig show the velocity distribution of boundry layer which is steeper mear walls and flatter through out reminder of layer.

The shear stress is greater in turbulant then in Luminer. thus;

$$\tau_0 = \rho f \frac{v^2}{8}$$

Where v is the average velocity to obtain relation between average and maximum

As we have

$$\frac{v}{U_{max}} = \frac{1}{1+1.33\sqrt{f}} \qquad f = 0.028$$

Pg (8)

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{0.028}}$$

$$U = 1.235\ V$$

$$\boxed{V = \frac{U}{1.235}}$$

Now

$$f = \frac{0.316}{(Rh)^{3/4}}$$

$$\tau_0 = f \rho \frac{V^2}{8}$$

$$\tau_0 = \frac{0.316}{D/\nu \left(\frac{U}{1.235}\right)^{1/4}}$$

$$\tau_0 = \frac{0.023 \rho v^2}{\left(\frac{2S}{\nu}\right)^{1/4}} \longrightarrow eq\ 1$$

As we have general equation

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \quad \text{—— (2)}$$

Eq (1) & eq (2)

$x = 0 \quad , \delta = 0$

$$\boxed{\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} \cdot x}$$

$\alpha = 0.0972$

$$\boxed{\delta = \frac{0.377}{(R_n)^{1/5}} \cdot x} \quad \text{—— eq (3)}$$

$$\tau_0 = 0.0587 \rho \frac{v^2}{2} \left(\frac{v}{Ux}\right)^{1/5}$$

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

$$\boxed{F_f = 0.0735 \rho \frac{v^2}{2} \left(\frac{v}{Ux}\right)^{1/5} \quad \text{B.L}}$$

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot B\alpha$$

$$C_f = \frac{0.0735}{(R)^{1/5}} \quad \because 500{,}000 < R < 10^7$$

For

$R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.52}}}$$

Que :- 1 b

Derive equation . . . .
. . . . Channel?

Answer :-

As
Specific energy
$$E = y + \frac{V^2}{2g}$$

The flow Q per unit width b can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

$$V = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y}$$

Thus

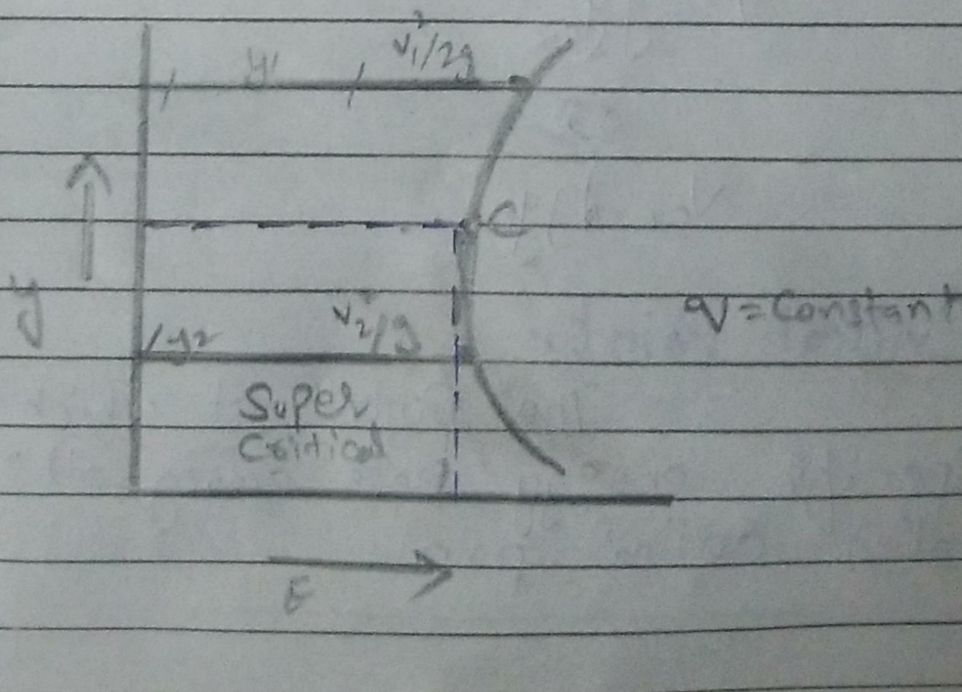
$$E = y + \frac{V^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E - y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

