Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: Engr Pir Meher Ali Shah **Total Marks:** 20

Student Details

Name: irshad khan **Student ID:** 12403

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for N ≥ L	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0\ \ 1\ \ 2\ \ 3)$$	

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Q1 (a) Determine the response $y(n), n \geq 0$ of the system described by the second order difference equation.

soln:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

To the input $x(n) = 4^n u(n)$.

sol: First we determine the solution to the homogenous equation.

$$y_h(n) = \lambda^n$$

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\lambda = -1, 4$$

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n$$

$$= C_1(-1)^n + C_2(4)^n$$

Initial conditions $y(-1)$ and $y(-2)$.

So,

$$y(0) = 3y(-1) + 4y(-2)$$

$$y(1) = 3y(0) + 4y(-1)$$

$$= 3[3y(-1) + 4y(-2)] + 4y(-1)$$

$$= 13y(-1) + 12y(-2)$$

&

$$y_h(n) = C_1(-1)^n + C_2(4)^n$$

so,

$$y_p(n) = K(4)^n u(n)$$

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$Kn(4)^n u(n) - 3K(n-1)(4)^{n-1} u(n-1)$
$- 4K(n-2)(4)^{n-2} u(n-2) = (4)^n u(n)$
$+ 2(4)^{n-1} u(n-1)$

To determine K, we evaluate this equation for any $n \geq 2$.
we select $n = 2$
we obtain $K = 6/5$. Therefore

$y_p(n) = 6/5\, n(4)^n u(n)$.

$y(n) = C_1(-1)^n + C_2(4)^n + 6/5\, n(4)^n$
$n \geq 0$

C_1 and C_2 are constants.

we obtain

$y(0) = 3y(-1) + 4y(-2) + 1$
$y(1) = 3y(0) + 4y(-1) + 6$
$= 13y(-1) + 12y(-2) + 9$

For $n = 0$ and $n = 1$ yields.

$y(0) = C_1 + C_2$
$y(1) = -C_1 + 4C_2 + 24/5$

$y(-1) = y(-2) = 0$.

$C_1 + C_2 = 1$
$-C_1 + 4C_2 + 24/5 = 9$

Hence.

$c_1 = -1/25$ and $c_2 = 26/25$ for function $x(n) = (4)^n u(n)$ in the form of

$$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n$$

$n \geq 0$

The total response of the system which includes the response to arbitrary initial condition.

Q1
(b) $y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$.

Soln. The system function is.

$$H(z) = \frac{1}{1 - 0.6z^{-1} + 0.8z^{-2}}$$

the system has two complex-conjugate.

$$p_1 = 0.6e^{j\pi\beta} \qquad p_2 = 0.6e^{-j\pi\beta}$$

The ~~z-transform~~

$$X(z) = \frac{1}{1 - z^{-1}}$$

Therefore.

$$y_{zs}(z) = \frac{1}{(1 - 0.6e^{j\pi\beta}z^{-1})(1 - 0.6e^{-j\pi\beta}z^{-1})(1 - z^{-1})}$$

$$= \frac{0.342 - j0.046}{1 - 0.6e^{j\pi/3}z^{-1}} + \frac{0.342 + j0.046}{1 - 0.6e^{-j\pi/3}z^{-1}} + \frac{1.066}{1 - z^{-1}}$$

$$y_{zs}(n) = \left[1.066 + 1.08(0.6)^n \cos\left(\frac{\pi}{3}n - 37\right)\right] u(n).$$

conditions $y(-1)$ and $y(-2)$.

$$y(0) = 0.6y(-1) - 0.8y(-2)$$

$$y(1) = 0.6y(0) - 0.8y(-1)$$

$$0.6[0.6y(-1) + 0.8y(-2)] + 0.8y(-1)$$

$$0.12y(-1) + 0.8y(-2).$$

Q9 Determine the casual signal $x(n)$ having

(a) the Z-Transform

$$X(Z) = \frac{1}{(1-9z^{-1})(1-z^{-1})^2}$$

Take inverse Z Transform using partial fraction method.

