Final term Exam

Name: Aazaz Ahmad

ID NO: 7705

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Submitted to: Mam Shumaliya mazhar

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(1)

Q.1 Find PQ where P is the point in three-dimensional Space with co-ordinates (4,3) & the point Q with Coordinates (1,2,4). Find the distance b/w P & Q. Further, find the position vector of the point dividing PQ in the ratio 1:3

Sol:

Co-ordinate of p = (4,1,3)

$OP = 4i + 1j + 3K$

or

$OQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 4K) - (4i + 1j + 3K)$

$= -3i + 1j + 1K$ — (1)

2

Now distance between P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \quad \text{— } (2)$$

Let M be the point which divided PQ in ratio 1:3, then by ratio theorm

Position vector of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

3

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \quad \text{— } 3$$

Hence eq 1, 2 & 3 are the required sol.

Q. NO (02) 4

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Sol:

$$= \int \frac{4x^3+10x+4}{2x^2+x}$$

$$\begin{array}{r|l}
 & 2x-1 \\
\hline
2x^2+x & \cancel{4x^3}+10x+4 \\
 & \pm\cancel{4x^3} \quad \pm 2x^2 \\
\hline
 & -\cancel{2x^2}+10x+4 \\
 & \mp\cancel{2x^2} \mp x \\
\hline
 & 11x+4
\end{array}$$

So

$$2x-1+\frac{11x+4}{2x^2+x}=\frac{4x^3+10x+4}{2x^2+x}$$

$$=\int\frac{4x^3+10x+4}{2x^2+x}=\int 2x-1+\int\frac{11x+4}{2x^2+2} \quad (1)$$

$$=2\int x\,dx-\int 1\,dx+\int\frac{11x+4}{2x^2+x}\,dx$$

$$=\frac{2x^2}{2}-x+\int\frac{11x+4}{x(2x+1)}\,dx \quad (2)$$

Now find

$$\int\frac{11x+4}{x(2x+1)}\,dx=?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad — \text{A}$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1)+Bx \rightarrow 3$$

put $x=0$ in 3

$$4 = A$$

Now put $x = -1/2$ in 3

$$11(-1/2)+4 = B(-1/2)$$

$$= \frac{-11+8}{2} = \frac{-B}{2}$$

$$-3 = -B \Rightarrow B = 3$$

putting the value A & B in

A

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x} + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \tfrac{3}{2}\ln|2x+1|$$

putting values in 2

$$= x^2 - x + 4\ln|x| + \tfrac{3}{2}\ln(2x+1)$$

Now put these val in 1

$$\frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x|$$

(9)

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Ans

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Q. NO (3) Part (A)

(a) $\int_0^2 x^2 e^x \, dx$

Now first find integration

$\int x^2 e^x \, dx$

$= x^2 \int e^x \, dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) \, dx$

$= x^2 e^x - 2 \int x e^x \, dx$

$= x^2 e^x - 2 \left(x \int e^x \, dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right)$

$= x^2e^x - 2\left[xe^x - \int e^x\,dx\right]$

$= x^2e^x - 2xe^x + 2e^x$

Now put limits

$= \left| x^2e^x - 2xe^x + 2e^x \right|_0^2$

$= \left(2^2e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$

$= \cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2$

$= 2e^2 - 2$ Am

b) $$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Sol:

First Find integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \quad (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$2dy = \frac{1}{\sqrt{x}}\, dy$ put in eq (1)

$$\int \sin(y)(2\,dy) = 2\int \sin(y)\, dy$$

$$= 2(-\cos y)$$

$$= -2\cos y \quad \text{put } y = \sqrt{x}$$

$$= -2\cos\sqrt{x}$$

put limit

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{x} - \cos 1)$$

$$= -2\cos\sqrt{2} + 2\cos(1) \text{ Ans}$$

Q.4) Verify that (14)

$$u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three dimensional laplace equation

Soln.

The laplace eq in 3 dis

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (1)$$

So $$u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$u(x, y, z) = (x^2+y^2+z^2)^{-1/2}$$

15

$$\frac{\partial u}{\partial x} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial^2 u}{\partial x^2} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(-3/\cancel{2}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x)+(x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \quad \text{— } 1$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

16

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \quad (2)$$

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \quad (3)$$

putting eq (1), (2) & (3) in eq (A)

(17)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$=(x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$=(x^2+y^2+z^2)^{-5/2}\left[3x^2-x^2-y^2-z^2+3y^2-x^2-y^2-z^2+3z^2-x^2-y^2-z^2\right]$$

$$=(x^2+y^2+z^2)^{-5/2}(0)=0$$

So the given $u(x,y,z)$ is solution of laplace equation