Date. 12290

Calculas & Analytical Geometry

Final Term Paper

Student Name => Aamir Saleem
Student ID => 12290
Instructor => Muhammad Abrar khan
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Q.1 Differentiate (Part a)

$$\frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

Solution:-

$$\frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

$$= \frac{d}{dx}\left(\frac{3x^4 - 2x^3 + 5}{x^3 + 1}\right)$$

$$= \frac{x^3 + 1\left(d/dx\left(3x^4 + 2x^3 + 5\right)\right) - \left(3x^4 - 2x^3 + 5\right)\left(d/dx\left(x^3 + 1\right)\right)}{\left(x^3 + 1\right)^2}$$

$$= \frac{\left(x^3 + 1\right)\left(12x^3 - 6x^2\right) - \left(3x^4 - 2x^3 + 5\right)\left(3x^2\right)}{\left(x^3 + 1\right)^2}$$

Date: 12290

Simplify $\dfrac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$

$$= \frac{3x^6+12x^3-21x^2}{(x^3+1)^2}$$

$$= \frac{3x^2(x^4+4x-7)}{(x^3+1)^2}$$

$$\Rightarrow \frac{3x^2(x^4+4x-7)}{(x^3+1)^2}$$ Answer.

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Date: 12290

Q:- 5 part (a)

Solution:-

Finding $A^2 + BC$

$$A^2 = A \cdot A \Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 4\times2 & 1\times4 + 4\times1 \\ 2\times1 + 1\times2 & 2\times4 + 1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

Now

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3\times1 + 2\times0 & -3\times0 + 2\times2 \\ 4\times1 + 0\times0 & 4\times0 + 0\times2 \end{bmatrix}$$

Date: 12290

$$BC = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 \\ 4 & 4 \end{bmatrix}.$$

Now

$A^2 + BC$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 9+(-3) & 8+4 \\ 4+4 & 9+4 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 6 & 12 \\ 8 & 13 \end{bmatrix}$$ Answer

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Date: 12290

Q.1 part (b)

Differentiate

$$\frac{(x^3+1)^2}{x^3-1}$$

Solution:-

$$\frac{d}{dx}\left(\frac{(x^3+1)^2}{x^3-1}\right)$$

$$= \frac{2x(x^2-x+1)(2x^2-2x-1)}{(x-1)^2}$$

By Quotient Rule

$$= \frac{\frac{d}{dx}\left((x^3+1)^2\right)(x^2-1) - \frac{d}{dx}(x^2-1)(x^3+1)^2}{(x^2-1)^2}$$

$$= \frac{d}{dx}\left((x^3+1)^2\right) = 6x^2(x^3+1)$$

$$= \frac{d}{dx}(x^2-1) = 2x$$

Date: 12290

$$= \frac{6x^2(x^3+1)(x^2-1) - 2x(x^3+1)^2}{(x^2-1)^2}$$

Simplifying:-

$$= \frac{6x^2(x^3+1)(x^2-1) - 2x(x^3+1)^2}{(x^2-1)^2}$$

$$\Rightarrow \frac{2x(x^2-x+1)(2x^2-2x-1)}{(x-1)^2}$$ Answer.

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Date:

Q.3 part (a)

Find the integration

$\int \frac{-x+9}{2x^2-8x+6}\,dx$ Partial fractions.

Solution:-

$$2x^2-8x+6$$

$$2x^2-6x-2x+6$$

$$2x(x-3)-2(x-3)$$

$$(x-3)(2x-2)$$

let

$$\frac{-x+9}{(x-3)(2x-2)} = \frac{A}{(x-3)} + \frac{B}{(2x-2)}$$

multiply by $(x-3)(2x-2)$ on both sides

$$-x+9 = A(2x-2) + B(x-3)$$

Putting $x=3$

Date: 12290

$$-3+9 = A(2(3)-2) + B(3-3)$$

$$6 = A(4) + B(0)$$

$$6 = A(4) + B(0)$$

$$A = \frac{\cancel{4}2}{\cancel{6}3} \Rightarrow A = 2/3$$

Now putting $2x-2=0$

$$x = 1$$

$$-1+9 = A(2(1)-2) + B(1-3)$$

$$8 = A(2-2) + B(-2)$$

$$8 = A(0) + B(-2)$$

$$B = \frac{-\cancel{2}1}{\cancel{8}4} \Rightarrow B = \frac{-1}{4}$$

Thus

$$\int \frac{-x+9}{(x-3)(2x-3)}\,dx = \int \frac{2/3}{x-3} + \frac{-1/4}{(2x-2)}\,dx$$