Soln

$$X(Z) = \sum_{n=-\infty}^{\infty} C_n Z^{-n}$$

$$X(Z) = \frac{1}{4}\frac{1}{(1-9z^{-1}} + \frac{3}{4}\frac{1}{1-9z^{-1}} + \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

~~By applying the inverse transform relations.~~

By using the formula.

$$Z^{-1}\left\{\frac{1}{1-P_k z^{-1}}\right\} = \begin{cases} (P_k)^n u(n) \\ -(P_k)^n u(-n-1). \end{cases}$$

$ROC: |Z| > |P_k|$.

$ROC: |Z| < |P_k|$

$$Z^{-1}\left\{\frac{pz^{-1}}{(1-pz^{-1})^2}\right\} = np^n u(n).$$

The ROC is $|Z| > |P|$.

By applying the inverse Transform relations.

$$x(n) = \frac{1}{4}(-1)^n u(n) + \frac{3}{4}u(n) + \frac{1}{2}nu(n) = \left[\frac{1}{4}(-1)^n + \frac{3}{4} + \frac{n}{2}\right]u(n).$$

Q2 (b) Determine the partial-fraction expansion of the proper function.

$$X(Z) = \frac{1}{1-1.5Z^{-1}+0.5Z^{-2}}$$

Sol: First we eliminate the negative powers by multiplying both numerator and denominator by Z^2. Thus.

$$X(Z) = \frac{Z^2}{Z^2-1.5Z+0.5}$$

The poles of $X(Z)$ are $p_1 = 1$ and $p_2 = 0.5$.

$$\frac{X(Z)}{Z} = \frac{Z}{(Z-1)(Z-0.5)} = \frac{A_1}{Z-1} + \frac{A_2}{Z-0.5}$$

A_1 and A_2 is to multiply the equation by denominator term $(Z-1)(Z-0.5)$. Thus we obtain

$$Z = (Z-0.5)A_1 + (Z-1)A_2$$

Now if we set.

$Z = p_1 = 1$. We eliminate the term involving A_2. Hence

$$1 = (1-0.5)A_1$$

$$A_1 = 2$$

$$Z = p_2 = 0.5$$

$$0.5 = (0.5-1)A_2$$

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and hence $A_2 = -1$.

Therefore the result of the partial fraction expansion is.

$$\frac{X(Z)}{Z} = \frac{2}{Z-1} - \frac{1}{Z-0.5}.$$

we can determine the coefficients $A_1, A_2 ----- A_N$.

$K = 1, 2 ---- N$.

In general form

$$\frac{(Z-P_K)X(Z)}{Z} = \frac{(Z-P_K)A_1}{Z-P_1} + ----- + A_K + ---- + \frac{(Z-P_K)A_N}{Z-P_N}.$$

$Z = P_K$.

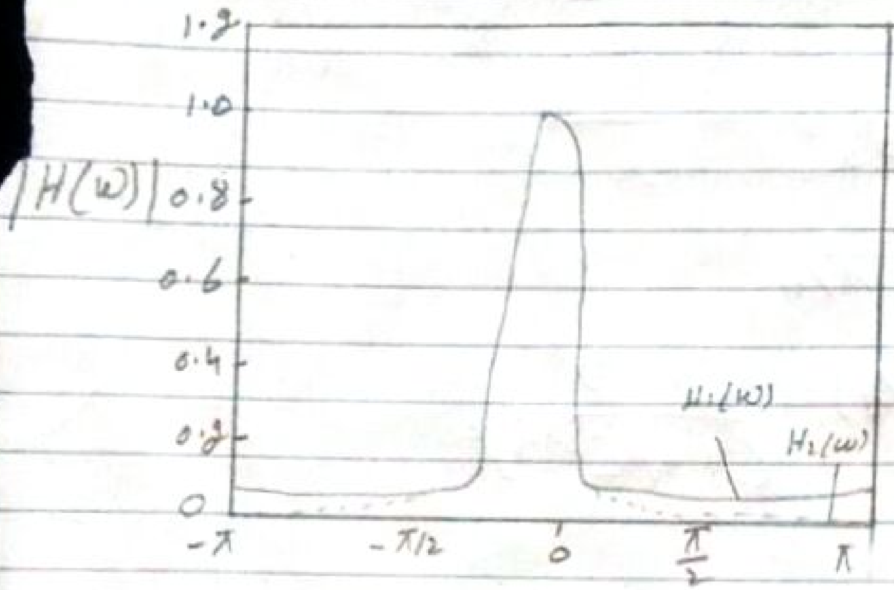
$$A_K = \frac{(Z-P_K)X(Z)}{Z}\Big|_{Z=P_K}$$

