

Linear Algebra
Mid Assignment

Note:

- *If your student ID is e.g. 14589 then $ID1 = 1$, $ID2 = 4$, $ID3 = 5$ etc*
- *Submission time 21-08-2020 before 6:00 pm (4 Hrs)*
- *Copied papers will both be marked zero*

ID= 12994 Name = shabban khan

Question No: 1

Q No. 1

12994

Part 1

Solve the system of equation that corresponds to this augmented Matrix

Ans

$$\begin{bmatrix} 1 & -3 & 4 & -10 & 1 \\ 3 & -7 & 7 & -10 & 9 \\ -4 & 6 & -1 & -10 & 3 \end{bmatrix}$$

$10R_1 = 1$
 $10R_2 = 2$
 $10R_3 = 9$
 $10R_4 = 4$

$$\Rightarrow A = \begin{bmatrix} 1 & -3 & 4 & -2 \\ 3 & -7 & 7 & -9 \\ -4 & 6 & -1 & -9 \end{bmatrix}$$

Now

$$A \begin{bmatrix} 1 & -3 & 4 & -2 \\ 0 & -2 & -5 & -3 \\ 0 & -6 & 15 & 7 \end{bmatrix} \begin{matrix} R_2 - 3R_1 \\ R_3 + 4R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -3 & 4 & -2 \\ 0 & 2 & -5 & -3 \\ 0 & 0 & 0 & 8 \end{bmatrix} \begin{matrix} R_3 + 3R_2 \end{matrix}$$

②

$$x - 3y + 4z = -2 \quad (i)$$

$$0x + 2y - 5z = -3 \quad (ii)$$

$$0x + 0y + 0z = 8 \quad (iii)$$

Put z value in (ii)

$$x - 3y + 4z = -2$$

$$+2y - 5(8) = -3$$

$$2y - 40 = -3$$

$$2y = -3 + 40$$

$$y = \frac{-3 + 40}{2}$$

$$y = 18.5$$

Put y value in (i)

$$x - 3(18.5) + 4(8) = -2$$

$$x - 55.5 + 32 = -2$$

$$x = 55.5 - 32 - 2$$

$$x = 21.5$$

$$x = 21.5 \quad y = 18.5 \quad z = 8$$

Question No: 2

Q2 Find inverse

$$A \Rightarrow A^{-1} = [A | I_3]$$

- 12944
- 1D1=1
- 1D2=2
- 1D3=9
- 1D4=9
- 1D5=4

$$\left[\begin{array}{ccc|ccc} 9 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 9 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$R_1/9 \rightarrow R_1$ div Row 1 by 9

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{9} & 0 & \frac{1}{9} & 0 & 0 \\ 0 & 1 & 9 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$R_3 - 1R_1 - R_2$
~~multiply Row 1 by 9 and~~

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{9} & 0 & \frac{1}{9} & 0 & 0 \\ 0 & 1 & 9 & 0 & 1 & 0 \\ 0 & \frac{10}{9} & 0 & -\frac{1}{9} & 0 & 1 \end{array} \right]$$