Date: 12290

$$= \frac{2}{3}\int \frac{1}{(x-3)}\,dx - \frac{1}{4}\int \frac{1}{(2x-2)}\,dx$$

$$= \frac{2}{3}\ln(x-3) - \frac{1}{4}\ln(2x-2) + C$$

Answer

Date: 12290

Q.2 part (a)

Find the integrals

$$\int \frac{1}{\sqrt{x^5}}\,dx$$

Solution:-

$$\int \frac{1}{\sqrt{x^5}}\,dx = -\frac{2}{3x^{3/2}} + C$$

$\sqrt{x^5} = x^{5/2}$, assuming $x \geq 0$

Applying exponent rule.

$$\frac{1}{x^{5/2}} = x^{-5/2}$$

$$= \int x^{-5/2}\,dx$$

Apply power rule

$$= \frac{x^{-5/2+1}}{-5/2+1}$$

Date

Simply

$$= \frac{x^{-5/2+1}}{-5/2+1}$$

$$= -\frac{2}{3x^{3/2}}$$

$$= \frac{-2}{3x^{3/2}}$$ Answer

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12290 Date:

Q.2 part (b)

Find the integrals

$$\int \frac{1}{(8x+7)^8} dx$$

Solution:-

$$\int \frac{1}{(8x+7)^8} dx$$

Apply u- substitution $u = 8x+7$

$$= \int \frac{1}{8u^8} du \Rightarrow \frac{1}{8} \cdot \int \frac{1}{u^8} du$$ (Taking constant out)

Apply exponent rule

$$\frac{1}{u^8} = u^{-8}$$

Applying power rule

$$= \frac{1}{8} \cdot \frac{u^{-8+1}}{-8+1}$$

Date:

Substitute back $u = 8x+7$

$$= \frac{1}{8} \cdot \frac{(8x+7)^{-8+1}}{-8+1}$$

Simplifying

$$\frac{1}{8} \cdot \frac{(8x+7)^{-8+1}}{-8+1} : -\frac{1}{56(8x+7)^7}$$

$$= -\frac{1}{56(8x+7)^7} + C$$ Answer

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Date:

Q. No. 4 Part (a)

Solution:-

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5-3 & , & 1-(-1) \\ -3-2 & , & 1-2 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$$ Answer

PART (b) Solution

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 8 \\ -2 & 0 \end{bmatrix}$$

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+(-4) & 6+8 \\ 1+(-2) & 5+0 \end{bmatrix}$$

$$n+\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix}$$

$$\cancel{n=\begin{bmatrix} -2+1 \\ -1 & -0 \end{bmatrix}}$$

$$n=\begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5\ 2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

Answer

Part (C)

Solution

$$X+2I=\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$x=\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} . 2I$$

$$X=\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Date:

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2\times1 & 2\times0 \\ 2\times0 & 2\times1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3-2) & (-1-0) \\ (1-0) & (2-2) \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

Answer

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Date.

Q.3 part (b)

Find the integration of

$\int \frac{4x^2+8x}{(x^{2+1})(x^2+2x+3)}\,dx$ by partial fractions.

Solution:-

Partial fraction for each factors

$$\therefore \frac{4x^2+8x}{x^3(x^2+2x+3)} = \frac{A}{x} + \frac{B}{(x)^2} + \frac{C}{(x)^3} + \frac{Dx\ E}{x^2+2x+3}$$

→ Multiplying through by the common denominator $x^3(x^2+2x+3)$

$$\therefore 4x^2+8x = A\times\left(x^2\left(x^2+2x+3\right)\right)+B\times\left(x\left(x^2+2x+3\right)\right)+C\times\left(x^2+2x+3\right)$$
$$+(Dx+E)\times(x^3)$$

$$\therefore 4x^2+8x = A\times(x^4+2x^3+3x^2)+B\times(x^3+2x^2+3x)+C\times(x^2+2x+3)+(Dx+E)\times(x^3)$$

$$\therefore 4x^2+8x = Ax^4+2Ax^3+3Ax^2+Bx^3+2Bx^2+3Bx+Cx^2+2Cx+3C+Dx^4+$$
$$+Ex^3$$

Grouping the x-terms & constant terms.

$$\therefore 4x^2+8x = (A+D)x^4+(2A+B+E)x^3+(3A+2B+C)x^2+(3B+2C)x+3C$$

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Co-efficient of the two polynomials must be equal, so we get equation

$$A + D = 0$$

$$2A + B + E = 0$$

$$3A + 2B + C = 4$$

$$3B + 2C = 8$$

$$3C = 0$$

Answer

α