$K = 1, 2, ---- N$.

Q3 (a) A two pole lowpass filter has the system function

$$H(z) = \frac{b_0}{(-pz^{-1})^2}$$

Determine the values of b_0 p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $|H(\frac{\pi}{4})|^2 = \frac{1}{2}$

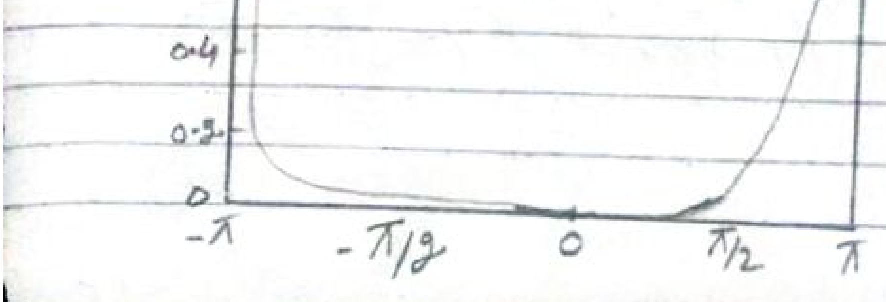


Solution:- At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$



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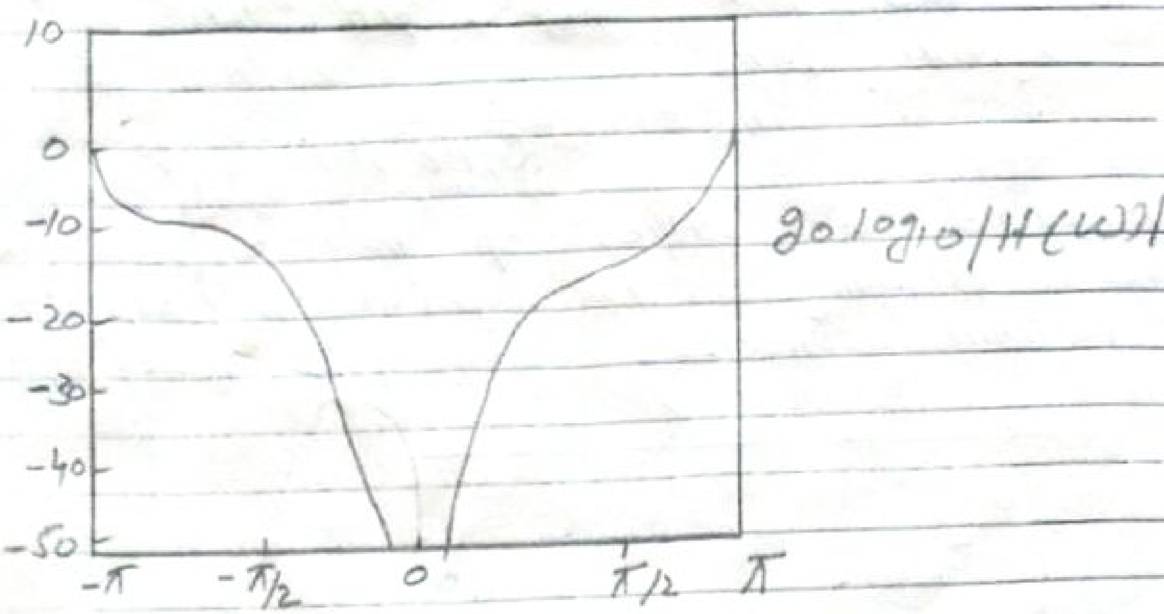


