$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\2 & 8 & -1\\-2 & 0 & 0\\1 & -4 & -2\end{bmatrix} R_2 - 2R_1 \text{ and } R_4 - R_1$$

$$R_2 = 2 \quad 8 \quad -1$$
$$-2R_1 = -2 \quad -12 \quad -16$$
$$R_2 + (-2R_1) = 0 \quad -4 \quad -17$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\0 & -4 & -17\\-2 & 0 & 0\\0 & -10 & -10\end{bmatrix} R_4 \div \text{ing by } -10$$

and

$$R_4 = 1 \quad -4 \quad -2$$
$$-R_1 = -1 \quad -6 \quad -8$$
$$R_4 + R_1 = 0 \quad -10 \quad -10$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\0 & -4 & -17\\-2 & 0 & 0\\0 & 1 & 1\end{bmatrix} R_3 + 2R_1$$

$$R_3 = -2 \quad 0 \quad 0$$
$$2R_1 = 2 \quad 12 \quad 16$$
$$R_3 + 2R_1 = 0 \quad 12 \quad 16$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\0 & -4 & -17\\0 & 12 & 16\\0 & 1 & 1\end{bmatrix} R_3 \longleftrightarrow R_4$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\0 & -4 & -17\\0 & 1 & 1\\0 & 12 & 16\end{bmatrix} R_4 - 12R_3$$

$$R_4 = 0 \quad 12 \quad 16$$
$$-12R_3 = 0 \quad -12 \quad -12$$
$$R_4 + (-12R_3) = 0 \quad 0 \quad 4$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\0 & -4 & -17\\0 & 1 & 1\\0 & 0 & 4\end{bmatrix} R_4 \div \text{ing by } 4$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 6 & 8\\0 & -4 & -17\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$ which is the Required echelon Form.

Q3 part (b)

Find an echelon form for the below matrix using row operations. where ID2 is 2nd digit in your ID, ID3 is 3rd digit in your ID and ID Firs and Last is the first and last digit in your ID.

Given

$$\begin{pmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -D3 & 0 & 0 \\ 1 & -4 & IDFirs\_Last \end{pmatrix}$$

~~1 6~~ 8

ID2 = 6, -ID3 = -2, ID First = 1 and ID Last = 3.

$$\Rightarrow \begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -2 & 0 & 0 \\ 1 & -4 & 1-3 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -2 & 0 & 0 \\ 1 & -4 & -2 \end{bmatrix}$$

Use elementary row opration To Reduce the given matrix on to echelon form

p.t.o

Q No 3 Part (A)

(1) Row Echelon form:

A matrix is said to be row echelon form has zero below each leading 1. For example the following matrix is an row echelon form.

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$ example

(2) Reduced Echelon Forms

Reduced row echelon form has zero below and above each leading 1.

• Example: The following matrix is the Example of Reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) $\begin{pmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{pmatrix}$ Reduce row echelon form.

The given matrix is in reduced row echelon, because the leading (Pivots) has zeros above and below. So the given matrix is in reduced row echelon form.

(d) $\begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}$ Reduced row echelon form.

The given matrix in reduced row echelon form, because the leading elements (Pivots) of the given matrix has zeros above and below. So due to definition of a reduced row echelon form the given matrix is a reduced echelon form.

are, e, π, −π and e and below it has zeros. We can reduced it into Reduced echelon form by using elementry row operation.

$$\underset{\sim}{R}\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

R_1 ÷ing by e.
R_2 ÷ing by π
R_3 ÷ing by −π
R_4 ÷ing by e.

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is requered reduce row echelon form.

(b)
$$\begin{pmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

It is in echelon form because the leading elements (pivots) has zeros below. So the given matrix is an echelon form.

Now by reverse Row operation as

$$\underset{\sim}{R} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} R_3 + 2R_2$$

$R_3 = 0\ 0\ 3\ -5$
$2R_2 = 0\ 2\ -8\ 4$

$R_3 + 2R_2 = 0\ 2\ -5\ -1$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$ Hence the required 1st matrix.

Q No 2 Part (B) Below given are the (a) some matrics. Find which one is the row echelon form and which is reduced row echelon form. Explain in your own words for each of the section in details.

(a) $$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$ is an echelon form.

Ans

Yes the above matrix is in echelon form because, The leading element (Pivot)

Q No 2 Par (A)

Find the elementry raw operation that transfom the first matrin to second matrin and reverse row operation that trans form the second matrin to first matrin.

Salution

1st matrin = $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$

Second matrin = $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

~~To make~~ To transfom matin ist into second - we use elementry row operation on matrin fist.

$\underset{\sim}{R} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} R_3 - 2R_2$

$R_3 = 0 \quad 2 \quad -5 \quad -1$

$-2R_2 = 0 \quad -2 \quad 8 \quad -4$

$R_3 + (-2R_2) = 0 \quad 0 \quad 3 \quad -5$

$\underset{\sim}{R} = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

which is the requered Second matrin

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$$\underset{\sim}{R}\left[\begin{array}{cccc|c}1&2&3&0&5\\0&1&0&0&-11\\0&0&1&0&-6\\0&0&0&1&2\end{array}\right] R_1-2R_2$$

$$\Rightarrow \begin{array}{l} R_1 \Rightarrow 1\ 2\ 3\ 0 \mid 5 \\ -2R_2 = 0\ -2\ 0\ 0 \mid 22 \\ \hline R_1+(-2R_2) = 1\ 0\ 3\ 0 \mid 27 \end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{cccc|c}1&0&3&0&27\\0&1&0&0&-11\\0&0&1&0&-6\\0&0&0&1&2\end{array}\right] R_1-3R_3$$

$$\begin{array}{l} R_1 = 1\ 0\ 3\ 0 \mid 27 \\ -3R_3 = 0\ 0\ -3\ 0 \mid 18 \\ \hline R_1+(-3R_3) = 1\ 0\ 0\ 0 \mid 45 \end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{cccc|c}1&0&0&0&45\\0&1&0&0&-11\\0&0&1&0&-6\\0&0&0&1&2\end{array}\right] -$$

Solution ~~we~~ let x_1, x_2, x_3 and x_4 are the unknown variables then from the above Reduce echelon form the unknown variables as

use forward substitution.

$x_1 = 45$, $x_2 = -11$, $x_3 = -6$ and $x_4 = 2$

Page No 1

Student Name: M. Naeem

Student ID: 16213

Sbybject : Linear Algebra

Instructor : Masoor Qadir

Q NO 1

$$\begin{pmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID,last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{pmatrix}$$

ID3 = 2, -ID Last = -3.

$$\begin{pmatrix} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & -3 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

The given augumented matrix as.

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & -3 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

The next Raw posible opration are.

$$R \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & -3 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] R_2+3R_3$$

$$\begin{array}{rcccc|c} R_2 = & 0 & 1 & -3 & 0 & 7 \\ 3R_3 = & 0 & 0 & 3 & 0 & -18 \\ \hline R_2+3R_3 = & 0 & 1 & 0 & 0 & -11 \end{array}$$

P.T.O