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Summer Paper Final

Linear Algebra

Q2 :-

$L(x,y) = (x+1), y, x+y)$ illustrate that L is linear transformation.

Sol

$L(x,y) = \cancel{(x+1)}\ (x+1, y, x+y)$

let $u = (x_1, y_1)$ $v = (x_2, y_3)$

$U+v = (x_1, y_1) + (x_2, y_2)$

$U+v = (x_1 + x_2, y_1 + y_2)$

$L(U+v) = L(x_1 + x_2, y_1 + y_2)$

$L(u+v) = (x_1 + x_2 + 1, y, x_1 + x_2 + y_1 + y_2)$

given that $u = (x+y)$

$L(u) = L(x_1, y_1) = (x_2 + 1, y_i, x_2 + y_2)$

$L(u) = L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$

$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

Since $1 \neq 2$

Assignment 1

Q^1 :- (b)

Explain a pair of plain whose intersect is the given line.

$\therefore x = 2 - 3t \ , \ y = 3 + t \quad z = 2 - 4t.$

Sol

$$\frac{x-2}{-3} = t \ , \ \frac{y-3}{1} = t \ , \ \frac{z-2}{4} = t$$

$$\frac{x-2}{3} = \frac{y-3}{1} = \frac{z-2}{4}$$

taking find and Second terms.

$$\frac{x-2}{3} = \frac{y-3}{1}$$

$x - 2 = -3y + 9$

$x - 2 = -3y + 9$

$x - 2 + 3y - 9 = 0$

$\boxed{x + 3y - 11 = 0}$

Taking 1^{st} & 3^{rd} term

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$-4x + 8 = -3z + 6$

$-4x + 8 + 3z - 6 = 0$

$\boxed{-4x + 3z + 2 = 0}$

Question 1)
part a:

Express the equation of Plane passing through the points A (2, -2, 1), B (-1, 0, 3), C (5, 3, 4)

Solution

Then non parallel vectors

$\overline{P_1P_2} = (-3, 2, 3)$ $(-1, 0, 3) - (2, -2, 1)$

$P_2 P_3 = (3, -1, 3)$ $(-1, 0, 3) - 2, 2, -1$

$(-3+2, 2)$

The perpendicular vector is

$m = \overleftrightarrow{P_1P_2} \times \overrightarrow{P_1 P_3}$

$$n = \begin{vmatrix} i & j & K \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$n = i(6+2) - j(-9-6) + k(3-6$

$n = 8i + 15j - 3k$

$m = (2, 15, -3)$

Now

$P_1 (x_0, y_0, z_0) = (2, -3, 1)$

$m(a, b, c) = 8, 15, -3$

So equation of plane is

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

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R (4)

$8(x-2)+15(y+2)-3(z-1)=0$

$8x+15y-3z-16+30+3=0$

$\boxed{8x+15-3z+17=0}$

Question 4:

find an equation of the passing through the point $(-1,3,2)$ and perpendicular to the vector $n=(0,1,-3)$.

Solution

$(-1,3,2)$ $n=(0,1,-3)$

~~Sol~~. equation of plane

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Given that

$p=(x_0,y_0,z_0)=(-1,3,2)$

$n=(a,b,c)=(0,1,-3)$

So

$0(x-(-1)+1(y-3)-3(z-2)$

$0(x+1)+1(y-3)-3(z-2)$

$0+y-3-3z+6$

$y-3z-3+6$

$\boxed{y-3z+3}$ Ans

Question 5:-

find an Eigen value vectors of matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

Solution

We know that

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = \lambda x_1 - ①$$
$$-2x_1 + 4x_2 = \lambda x_2 - -$$

So

$$x_1 - \lambda x_1 + x_2 = 0$$

$$= (1-\lambda)x_1 + x_2 = 0$$

&

$$-2x_1 + 4x_2 - \lambda x_2 = 0$$
$$-2x_1 + (4-\lambda)x_2 = 0$$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$= -2x_1 + 4x_2 = 2x_2 - ②$$

$$= -2x_1 + 2x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$x_1 = r$ then $x_2 = r$

So

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$$

$\Rightarrow 2x_1 - x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x^2 - (ii)$

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = \frac{1}{2} x_2$

let $x^2 = r$

where $r \neq 0$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} r \\ r \end{bmatrix}$

eigen vector for $\lambda_2 = 2$ put.

$x_1 + x_2 = 2x_1 - (i)$

$-2x_1 + 4x_2 = 2x_2 - (ii)$

$= -x_1 + x_2 = 0$

$= x_1 - x_2 = 0$

$= x_1 = x_2$

$= -2x_1 + 4x_2 = 2x_2 - (ii)$

$= -2x_1 + 2x_2 = 0$

$x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

$x_1 = r$ then $x_2 = r$

$$\boxed{\text{So} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}}$$

Question 3a

Solutn.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

So

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ -2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 & +1 & -1 \\ 0 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0+54-38 \\ 154+108-54 \\ -77+55+38 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 18 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

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