At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^4}{\left[(1-p/\sqrt{2})^2+p^2/2\right]^2} = \frac{1}{2}$$

equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$ satisfies this equation. consequently the System function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principles can applied for the design of bandpass filters. Basically, the band pass filter Should contain one or more pairs of complex - conjugate poles near the unit circle in the vicinity of the frequency band that constitues the passband of the filter. The following examples serves to illustrate the basic ideas.

23 (b) Design a two pole bandpass at $\omega = \pi/2$ at $\omega = 0$ and $\omega = \pi$ and its magnitude response is $2/\sqrt{2}$ at $\omega = 4\pi/9$.

Soln:- Clearly the filter must have poles at

$$p_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z = 1$ and $z = -1$. Consequently the system function is:

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2 - 1}{z^2 + r^2}$$

Frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have.

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1 - r^2} = 1$$

$$G = \frac{1 - r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$|H(\frac{4\pi}{9})|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = 1/2$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

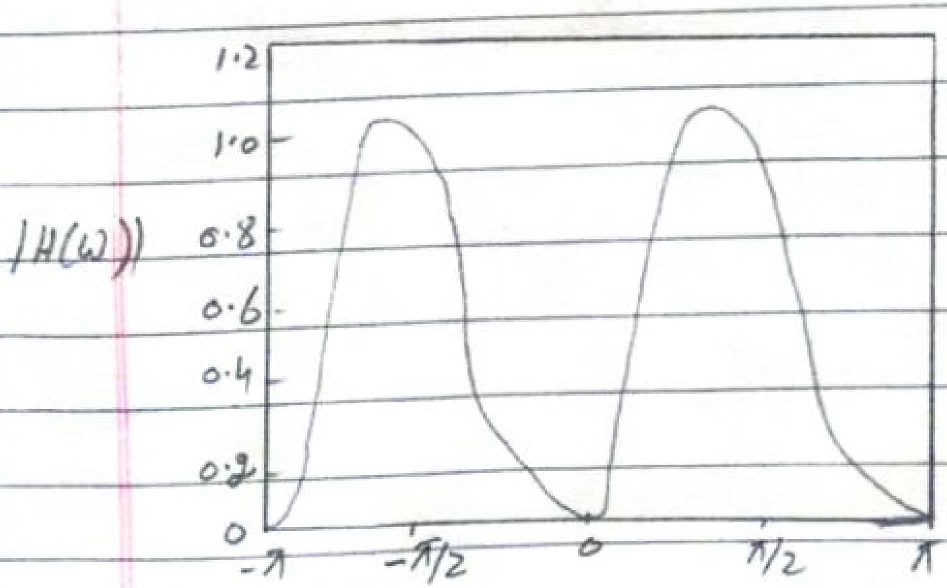
The value of $r^2 = 0.7$ satisfies the equation.

For the desired Filter.

$$H(Z) = 0.15 \frac{1-Z^{-2}}{1+0.7Z^{-2}}$$

so,

$$H(Z) = 0.15[(1-Z^{-2})/(1+0.7Z^{-2})].$$



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It should be emphasized that the main purpose of foregoing methodology of for designing simple digital filters by pole-zero placement is to provide insight into the effect the poles and zeros have on the frequency response characteristic of systems. The methodology is not intended as a good method for designing digital filters with well-specified passband and stopband characteristics. systematic methods for the design of sophisticated digital filters for practical application are discussed.

