Question No #01
ANSWER

Given data :-

distance = 1/2"
E = 29000 Ksi
I = 150 in⁴
δ = deflection due to 7684 lb

Required data :-

natural time period = ?
equation of motion = ?
amplitude = ?
graph of vibration of displacement with time = ?
graph of vibration of equivalent static force with time = ?

Solution :-

As we know that from general equation of motion for single degree of freedom system

Now we also know that from general solution to the Eom for undamped free vibration

$u(T) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$

$u(0) = \frac{1}{2}'' = \frac{1}{24}'$

$\dot{u}(0) = 0$

$u(T) = (\frac{1}{24}) \times \cos(90.1T)$

Equivalant static force at any time "T" is

$$F_s(T) = K \cdot u(T) = \frac{90625 \times \cos(54.97)}{24}$$

$$= 3776.042 \cos 54.97$$

$$Ku + c\dot{u} + m\ddot{u} = P(T)$$

here

In our Case system is undamped

$c = 0$ so under ~~damped~~ going free

vibration $P(T) = 0$

Hence general EOM became

$$Ku + m\ddot{u} = 0 \longrightarrow (1)$$

Now we know that

$$K = \frac{3EI}{L^3}$$

Put value

$$= \frac{3 \times 2900 \times 150}{(10 \times 12)^3}$$

$$\boxed{= 7.55 \ K/IN}$$

$$\boxed{= 906.25 \ lb/FT}$$

Now we mass (m)

$$m = F/a = 7684/32.2 = \boxed{238.64 \ slug}$$

Now we find w_n

$$w_n = \sqrt{K/m} = \sqrt{\frac{96025}{238.64}}$$

$$\boxed{= 20.1 \ rad/sec}$$

Now put value in equation,

$$Ku + m\ddot{u} = 0$$

$$90625u + 31.064 = 0$$

Now find time period

$$T_n = 2\pi/w_n = 2\pi/20.1 \Rightarrow \boxed{0.313 \ sec}$$

Now we find aplitude of dynamic displacement u_0 for undamped free vibration

$$u_0 = \sqrt{(u(0))^2 + (\dot{u}(0)/\omega_n)^2}$$

$$= \sqrt{(1/24)^2 + 0} = \sqrt{(1/24)^2}$$

$$= 1/24 = 0.042 \text{ ft}$$

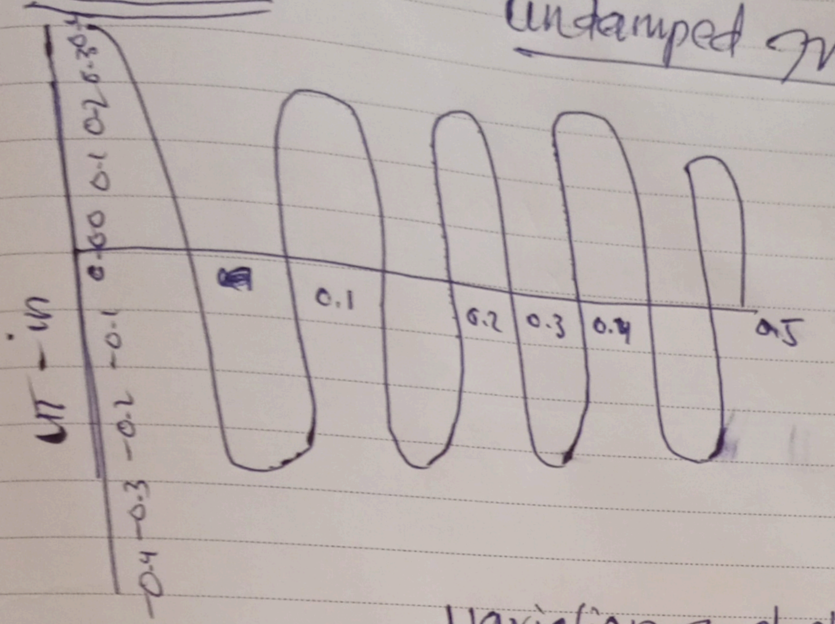
Now amplitude of equivalant static force

