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PAPER FLUID II

SUBMITTED TO ENGR. WAHEED

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Q no 1:

a) Define Drag with its components. write down the equations for friction drag co-efficient both in Laminar and turbulant boundry layer.

b) Derive equation for critical depth, critical velocity of rectangular section of a channel.

Answer:

Forced on Immersed Bodies:

A body which is wholly immersed in a homogeneous fluid may be subjected to two kind of forces araising from relative motion b/w body & fluid. These forces are termed as drag & lift, depending on forces either parallel or right angle to motion.

Drag force on submerged body can have two components.

1): **Pressure Drag: (Fp)**

It is equal to the Integration of component in direction of motion of all pressure forces exerted on surface of body.

$$F_p = C_p \cdot \rho \cdot \frac{v^2}{2} \cdot A$$

Where "C_p" depend's on "shape".

2): **Friction Drag:**

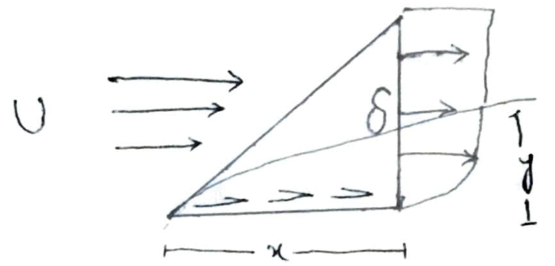
It is equal to the integration at component of all shear stress along the surface in direction of motion.

$$F_p = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL$$

Where C_f depend on "viscosity".

Friction drag on boundry layer:

$$\tau_o = \mu u \frac{df(\eta)}{\delta d(\eta)}$$



$$\tau_o = \frac{\mu U B}{\delta}$$

As we have $\tau_o = \rho u^2 \times \frac{d\delta}{dx}$

Compare both

$$\rho u^2 \times \frac{d\delta}{dx} = \frac{\mu u B}{\delta}$$

$$\delta d\delta = \frac{\mu B}{\rho u \alpha} dx$$

Integration on both sides;

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} dx + c$$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho u}} \qquad \because c = 0$$

$B = 1.63$, $d = 0.135$

$$\delta = \frac{4.91\ x}{\sqrt{R_n}}$$

Where "R_n" is local Reynald number.

As we have;

$$F_x = \int B U^2 \delta d$$

Where d is a function at boundry layer velocity distribution.

Now to find shear stress:

$$\tau = \frac{F_x}{A} = \frac{d F_x}{B d x} = \frac{d F_x}{B d x}$$

$$\tau_0 = \frac{\int B U^2 d\ d\delta}{B dx} = \frac{\int U^2 \alpha\ d\delta}{dx}$$

$$\tau_0 = \frac{\int U^2 \alpha\ d\delta}{dx} \longrightarrow \text{general Equation}$$

Laminar boundry Layer:

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad \text{—— eq ①}$$

$$\frac{y}{\delta} = \eta \qquad \Rightarrow y = \delta\eta$$

$$dy = \delta\, d\eta \longrightarrow \text{eq } (2)$$

$$\frac{u}{U} = f(\eta)$$

$$du = U\, df(\eta) \longrightarrow \text{eq } (3)$$

For Laminar flow;

$$\tau_0 = \mu \frac{du}{dy} \longrightarrow \text{eq } (4)$$

$$\tau_0 = \mu . \frac{U\, df(\eta)}{\delta\, d(\eta)}$$

$$\tau_0 = \frac{\mu U \beta}{\delta} \longrightarrow \text{eq } (5)$$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Comparing both

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U \beta}{\delta}$$

$$\delta\, d\delta = \frac{\mu \beta}{\rho U \alpha} dx$$

$$\frac{\delta^2}{2} = \frac{\mu\beta}{\rho U \alpha} x + c$$

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$$\delta = \frac{4.91}{\sqrt{R_x}} \longrightarrow \text{eq } 6$$

put eq 6 in eq 5

$$\boxed{\tau_o = 0.332 \frac{\mu u}{x} \sqrt{R_n}}$$

Now, $\boxed{F_f = B \int \tau_o \, dw}$

where $\tau_o = 0.332 \frac{\mu u}{x} \sqrt{R_n}$

$$\boxed{R_n = \frac{x u \rho}{\mu}}$$

then putting the values;

$$\boxed{F_f = 0.664 \sqrt{\rho \mu L U^3}}$$

As we have;

