Name Usama-Aziz
Id 16310
Section "B"
Paper Calculus
Department Civil Engineering
Instructor Name Shumaila Mazhar

QNo1

Solution:- $P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$
$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$

Now distance b/w PQ

So $|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

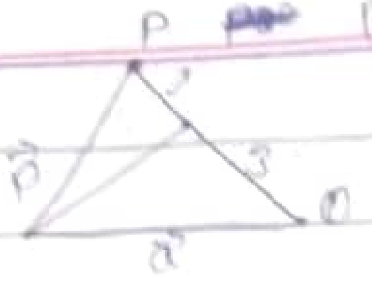
$= \sqrt{(4-1)^2 + (1-2)^2 + (3-4)^2}$

$= \sqrt{(3)^2 + (-1)^2 + (-1)^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11}$

Now find the position vector of the point dividing PQ in the ratio of 2:3



$a : b = 1 : 3$

$$\vec{r} = \frac{b\vec{p} + a\vec{a}}{b+a}$$

$$= \frac{3(4i + j + 3k) + 1(\hat{i} + 2\hat{j} + 4\hat{k})}{3+1}$$

$$= \frac{12\hat{i} + 3\hat{j} + 9k + \hat{i} + 2\hat{j} + 4\hat{k}}{4}$$

$$= \frac{13\hat{i} + 5\hat{j} + 13\hat{k}}{4}$$

$$\boxed{\vec{r} = \frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{13}{4}\hat{k}}$$ Ans

QNo4 Verify that

$$u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

Solution:

laplace for $u(x,y,z)$ is given by

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0$$

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{du}{dx} = \frac{d}{dx}(x^2+y^2+z^2)^{-1/2}$$

$$\frac{du}{dx} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$\frac{du}{dx} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\left[-x(x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{d^2u}{dx^2} = -\left[x\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-5/2}2x + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{d^2u}{dx^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\frac{du}{dy} = \frac{d}{dy}\left(x^2+y^2+z^2\right)^{-1/2}$$

$$\frac{du}{dy} = \frac{-1}{2}\left(x^2+y^2+z^2\right)^{-3/2}(2y)$$

$$\frac{du}{dy} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{d^2u}{dy^2} = \frac{d}{dy}\left[-y\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{d^2u}{dy^2} = -\left[y\left(\frac{-3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2y)+\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{d^2u}{dy^2} = -\left[3y^2\left(x^2+y^2+z^2\right)^{-5/2}-\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{du}{dz} = \frac{d}{dz}\left(x^2+y^2+z^2\right)^{-1/2}$$

$$\frac{du}{dz} = -\frac{1}{2}\left(x^2+y^2+z^2\right)^{-3/2}(2z)$$

$$\frac{du}{dz} = -2\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2u}{\partial z^2} = \frac{\partial}{\partial z}\left(-z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

Scanned with CamScanner

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

Now $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}$$
$$- (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$x^2+y^2+z^{2^{-5/2}}\left[3x^2 - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2\right.$$
$$\left.- x^2 - y^2 - z^2\right]$$

$$x^2+y^2+z^2\,(0) = 0$$

Hence $u(x,y,z)$ satisfies solution of Laplace solution

Q.No 3 Evaluate

P-No (a) $\int_0^2 x^2 e^x dx$

Solution:

$$\int_0^2 x^2 e^x dx$$

By part integration

$$= x^2 \int e^x - \int 2x \int e^x dx$$

$$= x^2 e^x - \int 2x e^x dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left[x \int e^x - \int 1 . \int e^x dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= \left| x^2 e^x - 2 \left[x e^x - e^x \right] \right|_0^2$$

$$= \left| x^2 e^x - 2x e^x + 2 e^x \right|_0^2$$

$$= \left((2)^2 e^2 - 2(2) e^2 + 2 e^2 \right) - \left(0^2 e^0 - 2(0) e^0 + 2 e^0 \right)$$

$$= \left(4e^2 - 4e^2 + 2e^2 \right) - \left(0 - 0 + 2(1) \right)$$

$$= 2e^2 - 2$$

$$= 2(e^2 - 1)$$

hence

$$\boxed{\int_0^2 x^2 e^x dx = 2(e^2 - 1)} \text{ Ans}$$

Scanned with CamScanner

Q.No 3 (b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

P.No (b)

Solution

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

let $u=\sqrt{x} \Rightarrow du=\frac{dx}{2\sqrt{x}} \Rightarrow 2du=\frac{dx}{\sqrt{x}}$

AS $x=1$ then $u=1$ and $x=2$

then $u=\sqrt{2}$

$$\int_1^2 \sin\sqrt{x}\,\frac{dx}{\sqrt{x}}$$

$$=\int_1^{\sqrt{2}} \sin u\, 2du$$

$$=2\int_1^{\sqrt{2}} \sin u\,du$$

$$=|2[-\cos u]\Big|^{\sqrt{2}}$$

$$=|-2\cos u\Big|^{\sqrt{2}}$$

$$=(-2\cos\sqrt{2})-(-2\cos 1)$$

$$=-2\cos\sqrt{2}+2\cos 1$$

$$\int_1^2 \frac{\sin\sqrt{x}\,dx}{\sqrt{x}} \Rightarrow 2(\cos 1 - \cos\sqrt{2})$$

$$= 2(0.3484)$$

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \boxed{0.7687} \text{ or } \boxed{0.769}$$ Ans:

Q NO 2 Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

Solution:

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

By long division

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \underset{(-)}{\cancel{4x^3}} \quad \underset{(-)}{\pm 2x^2} \\ \hline & -2x^2+10x+4 \\ & \underset{(+)}{\mp}\cancel{2x^2} \underset{(+)}{\mp} x \\ \hline & 11x+4 \end{array}$$

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int \left(2x-1+\frac{11x+4}{2x^2+x}\right)dx$$

$$= 2x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= x^2 - x + \int \frac{11x+4}{2x^2+x}\,dx$$

By using Partial fraction

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} \times \cancel{x(2x+1)} = \frac{A\,\cancel{x}(2x+1)}{\cancel{x}} + \frac{B\,x\cancel{(2x+1)}}{\cancel{2x+1}}$$

$$= 11x+4 = A(2x+1) + B(x)$$

$$= 11x+4 = 2Ax + A + Bx$$

By compairing

$\Rightarrow$ $A = 4$ and $B = 3$ So

$$\Rightarrow x^2 - x + \int \left(4/x + 3/(2x+1)\right)dx$$

$$\Rightarrow x^2 - x + \int 4/x\,dx + \int \frac{3}{2x+1}\,dx$$

$$\Rightarrow \boxed{x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C} \text{ Ans}$$