$$ku = 90625 \times \frac{1}{24}$$

$$\boxed{= 3776.042 \text{ lb}}$$

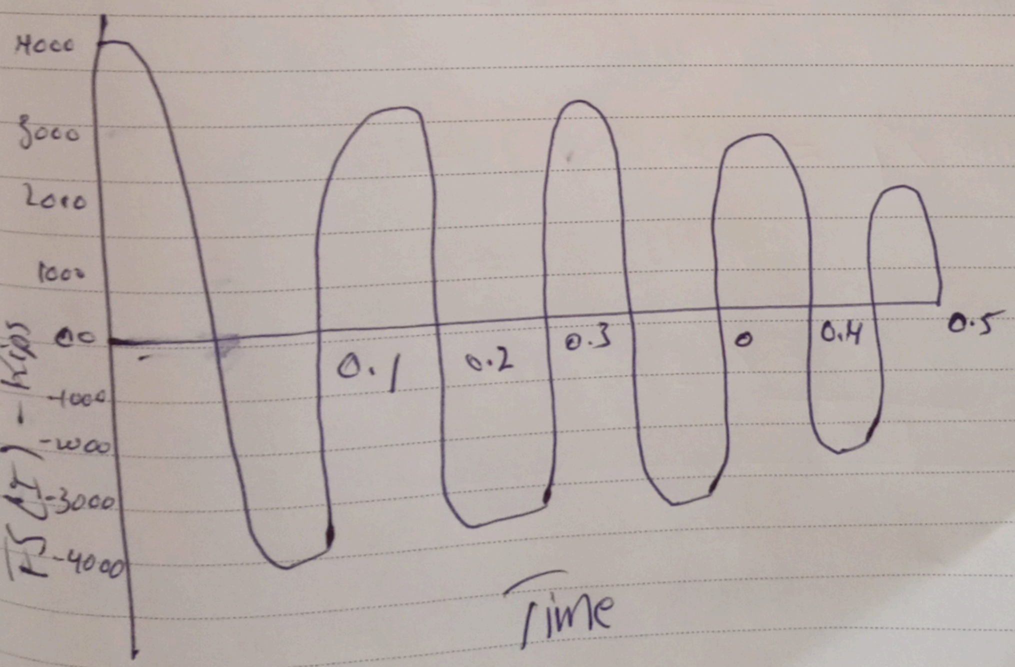
Q.1 Graph

Undamped free vibration



Variation of displacement with time

Undamped free vibration



Q.2 ANSWER

Given data :-

distance = 1/2"

$E = 29000$ ksi

$I = 150$ in^2

$\delta = 7684$ lb

$L = 10$ ft

Damping ratio = $\delta = 4\%$

Required data :-

1. develop and solve the equation of motion for vibration at free end = ?

2. also develop equation showing vibration in the equivalent of static force with time = ?

3. draw graph vibration of displacement with time and the variation of equivalent static force with time = ?

SOLUTION :-

For part 1 and 2

Now we know that from equation of motion for damped vibration

$$Ku + C\dot{u} + m\ddot{u} = 0 \rightarrow (i)$$

we know that

$$K = 3EI/L^3 = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$= 7.55 \text{ K/in}$$

$$= 90625 \text{ lb/Ft}$$

Now mass

$$m = F/a = \frac{7684}{32.2} = 238.64 \text{ slug}$$

Now we find C.

we know that

$$C = \xi \times 2 m\omega_n \rightarrow (ii)$$

$$= \frac{4}{100} \times 2 \times 238.64 \times \omega_n$$

First we find ω_n

$$w_n = \sqrt{K/m} = \sqrt{96025/238.64}$$

$$= 20.1 \text{ rad/sec}$$

Now put in eq (11)

$$C = \frac{4}{100} \times 2 \times 238.64 \times 20.1$$

$$\boxed{= 383.74 \text{ lb.sec/ft}}$$

Now put these value in eq (1)

$$Ku + C\dot{u} + M\ddot{u} = 0$$

$$96025\,u + 383.744 + 238.644\,\ddot{u} = 0$$

Now solution to the EOM for damped free vibration

$$u(t) = e^{-\xi w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left(\dot{u}(0) + u(0)\sin t \; \sin w_D t\right)\right]$$

$$w_D = 20.1 \text{ rad/sec}$$

$$w(t) = e^{-0.04 \times 20.1 t}\left[\frac{1}{24} \times \cos(20.1 t) + \frac{1}{20.1}\left(0 + \frac{1}{24} + 0.04 \times 20.1 \times \sin(20.1)\right)\right]$$

