Iqra National University, Peshawar
Department of Electrical Engineering

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Assignment
Date:20/4/2020

Course Code:	MTH 102			Course Title:	Calculus and analytic geometry
Prerequisite:				Instructor:	HIMAYATULLAH
Module:	3	Program:	BEE	Total Marks:	30

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Q1.	(a)	Identify $\lim_{h \to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$	Marks 5 CLO1 C1
Q2	(a)	A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 - 2$ have nay horizontal tangent if so where ?	Marks 10 CLO1 C1

Q1:->
a=>
Solution:-

Given that:

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

Now

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

now apply limit

$$= \frac{\sqrt{2+0}-\sqrt{2}}{0}$$

$$= \frac{\sqrt{2}-\sqrt{2}}{0}$$

$$= \frac{0}{0}$$

So,

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \times \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

multipaly and divide $\sqrt{2+h}+\sqrt{2}$

$$\Rightarrow \lim_{h\to 0} \frac{(\sqrt{2+h}-\sqrt{2})(\sqrt{2+h}+\sqrt{2})}{(h)(\sqrt{2+h}+\sqrt{2})}$$

$$\Rightarrow \frac{(\sqrt{2+h})^{\cancel{2}}-(\sqrt{2})^{\cancel{2}}}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\Rightarrow \frac{2+h-2}{h(\sqrt{2+h}+\sqrt{2})}$$

$$= \lim_{h\to 0} \frac{h+2-2}{h(\sqrt{2+h}+\sqrt{2})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

now apply limit.

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{4}} \Rightarrow \frac{1}{2\sqrt{2}}$$

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \boxed{\frac{1}{2\sqrt{2}}}$$ Answer

Q1:→
b:→
Solution:→
Given that

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

(3)

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

$$\frac{dy}{dx} = (x + x^{-1})\frac{d}{dx}(x - x^{-1} + 1) + (x - x^{-1} + 1)\frac{d}{dx}(x + x^{-1})$$

$$= (x + x^{-1})(1 + x^{-2}) + (x - x^{-1} + 1)(1 - x^{-2})$$

$$= \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x} + 1\right)\left(1 - \frac{1}{x^2}\right)$$

$$\Rightarrow x + x\frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^3} + x - x\frac{1}{x^2} - \frac{1}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^2}$$

$$\Rightarrow 2x + 1 - \frac{1}{x^2} + \frac{1}{x^3}$$

So

$$\left[\frac{dy}{dx} = 2x + \frac{1}{x^3} - \frac{1}{x^2} + 1\right] \text{Ans}$$

Q2 =>

a =>

Solution:->

Given that:-

$$S = 160t - 16t^2 \text{ ft}$$

i) $S = 160t - 16t^2$ ft

velocity is

$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$= \frac{d}{dt}(160t) - \frac{d}{dt}(16t^2)$$

$$v = 160 - 32t$$

maximum hight,

$$v = 0$$

Now

$$160 - 32t = 0$$
$$160 = 32t$$
$$\frac{160}{32} = t$$

$$t = 5\ sec$$

$$S_{max} = S(5) = 160(5) - 16(5)^2$$
$$S_{max} = 400 ft$$

$$S_{max} = 400 ft$$

ii)

Given that

$$S = 256 ft$$

Now

$$160t - 16t^2 = 256$$
$$160t - 16t^2 - 256 = 0$$

$$16(10t - t^2 - 16) = 0$$

÷ing both sides by $\boxed{16}$

$$\frac{\cancel{16}}{\cancel{16}}(10t - t^2 - 16) = \frac{0}{16}$$

$t^2 - 8t - 2t + 16 = 0$

$t(t-8) - 2(t-8) = 0$

$(t-8)(t-2) = 0$

$t - 8 = 0 \quad , \quad t - 2 = 0$

$t = 8 \quad , \quad t = 2$

$t_1 = 8sec \quad , \quad t_2 = 2\ sec$

Now

$v = 160 - 32t$

$t_1 = 8sec$

$V(t_1) = 160 - 32t$

$V(8) = 160 - 32(8)$

$V_{(8)} = 160 - 256$

$V t_1 = -96\ m/s$

$\boxed{V(t_1) = -96 m/s \quad \text{downward}}$

$t_2 = 2sec$

$V t_2 = 160 - 32t$

$V(2) = 160 - 32(2)$

$V_{(2)} = 160 - 64$

$V_2 = 96\ m/s$

$\boxed{v(t_2) = 96\ m/s \quad \text{upward}}$

iii) **As we know that:** $v = 160 - 32t$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$

$a = 0 - 32$

$\boxed{a = -32\ m/s^2}$

Scanned with CamScanner

Q3:->

a ->

Solution:-

Given that:-

$$y = x^4 - 2x^2 + 2$$

$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$= \frac{d}{dx}(x^4) - \frac{d}{dx}(2x^2) + \frac{d}{dx}(2)$$

$$= 4x^3 \cancel{\frac{dx}{dx}} - 2 \times 2x \cancel{\frac{dx}{dx}} + 0$$

$$= 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x + 0$$

Now,

if the tangent line is horizontal then

$$\frac{dy}{dx} = 0$$

There for,

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0, \quad x^2 - 1 = 0$$

$$x = \frac{0}{4}, \quad x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = 0, \quad x = \sqrt{1}$$

$$\boxed{x = 0}, \quad \boxed{x = \pm 1}$$

So,

$$x = 0, 1, -1$$

and the Corresponding points are;

For, $x = 0$.

$$y = x^4 - 2x^2 + 2$$

$$y = (0)^4 - 2(0)^2 + 2 \quad \text{putting } (x = 0)$$

$$y = 0 - 0 + 2$$

$$\boxed{y = 2}$$

For,

$$x = 1$$

$$y = x^4 - 2x^2 + 2$$

$$y = (1)^4 - 2(1)^2 + 2 \quad \text{putting } (x = 1)$$

$$y = 1 - 2 + 2$$

$$\boxed{y = 1}$$

For,

$$x = -1$$

$$y = x^4 - 2(x)^2 + 2$$

$$y = (-1)^4 - 2(-1)^2 + 2$$

$$y = 1 - 2 + 2$$

$$\boxed{y = 1}$$

Hence, $(0, 2)$, $(1, 1)$, $(-1, +1)$