$R_1 + \frac{1}{9}R_2 \rightarrow R_1, R_3 - \frac{10}{9}R_2 - R_3$

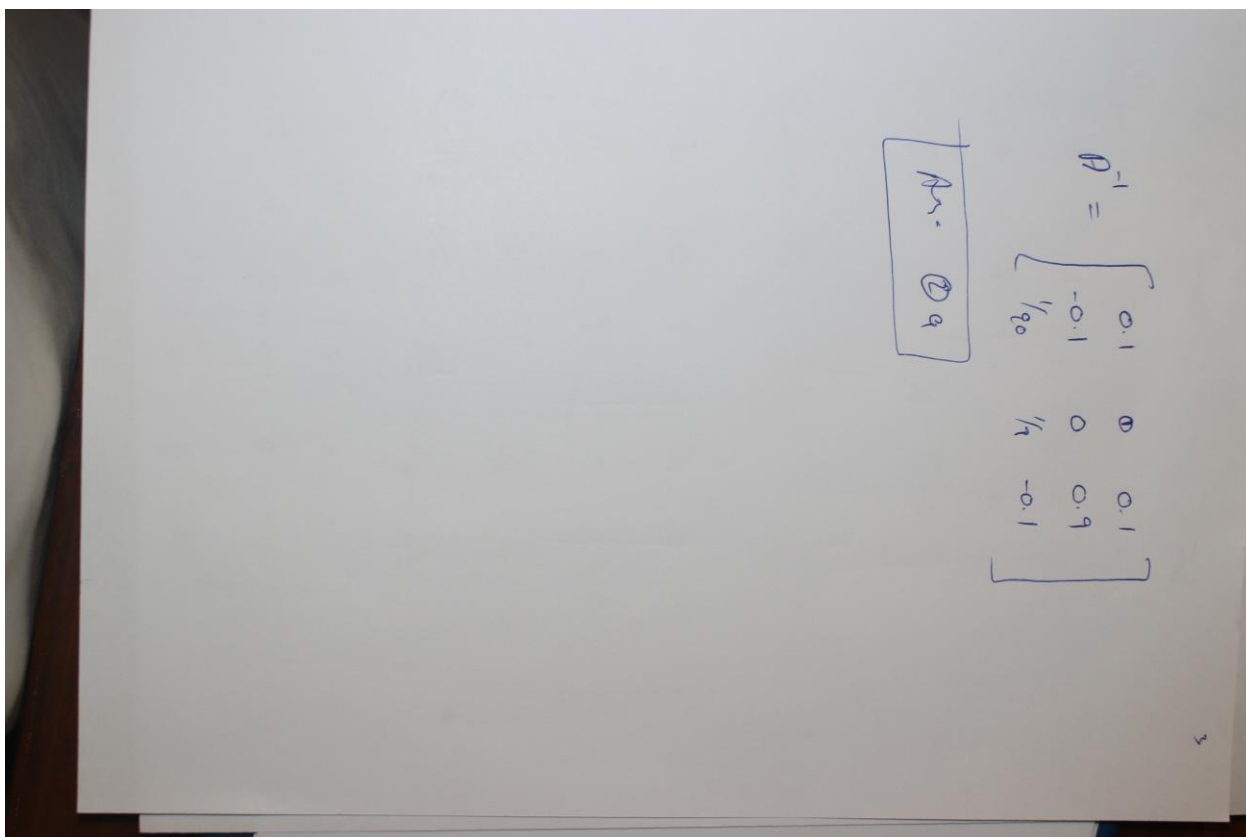
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & 1 & 9 & 0 & 1 & 0 \\ 0 & 0 & -10 & -\frac{1}{9} & -\frac{10}{9} & -1 \end{array} \right]$$

$R_3 / -10 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & 1 & 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{90} & \frac{1}{9} & -0.1 \end{array} \right]$$

$R_1 - R_3 \rightarrow R_1, R_2 - 9R_3 \rightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0.1 & 0 & 0.1 \\ 0 & 1 & 0 & -0.1 & 0 & 0.9 \\ 0 & 0 & 1 & \frac{1}{90} & \frac{1}{9} & -0.1 \end{array} \right]$$



a) Find an echelon form for the below matrix using row operations

$$\begin{bmatrix}
 \mathbf{1} & \mathbf{ID3} & \mathbf{8} \\
 \mathbf{2} & \mathbf{ID4} & \mathbf{-1} \\
 \mathbf{-3} & \mathbf{0} & \mathbf{0} \\
 \mathbf{1} & \mathbf{-ID3} & \mathbf{16}
 \end{bmatrix}$$

Ans:

Q No 2
a) b)

Ans

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 2 & 1 & 0 & -1 \\ -3 & 0 & 0 & 0 \\ 1 & 1 & -1 & 16 \end{bmatrix}$$

$ID_1 = 1$
 $ID_2 = 2$
 $ID_3 = -3$
 $ID_4 = 1$
 $ID_5 = 4$

Step 1 $-1R_1 \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 0 & -1 & -9 \\ -3 & 0 & 0 & 0 \\ 1 & 1 & -1 & 16 \end{bmatrix}$$

Step 2 $3R_1 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 0 & -1 & -9 \\ 0 & 3 & 3 & 24 \\ 1 & 1 & -1 & 16 \end{bmatrix}$$