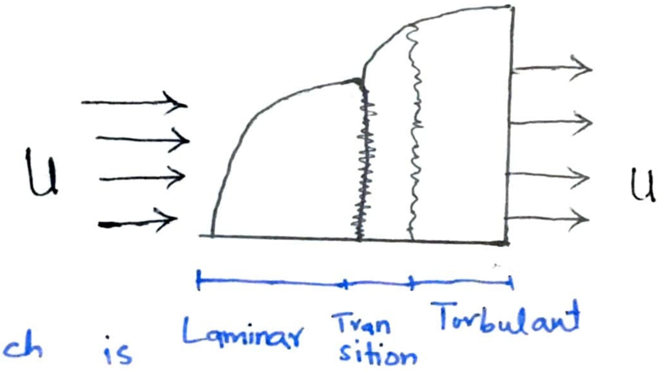
$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} BL$$

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{R_h}$$

For Laminar $R < 500,000$.

Turbulant Boundry Layer:

In this fig: show the velocity distribution of boundry layer which is steepers near walls and flatter through ad remainder of layer.



The shear stress is greater in turbulant than in Laminar thus;

$$\tau_o = \rho f \frac{v^2}{8}$$

Where "v" is the average velocity to obtain relative b/w average & maximum.

As we have;

$$\frac{V}{U_{maxi}} = \frac{1}{1+1.33\sqrt{f}} \qquad \because f = 0.028$$

$$\frac{V}{U_{maxi}} = \frac{1}{1+1.33\sqrt{0.028}}$$

$$U = 1.235V$$

$$\boxed{V = \frac{U}{1.235}}.$$

Now;

$$f = \frac{0.316}{(Rh)^{3/4}}$$

$$\tau_0 = f\rho\frac{V^2}{8}$$

$$\tau_0 = \frac{0.316}{\frac{D}{(\nu)}\left(\frac{U}{1.235}\right)^{1/4}}$$

$$\tau_0 = \frac{0.023\rho U^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}} \longrightarrow \text{eq } 1$$

As we have general equation

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \longrightarrow \text{eq } 2$$

Eq ① & eq ②

$x = 0 \quad , \quad \delta = 0$

$$\boxed{\delta = \left(\frac{0.0282}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} \cdot x}$$

$d = 0.0972$

$$\boxed{\delta = \frac{0.377}{(R_x)^{1/5}} \cdot x} \rightarrow \text{eq ③}$$

$$\tau_0 = 0.0587 \, \rho \frac{V^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

NOW;

$$F_f = B \int_0^L \tau_0 \, dx$$

$$\boxed{F_f = 0.0735 \, \rho \frac{V^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5} \cdot BL}$$

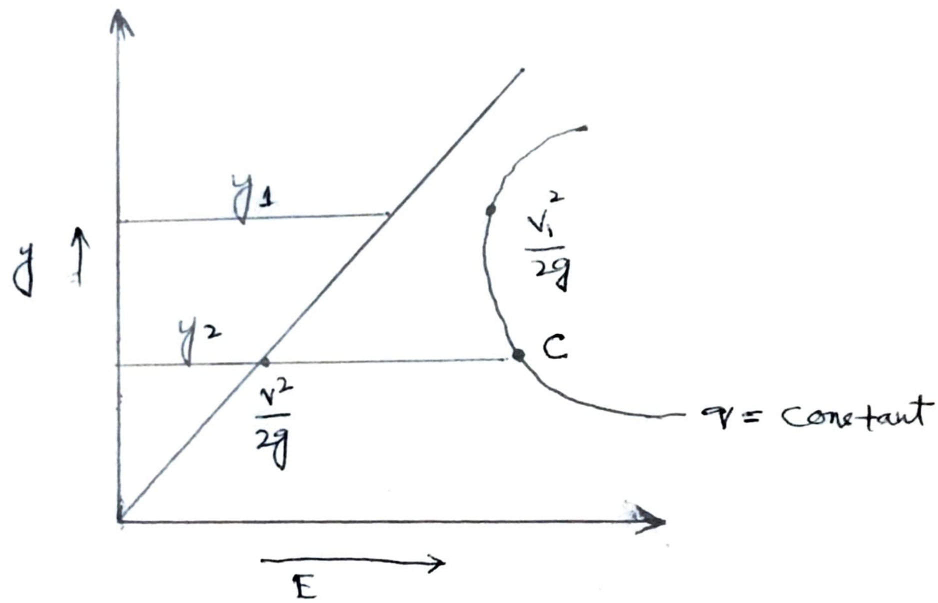
$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

$$C_f = \frac{0.0735}{(R)^{1/5}} \quad : (5,00,000 < R < 10^7)$$

For $R > 10^7$

$$C_f = \frac{0.455}{(\log_R)^{2.52}} \cdot$$

PART NO (B) :-



This is Specific Energy Equation :-

For particular q, there will be two kind of possible values of y for given E. The equation is

Cubic with three roots with third being negative giving no values. Thus two alternatives depth represents two totally different flow regimes; slow & deep. And an upper portion fast & shallow on lower portion.