Q4 "C" A finite duration sequence of length L is given as

$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the N-point DFT of this sequence for $N \ge L$.

Solution: The fourier Transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} =$$

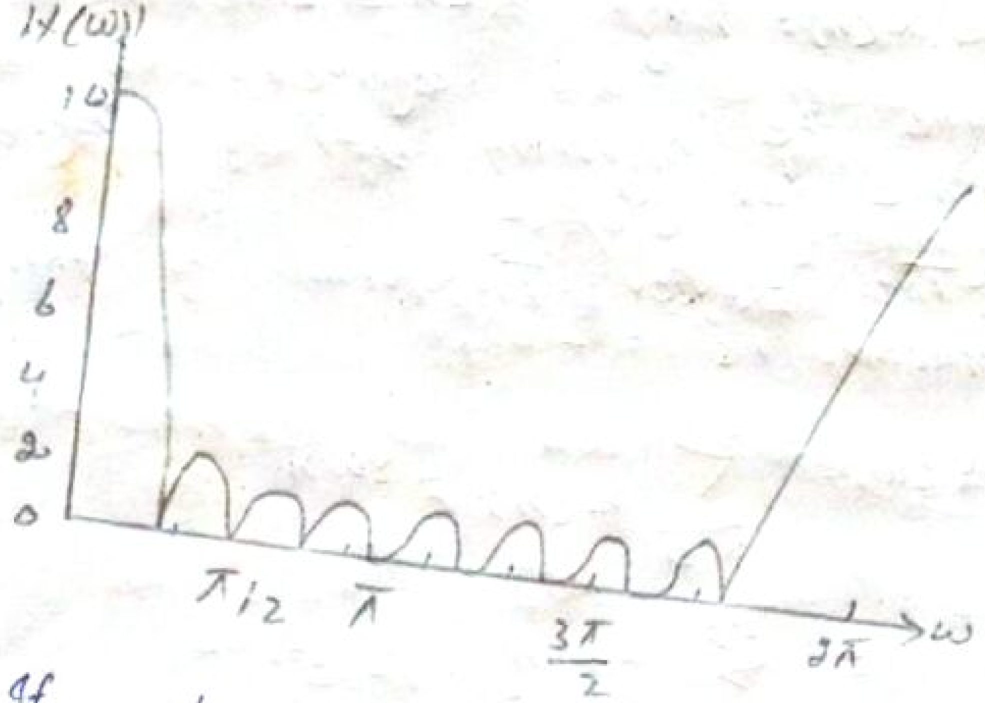
$$\frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated for $L = 10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies

$\omega_k = 2\pi k/N$, $k = 0, 1 \cdots N-1$. Hence

$$X(k) = \frac{1 - e^{j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \quad k = 0, 1 \cdots N-1$$

$$\frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$



If N is selected such that N = L then the DFT becomes.

$$X(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2 \dots L-1 \end{cases}$$

Thus there is only one nonzero value in the DFT.

since $X(\omega) = 0$ at the frequencies $\omega_K = 2\pi K/L$, $K \neq 0$. The reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT. In magnitude and phase for $L = 10$, $N = 50$ and $N = 100$ as one will conclude by comparing these spectra with the continious spectrum $X(\omega)$.

Q4 compute the DFT of the four 'd' point sequence.

$$x(n) = (0 \; 1 \; 2 \; 3)$$

soln: The first step is to determine the matrix W_4. By exploiting the periodicity property of W_4 and the symmetry property.

$$W_N^{k+N/2} = -W_N^k$$

The matrix W_4 may expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The IDFT of X_4 may be determined by conjugating the elements in W_4 to obtain W_4^* and then applying the formula.

The DFT and IDFT are computational tools that play a very important role in many digital signal processing applications such as frequency analysis (spectrum analysis) of signals, power spectrum estimation and linear filtering. The importance of the DFT and IDFT in such practical applications is due to a large extent on the existence of computationally efficient algorithms, known collectively as fast.