$e^{-0.804t}\left(0.042\cos(20.1t) + 0.0017\sin(20.1t)\right)$

$= e^{-0.804}\left\{0.042\cos(20.1t) + 0.0017\sin(20.1t)\right\}$

Now we know that

$F(s) = K\,u(t)$

$K = 90625$

$F_s = 90625\left(e^{-0.804} \times 0.042\cos(20.1t) + 0.0017\sin(20.1t)\right)$

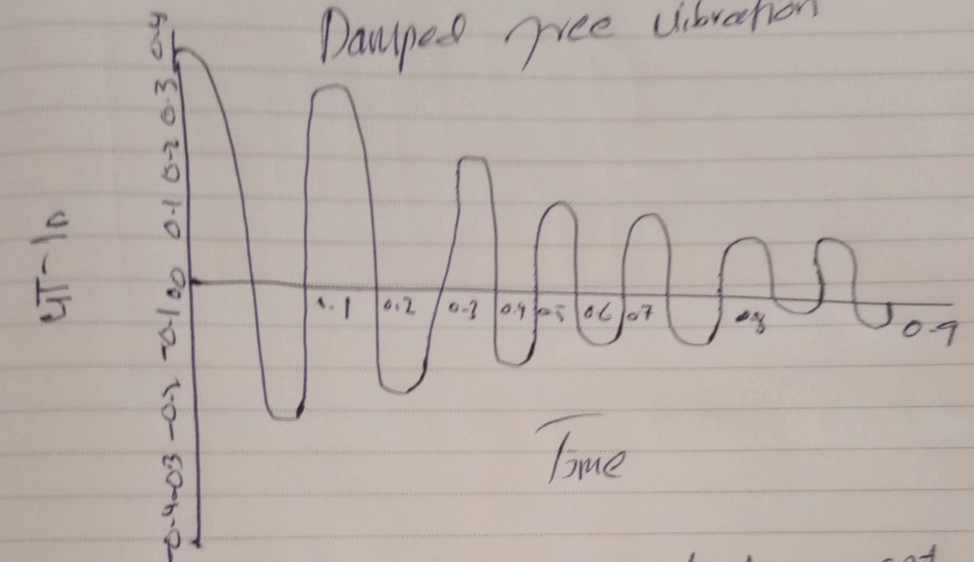
$= e^{-0.804}\left(90625 \times 0.042\cos(20.1t) + 90625 \times 0.0017\sin(20.1t)\right)$

$= e^{-804}\left(3806.25\cos(20.1t) + 154.0625\sin(20.1t)\right)$

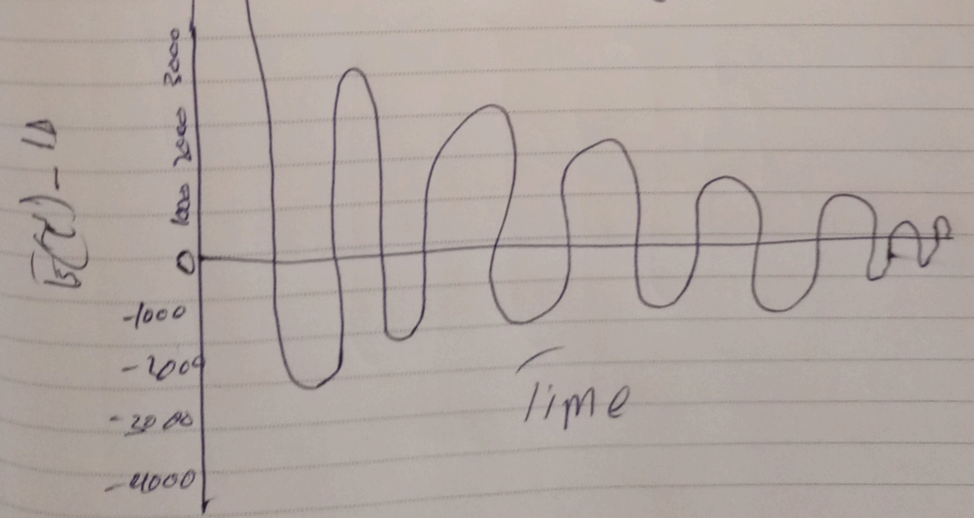
$$\boxed{e^{-804}\left(3806.25\cos(20.1t) + 154.0625\sin(20.1t)\right)}$$