Point represent dividing point b/w two regimes of flow. Thus for given "q" values of E is minimum & flow at this point is critical flow depth of flow at this point is critical depth & velocity at this point is critical velocity.

Thus relation of critical depth can be found as;

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum Specific energy

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 = \frac{q^2}{gy^3} \Rightarrow q^2 = gy^3$$

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $\quad q = vy \quad , V_c^2 = gy^3$

or $\quad V_c = \sqrt{gy_c}$

$$y_c = \frac{V_c^2}{g}$$

Now; $\quad \frac{y_c}{2} = \frac{V_c^2}{2g}$

$$E_{min} = y_c + \frac{V_c^2}{2g} \Rightarrow y_c + \frac{y_c}{2}$$

$$= \frac{3}{2} y_c \quad \text{OR} \quad y_{cr} = \frac{2}{3} \text{ constant.}$$

	Sub-critical	Critical	Super critical
Depth of flow	$y > y_c$	$y = y_c$	$y < y_c$
velocity slope	$V < V_c$ mild slope $S_o < S_c$	$V = V_c$ critical slope	$V > V_c$

Q NO 2:

Find depth of the rectangular channel if water flows at the rate of 3.5 m^3/s with bed slope of 0.0008 and $n = 0.0219$. Weidth of bed is your student ID number in mm. Also find the critical depth, critical velocity? Is flow sub-critical or super critical?

Given Data:

Flow rate $= Q = 3.5 \, m^3/sec$

Slope of bed $= S_o = 0.0008$

$n = 0.0219$

Width of ~~depth~~ bed $= 7614$ mm

$= 7.614$

Required:

1): To find depth of rectangular channel.

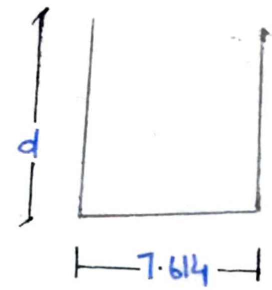
2): Is flow sub-critical or super critical = ?

Solution:

As we know that

$$\text{Area} = 7.614 \times d$$

$$= 7.614d$$



$$\text{Perimeter} = d + 7.614 + d$$

$$= 7.614 + 2d$$

$$\text{Hydraullic Radius} = Rh = A/p$$

$$= \frac{7.614d}{7.614 + 2d}$$

By using manning Equation

$$Q = \frac{1}{n} A R_h^{2/3} \cdot S_o^{1/2}$$

putting the values

$$3.5 = \frac{1}{0.0219} \times 7.614d \times \left(\frac{7.614d}{2d + 7.614}\right)^{2/3} \times (0.008)^{1/2}$$

$\boxed{d = 0.5688\ m}$

$AREA = 7.614\,(0.5688)$

$\boxed{Area = 4.3308\ m^2}$

~~PERIMETER = 7.614 + 2 ~~~~0.55~~ (4.3308)~~

$PERIMETER = 7.614 + 2\,(0.5688)$

$= 7.614 + 1.1376$

$\boxed{Perimeter = 8.7516}$

$$\text{Hydraullic Radius} = R_h = \frac{4.3308}{8.7516}$$

$\boxed{\text{Hydraullic Radius} = 0.494\ m}$

QNO 3:

Find the friction drag on one side of a smooth plate with 200 mm wide and 800 mm length placed longitudnally in stream of oil with specific gravity of 0.89. The undisturbed velocity is 5 m/s and kinematic viscosity is 0.93×10^{-4} m^2/s.

GIVEN DATA:

wide = d = 200 mm = 0.2 m

Length = L = 800 mm = 0.8 m

Specific gravity = G_s = 0.89

velocity = V = 5 m/sec

REQUIRED:

Kinematic viscosity = V_k = 0.93×10^{-4} m^2/s.

To find friction drag.

SOLUTION:

As we know that

Checking flow is Laminar or not.

By Reynold Number

$$R = \frac{DV}{\nu}$$

For smooth plate

$D = L$, $V = U$

So;

$$R = \frac{LU}{\nu}$$

putting the values

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}}$$

$$= 43010$$

$43010 < 500,000$ ⟶ Laminar

By using formula:

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL$$

where

$$C_f = \frac{1.328}{\sqrt{R}}$$

$$= \frac{1.328}{\sqrt{43010}} = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}}$$

$$0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 \text{ kg/m}^3$$

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL$$

putting the values

$$F_f = 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39\,N}.$$