Mam Shomaila.

NAME HARIS IQBAL

ID 7926 (A)

Subject Calculus

Exam Summer.

Date 25, sep, 20

=> **Question :— 1** Find PQ where

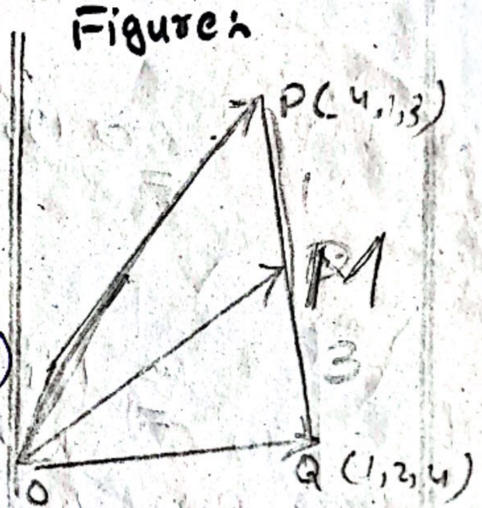
P is 1:3

=> **Solution:.**

coordinate of P = (4, 1, 3)

$\overline{OP} = 4i + 1i + 3k$

Figure:



or

$\overline{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k$ —— (1)

Now distance between P & Q

$|PQ| = \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9+1+1}$

$|PQ| = \sqrt{11}$ —— (2)

so distance b/w P & Q is $\sqrt{11}$

⇒ Let M be the point which divided P Q in ratio 1:3

Then by the ratio theoram position vector of $M = \vec{OM}$

$$\bar{OM} = \frac{3(\bar{OP}) + 1(\bar{OQ})}{1+3}$$

$$\Rightarrow \bar{OM} = \frac{3(4i + 2j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \quad -⑥$$

Hence eq 1, 2, & 3 are required solut.

Q2 $\int \frac{4x^3+10x+4}{2x^2+x}\,dx$

Solution:

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \underline{4x^3 \quad +2x^2} \\ & -2x^2+10x+4 \\ & \underline{-2x^2 \quad -x} \\ & 11x+4 \end{array}$$

So

$$2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int (2x-1)\,dx + \int \frac{11x+4}{2x^2+x}\,dx \quad (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow (2)$$

Now Find:

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow (1)$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1)}$$

$$11x+4 = A(2x+1) + Bx \rightarrow (3)$$

Put $x=0$ in eq (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in (3)

$$11(-1/2) + 4 = B(-1/2)$$

$$-11/2 + 4 = -B/2$$

$$\frac{-11+8}{2} = -B/2$$

(4)

$$-3 = -B \Rightarrow \boxed{B = 3}$$

Putting the values of A and B in A.

$$\frac{11x+4}{x(2x+1)} = 4/x + \frac{3}{2x+1}$$

$\Rightarrow$ Taking integral both sides

$$\int \frac{11x+4}{x(2x+1)} dx = \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$$

$$\Rightarrow 4 \int \frac{1}{x} dx + \frac{3}{2} \int \frac{2}{2x+1} dx$$

$$\Rightarrow 4 \ln |x| + 3/2 \ln (2x+1)$$

Now Put these value in (1)

$$\int \frac{4x^3+10x+4}{2x^2+x} dx = x^2 - x + 4 \ln |x| + 3/2 \ln |2x+1| + C$$

Q3 Part = A

6

$$\int_0^2 x^2 e^x \, dx$$

Solution:

$$\int_0^2 x^2 e^x \, dx$$

$$\Rightarrow x^2 e^x \Big|_0^2 - \int_0^2 e^x (2x) \, dx$$

$$= x^2 e^x \Big|_0^2 - 2\left\{ \int_0^2 e^x (x) \, dx \right\}$$

$$= x^2 e^x \Big|_0^2 - 2\left\{ x e^x \Big|_0^2 - \int_0^2 e^x (1) \, dx \right\}$$

$$\Rightarrow x^2 e^x \Big|_0^2 - 2x e^x \Big|_0^2 + 2\int_0^2 e^x \, dx$$

$$= x^2 e^x \Big|_0^2 - 2x e^x \Big|_0^2 + 2e^x \Big|_0^2$$

Putting Limits

$$\Rightarrow (2)^2 e^2 - (0) - \{2(2)e^2 - 0\} + 2e^2 - 2e^0$$

$\Rightarrow \cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2$

$\Rightarrow 2e^2 - 2$

$= \boxed{12.77}$

Q3 Part # B

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Let:

$\sqrt{x} = t$

Differentiate w.r.t x

$\frac{d}{dx}\sqrt{x} = \frac{dt}{dx}$

$\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{2\sqrt{x}}\, dx = dt$

$\frac{1}{\sqrt{x}} dx = 2dt$

R.W

$\frac{d}{dx}\sqrt{x}$

$\frac{d}{dx}(x)^{1/2}$

$= \frac{1}{2} x^{-1/2}$

$= \frac{1}{2} x^{-}$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

Now

$\sqrt{x} = t$

put $x=1$

$t = \sqrt{1}$

$t = 1$

Put $x = 2$

$t = \sqrt{2}$

$\Rightarrow$ **Now given integral Becomes as:**

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$= \int_1^{\sqrt{2}} \sin t \; 2\,dt$$

$$= 2\int_1^{\sqrt{2}} \sin t\,dt$$

$$= 2(-\cos t)\Big|_1^{\sqrt{2}}$$

$$= -2\cos t\Big|_1^{\sqrt{2}}$$

$\Rightarrow$ Putting Limit.

$\Rightarrow -2\cos\sqrt{2} - (-2\cos(1))$

Scanned with CamScanner

$= -2\cos\sqrt{2} + 2\cos(1)$

$= -2(0.155) + 2(0.540)$

$= -0.31 + 1.080$

$= 0.77$

=> Question 4

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

As three dimensional Laplace eqn

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

As $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$ $\Rightarrow U = \frac{1}{(x^2+y^2+z^2)^{1/2}}$

$$\Rightarrow U = (x^2+y^2+z^2)^{-1/2}$$

$$\Rightarrow \frac{\partial U}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}-1} \frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$\Rightarrow -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(2x - 0 + 0$$

$$\frac{\partial x}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 x}{\partial x^2} = \frac{\partial}{\partial x}\left[-x(x^2+y^2+z^2)^{-3/2}\right]$$

$$\Rightarrow -x\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-3/2} + (x^2+y^2+z^2)^{-3/2}\,\partial/\partial x(-x)$$

(11)

$$\Rightarrow -x\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-3/2-1}\frac{\partial}{\partial x}(x^2+y^2+z^2)+(x^2+y^2+z^2)^{-3/2}(-1)$$

$$\Rightarrow \frac{3x}{2}(x^2+y^2+z^2)^{-5/2}(2x)-(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2}=3x^2(x^2+y^2+z^2)^{-5/2}-(x^2+y^2+z^2)^{-3/2} \quad \text{— (1)}$$

As $\quad u=(x^2+y^2+z^2)^{-1/2}$

Now differentiate w.r.t y

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}$$

$$=-\tfrac{1}{2}(x^2+y^2+z^2)^{-3/2}\frac{\partial}{\partial y}(x^2+y^2+z^2)$$

$$=-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(0+2y+0)$$

$$\frac{\partial u}{\partial x}=-y(x^2+y^2+z^2)^{-3/2}$$

$\Rightarrow$ Now differentiate w.r.t y again

$$\Rightarrow \frac{\partial^2 u}{\partial x^2}=-y\left(-\frac{3}{2}(x^2+y^2+z^2)^{-5/2}(2y)\right)+(x^2+y^2+z^2)^{-3/2}(-1)$$

$$= \frac{\partial^2 u}{\partial y^2}=3y^2(x^2+y^2+z^2)^{-5/2}-(x^2+y^2+z^2)^{-3/2} \quad \text{— (2)}$$

$\Rightarrow$ Now differentiate w.r.t z

$$\frac{\partial u}{\partial z}=\frac{\partial}{\partial z}(x^2+y^2+z^2)^{-1/2}$$

Scanned with CamScanner

$= -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2z)$

$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$

=> Now differentiate w.r.t. z again.

$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(-z(x^2+y^2+z^2)^{-3/2}\right)$

$= -z\frac{\partial}{\partial z}(x^2+y^2+z^2)^{-3/2} + (x^2+y^2+z^2)^{-3/2}\frac{\partial}{\partial z}(-z)$

$= -z(-3/2)(x^2+y^2+z^2)^{-5/2}(2z) + (x^2+y^2+z^2)(-1)$

$\frac{\partial^2 U}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)$ — (3)

Adding eq 1, 2 and 3

=> $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$

$+ 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$

$= \quad + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$

$= 3(x^2+y^2+z^2)^{-5/2}\{(x^2+y^2+z^2)\}$

$- 3(x^2+y^2+z^2)^{-3/2}$

$= 3(x^2+y^2+z^2)^{1-5/2} - 3(x^2+y^2+z^2)^{-3/2}$

$= 3(x^2+y^2+z^2)^{-3/2} - 3(x^2+y^2+z^2)^{-3/2}$

$= 0$

So laplace eq is satisfied.