Q.2 Graph

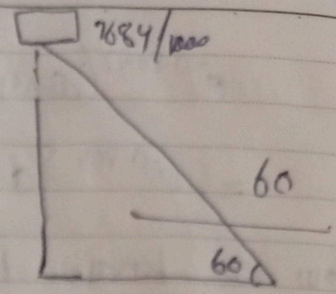
Damped free vibration



variation of displacement with time



QUESTION NO # 03
ANSWER



Given data :-

load = F = 60 kips
displace the tank by the force = $7684/1000$

= 7.684"

Time period = 3.57 sec

cycle = 7

amplitude of displacement = 2.286 cm

$= \frac{2.286}{2.54}$

= 0.9"

Required data :-

① Damping ratio = ?
② natural period of undamped variation = ?
③ Stiffness of structure = ?

(4) weight of tank = ?

(5) Damping coefficient = ?

(6) Number of cycle to reduce the displacement amplitude To 0.5"

Solution :-

(1) Damping ratio = δ =

as we know that

$$J = \frac{1}{2\pi\delta} \ln\left[\frac{u_i}{u_{j+1}}\right]$$

$$\delta = \frac{1}{2\pi J} \ln\left(\frac{u_i}{u_{i+1}}\right) \rightarrow (1)$$

$u = 7.684"$

a7Kr $J = 7$

$u_{J+1} = 7+1 = 8$

Now put these value in eq (1)

$$\delta = \frac{1}{2\pi \times 7} \ln\left(\frac{7.684}{0.9}\right)$$

$\boxed{\delta = 0.0487}$ $= 0.0487 \times \frac{100}{100} = 4.87\%$

Now put in eq (1)

$T_n = 0.51 \sqrt[4]{(0.0497)^2}$

$T_n = 0.5047 = \boxed{0.52 \text{ sec}}$

(3) Stiffness of structure

K

as we know that

$K = \frac{P\cos\theta}{2} = \frac{60 \times \cos 60}{2}$

$= 15 \text{ kips} \Rightarrow \boxed{180000 \text{ lb/ft}}$

(4) Weight of the tank

as we know that

$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{w/g}} = \sqrt{\frac{Kg}{w}}$

Now

$w = Kg/\omega_n^2$

$\omega_n = 2\pi/T_n$

$m = lb/in^2/in^2 = \frac{kg \cdot in^2}{s^2}$

$= \frac{19000 \times 0.51^2 \times 32.2}{4 \times (3.14)^2}$

$= \frac{1207537.6}{39.4384}$ $= 38222.18$

$\Rightarrow 38.2k$

(c) Damping co-efficient

as we known that

$\zeta = \frac{C}{2\sqrt{mn}}$

$C = \zeta \times 2\sqrt{mn} = \zeta \times 2\omega$ $(2\sqrt{km})$

Put value

$= \frac{\frac{0.0487 \times 2 \times 38222}{32.2}}{0.51}$

$m = 112.93$ lb.sec/ft

(6) No of cycle to reduce displacement amplitude from 7.684" to 0.9"

$$J = \frac{1}{2\pi\xi} \ln \left[\frac{u_1}{u_{J+1}} \right]$$

$$J = \frac{1}{2 \times \pi \times 0.087} \times \ln \left[\frac{7.684}{9} \right]$$

$$\boxed{J = 7.00 \text{ cycles}}$$

" ∞ "