OR

$$(E-y)\,y^2 = \frac{q^2}{2g}$$

Thus plot of E vs Y will be parabolic. For particular q, there will be two kind of possible values of y for given E.
The equation is cubic with three roots with 3rd root being negative point C represents dividing point between two regime of flow. Thus for given q & value of E is minimum and flow at that point is critical flow. Depth of flow at that point is critical depth y_c and velocity at that point is critical velocity V_c



~~Thus~~ Thus

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

for minimum specific energy $\frac{dE}{dy} = 0$

Thus

$$\frac{dE}{dy} = 1 - \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^3}\right) = 0$$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow$$

$$\boxed{y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}}$$

Now $q^2 = gy^3$

&

$$q = vy \Rightarrow v^2 y^2 = gy^3$$

or

$$v^2 = g y_{cr}$$

or

$$V_{cr} = \sqrt{g y_{cr}}$$

Critical Point:

The point at which specific energy least among all is called critical point.

Que :- 2:-

Find depth . . . . . .
. . . . critical?

Given data:-

Depth of rectangular channel(d) = ?

Flowrate (Q) = 3.5m/sec

Slop of bed = 0.0008

$n = 0.0219$

Width of Bed = 7666 mm

" $\Rightarrow \frac{7666}{1000} \Rightarrow 7.666$

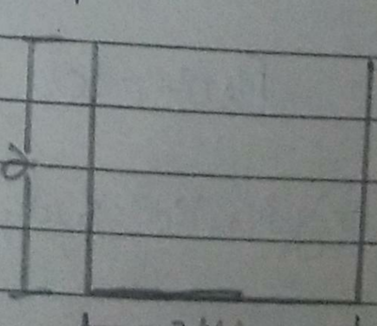
Critical depth = ?

Flow sub critical or super critical = ?

Sol:-

Area = $7.666 \times d$

$7.666d$



Perimeter = $d + 7.666 + d$

$7.666 + 2d$

Hydraulic Radius (R_h) = A/P

$$\Rightarrow \frac{7.666}{7.666+2d}$$

By using Manning Equation

$$Q = \frac{1}{n} A \cdot R_h^{2/3} \cdot S_0^{1/2}$$

Putting values

$$3.5 = \frac{1}{0.0219} \times 7.666d \times \left(\frac{7.666d}{2d+7.666}\right)^{2/3} \times (0.008)^{1/2}$$

$$\boxed{d = 0.5662}$$

Area $= 7.666(0.5662)$

$$\boxed{\text{Area} = 4.3404\,m^2}$$

Perimeter $= 7.666 + 2(0.5662)$

$7.666 + 1.1324$

$$\boxed{\text{Perimeter} = 8.7984}$$

Hydraulic radius $= \frac{4.3404}{8.7984}$

$$\boxed{\text{Hydraulic radius} = 0.4933\,m}$$

Find critical depth:-

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = Q/B$

$$= \frac{3.5}{7.666}$$

$\Rightarrow 0.456\ m^2/sec$

$$y_{cr} = \left(\frac{(0.456)^2}{9.81}\right)^{1/3}$$

$$y_{cr} = \left(\frac{0.207}{9.81}\right)^{1/3}$$

$$\boxed{y_{cr} \Rightarrow 0.2763}$$

As $y > y_{cr}$

$0.5662 > 0.2763$

So flow is Sub-critical

Que 3:-

Find . . . . 0.93×10⁻⁴ m²/s

Given data:-

width (B) = 200 mm ⇒ 0.2 m
Length (L) = 800 ⇒ 0.8 m
Specific gravity = 0.87
Undisturbed velocity (U) = 5 m/sec
Kinemetic velocity = 0.93×10^{-4} m²/sec

Req:-

Friction Drag (FD) = ?

Sol:-

Checking whether flow is laminar or not by reynold Number.

$$R = \frac{DV}{\nu}$$

For Smooth flat plate

$D = L$, $V = U$

So

$$R = \frac{LU}{\nu}$$

Putting Values

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

43010 < 500,000 → laminar

By using formula

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

where

$$C_f = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{43010}} \Rightarrow 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} = 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$
$$\rho_{oil} = 890 \ kg/m^3$$

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

Putting values

$$= 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39 \ N}$$