Step 3 $-1R_1 \rightarrow R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 0 & -1 & -9 \\ 0 & 3 & 3 & 24 \\ 0 & 0 & -2 & 8 \end{bmatrix}$$

Step 4 Put R_2 replace R_3

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 0 & -1 & -9 \\ 0 & 3 & 3 & 24 \\ 0 & 0 & -2 & 8 \end{bmatrix}$$

Step 5 $(1/2)R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 1/2 & -9/2 \\ 0 & 0 & -1 & -9 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

Step 6 $R_3 + R_2 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 1/2 & -9/2 \\ 0 & 0 & -1 & -9 \\ 0 & 0 & 0 & 21 \end{bmatrix}$$

Step 7 $(-1/4)R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 1/2 & -9/2 \\ 0 & 0 & 1/4 & 9/4 \\ 0 & 0 & 0 & 21 \end{bmatrix}$$

Step 8 $-24R_3 \rightarrow R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 1/2 & -9/2 \\ 0 & 0 & 1/4 & 9/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 9 $-4/4 R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 1/2 & -9/2 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 10 $-8R_3 \rightarrow R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 1/2 & -9/2 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 11 $-9R_2 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans

Question No: 3

Find the Eigen values and Eigen vectors of the below Matrix

$$\begin{bmatrix} \text{ID3} & -6 & 2 \\ -6 & \text{ID2} & -4 \\ 2 & -4 & \text{ID4} \end{bmatrix}$$

(3)

$\Rightarrow (9-x) \int x^2 - 11x + 2 \int 6 \int 6x - 46 \int + 2(2x+20) \int$

$\Rightarrow 9x^2 - 99x + 18 - x^3 + 11x^2 - 2x + 36x - 276 + 4x + 40$

$\Rightarrow -x^3 + 20x^2 - 61x - 210 = 0$

by using Synthetic Division by
Put $x = -2$ we have

$-(2)^3 + 20(-2)^2 - 61(-2) - 210 = 0$

$8 + 80 + 122 - 210 = 0$

have form $(x+2)$

-2	-1	20	-61	-210
	12	-44	+210	
	-1	22	-105	0

Q.4

$$(x-2)(-x^2+22x-105) = 0$$

$$x^2 - 22x + 105 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{22 \pm \sqrt{22^2 - 4(-1)(-105)}}{-2}$$

$$= \frac{22 \pm \sqrt{484 - 420}}{-2}$$

$$= \frac{22 \pm \sqrt{64}}{-2}$$

$$= \frac{22+8}{-2} \quad \frac{22-8}{-2}$$

$$x = -15 \quad x = -7$$

Eigen value

$$\boxed{x = 2, -15, -7}$$

Q.5

Q. No. 3

$$\begin{bmatrix} 9 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 9 \end{bmatrix}$$

Find the Eigen value

$$= \begin{bmatrix} 9 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 9 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-\lambda & -6 & 2 \\ -6 & 2-\lambda & -4 \\ 2 & -4 & 9-\lambda \end{bmatrix}$$

$$\begin{pmatrix} 9-\lambda & -6 & 2 \\ -6 & 2-\lambda & -4 \\ 2 & -4 & 9-\lambda \end{pmatrix}$$

$$= 9-\lambda \begin{vmatrix} 2-\lambda & -4 \\ -4 & 9-\lambda \end{vmatrix} + 6 \begin{vmatrix} -6 & -4 \\ 2 & 9-\lambda \end{vmatrix} + 2 \begin{vmatrix} -6 & 2-\lambda \\ 2 & -4 \end{vmatrix}$$

$$9-\lambda \left[(2-\lambda)(9-\lambda) - (-4)(-4) \right] + 6 \left[(-6)(9-\lambda) - (-4)(2) \right]$$

$$\Rightarrow +2 \left[(-6)(-4) - (2-\lambda)(2) \right]$$

$$9-\lambda \left[18\lambda - 2\lambda^2 - 9\lambda + \lambda^2 + 16 \right] + 6 \left[-54 + 6\lambda + 8 \right]$$

$$+ 2 \left[24 - 4 + 2\lambda \right]$$