

Date: _____

Name :- Ali Davish Kayani

Subject :- Linear Algebra

ID NO :- 15243

Instructor :- Himayatullah.

Question No 1:-
(A part):-

The non parallel vectors:-

$$\vec{P_1 P_2} = (-3, 2, 3)$$
$$\vec{P_1 P_3} = (3, -1, 3)$$

$$\begin{aligned} &(-1, 0, 3) - (2, -2, 1) \\ &(-1, 0, 3) - 2, +2, -1 \\ &(-3, +2, 2) \end{aligned}$$

The perpendicular vector is:-
 $m = \vec{P_1 P_2} \times \vec{P_1 P_3}$

$$m = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$\begin{aligned} \vec{P_1 P_2} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ \vec{P_1 P_3} &= \sqrt{(-1 - 2)^2} \end{aligned}$$

$$m = i(t+2) - j(-9-6) + k(3-6)$$

$$m = 8i + 15j - 3k$$

$$m = (8, 15, -3)$$

Now, $P_1 (x_0, y_0, z_0) = (2, -3, 1)$

$$m(a, b, c) = 8, 15, -3$$

So, equation of plane is
 $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$



$$\textcircled{2} \\ 8(x-2) + 15 \left(\begin{matrix} y+2 \\ y+3 \end{matrix} \right) - 3(z-1) = 0$$

$$8x + 15y - 32 - 16 + \frac{30}{17} + 3 = 0$$

$$8x + 15y - 32 + \frac{30}{17} = 0$$

(Q1 (b))

$$x = 2 - 3t$$

$$y = 3 + t$$

$$z = 2 - 4t$$

Solution:-

$$2 - 2 = -3t \Rightarrow t = \frac{x-2}{-3}$$

$$y = 3 + t \Rightarrow t = \frac{y-3}{1}$$

$$2 - 2 = -4t \Rightarrow t = \frac{z-2}{-4}$$

$$\text{So } \frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

For 1st plane takes
1st and 2nd

$$\frac{x-2}{-3} \times \frac{y-3}{1}$$

$$x-2 = -3y+9$$

$$x + 3y = 11 = 0$$

Date: _____

③

For 2nd plane take 1st Eq
3rd

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$$-4x+8 = -3z+6$$

$$-4x+3z+2=0$$

or

$$4x-3z+2=0$$

Question:- 2

$$L(x, y) = (x+1, y, x+y)$$

Solution:-

$$L(x, y) = (x+1, y, x+y)$$

$$\text{Let } u = (x_1, y_1) \quad v = (x_2, y_2)$$

$$u+v = (x_1, y_1) + (x_2, y_2)$$

$$u+v = (x_1 + x_2, y_1 + y_2)$$

$$L(u+v) = L(x_1 + x_2, y_1 + y_2)$$

$$L(u+v) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$$

given that $u = (x_1, y_1)$

$$L(u) = L(x_1, y_1) = (x_1 + 1, y_1, x_1 + y_1)$$

$$L(v) = L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$$

$$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$$

Since $1 \neq 2$

Question No 3

Using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

then interpret to decode the message.

Answer:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

To decode the above message we have to break it into five vectors in \mathbb{R}^3 .

$$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}, \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}, \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix}, \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}, \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

So solve the equation.

$$L(x_1) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax_1$$

for x_1 . Since A is nonsingular

$$x_1 = A^{-1} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

Similarly...

$$x_2 = A^{-1} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

Using our corresponding between letters & number, we received the following message.

PhotoGRAPH PLANS
Answer.

Question no 4:-

$$(-1, 3, 2) \quad n = (0, 1, -3)$$

Solution:-

Equ of Plane

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

Given that:-

$$P = (x_0, y_0, z_0) = (-1, 3, 2)$$

$$n = (a, b, c) = (0, 1, -3)$$

$$\text{So, } 0(x - (-1)) + 1(y - 3) - 3(z - 2)$$

$$0 \frac{(x+1)}{b} + 1(y-3) - 3(z-2)$$

$$0 + y - 3 - 3z + 6$$
$$\Rightarrow y - 3z - 3 + 6$$

$$\Rightarrow y - 3z + 3 \text{ Ans.}$$



Question no :- 5

Eigen values.

Sol:-

We know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} 1x_1 \\ 1x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = 1x_1 \quad \text{--- (i)}$$

$$-2x_1 + 4x_2 = 1x_2 \quad \text{--- (ii)}$$

$$\text{So } x_1 - 1x_1 + x_2 = 0$$

$$= (1-1)x_1 + x_2 = 0$$

$$\begin{aligned} \text{Eq } -2x_1 + 4x_2 - 1x_2 &= 0 \\ &= -2x_1 + (4-1)x_2 = 0 \end{aligned}$$

$$\begin{bmatrix} 1-1 & 1 \\ -2 & 4-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Characteristic equation.

$$\begin{bmatrix} \lambda - 1 & 1 \\ -2 & \lambda - 1 \end{bmatrix} = 0$$

$$(\lambda - 1)(\lambda - 1) + 2 = 0$$

$$\lambda - 1 - \lambda + 1 + \lambda^2 + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$$

$$\lambda(\lambda - 3) - 2(\lambda - 3) = 0$$

$$(\lambda - 3)(\lambda - 2) = 0$$

$$(\lambda - 3)(\lambda - 2) = 0$$

$$\lambda - 3 = 0 \quad \lambda - 2 = 0$$

$$\lambda = 3 \quad \lambda = 2$$

are eigen values.

Now find the eigen vector
of $\lambda = 3$ put in (1) & (2)

$$\text{Then } x_1 + x_2 = 3x_1 \quad \text{--- (1)}$$

$$-2x_1 + x_2 = 0$$

$$\Rightarrow 2x_1 - x_2 = 0$$

$$\Rightarrow -2x_1 + 4x_2 = 3x_2 \quad \text{--- (i)}$$

$$\Rightarrow -2x_1 + x_2 = 0$$

$$\Rightarrow 2x_1 - x_2 = 0$$

$$x_1 = \frac{1}{2} x_2$$

$$\text{let } x_2 = \gamma$$

where $\gamma \neq 0$

$$\text{So } u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \gamma \\ \gamma \end{bmatrix}$$

eigen vector for $\lambda_2 = 2$ put
in eq 2.

$$x_1 + x_2 = 2x_1 \quad \text{--- (i)}$$

$$-2x_1 + 4x_2 = 2x_2 \quad \text{--- (ii)}$$

$$= -x_1 + x_2 = 0$$

$$\Rightarrow x_1 - x_2 = 0$$

$$= x_1 = x_2$$

$$= -2x_1 + 4x_2 = 2x_2 \quad \text{--- (1)}$$

$$= -2x_1 + 2x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$x_1 = \gamma$ then $x_2 = \gamma$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \end{bmatrix}$