Subject : Structural Analysis – II

Submitted To : Engr Adeel Khan

Submitted By : Muhammad Salman

ID # 7759

Date : 25/09/2020

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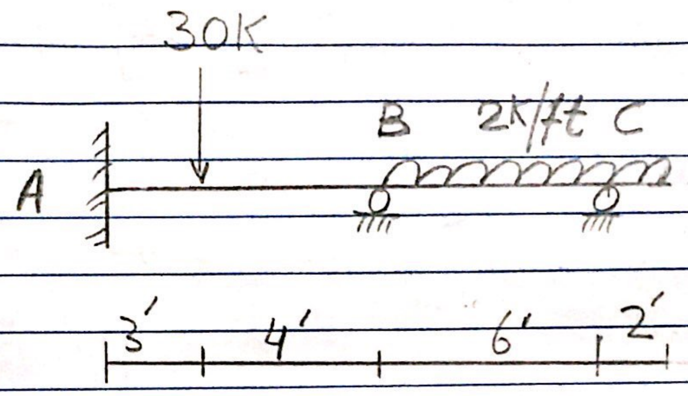
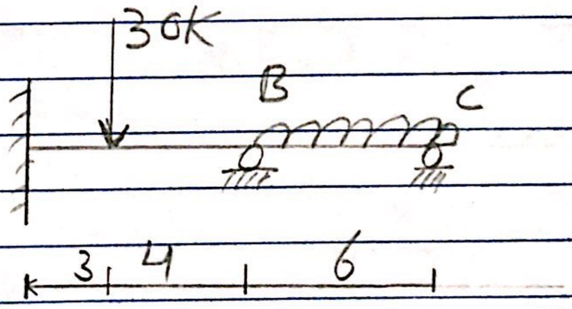
Q No. (01) Analyze the beam shown in figure - 1 by stiffness method. Assume EI is constant.

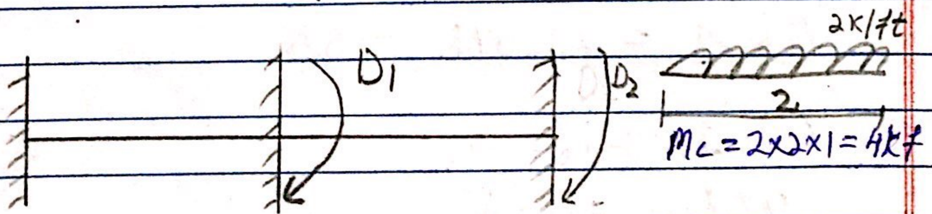


Solution:

Step # 01 K.I = 2° (Neglecting Axial effects)

Step # 02: Select the Unknown joint displacement.





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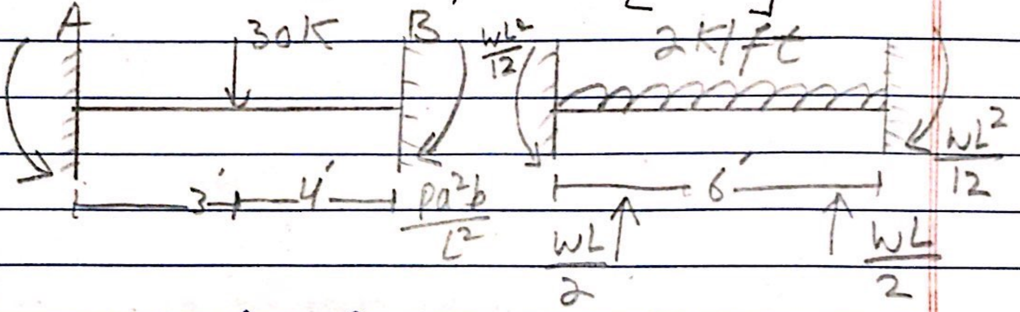
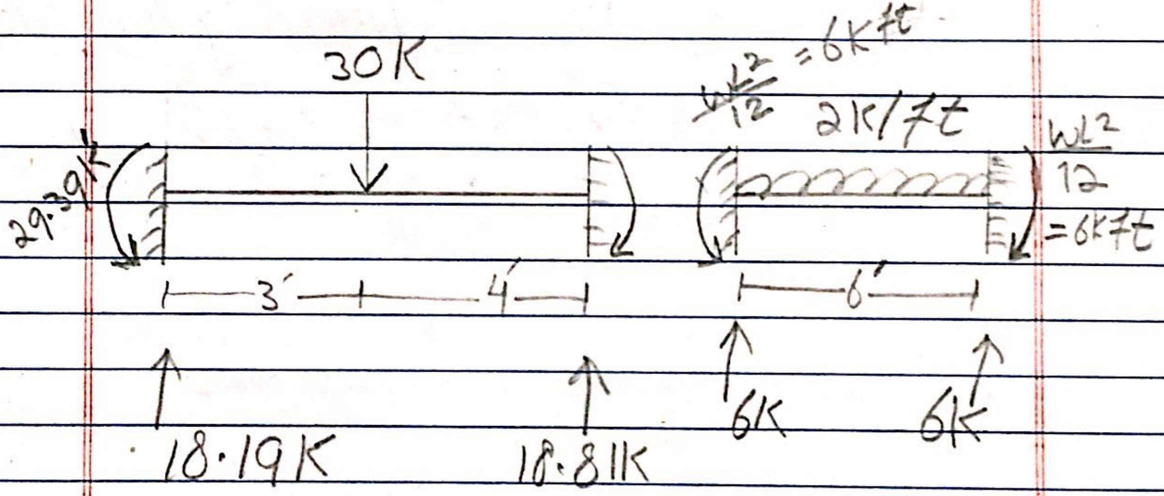
$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad [AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step # 03:

Compute [ADL] matrix



$$\frac{Pb^2(3a+b)}{L^3} \qquad \frac{Pa^2(a+3b)}{L^3}$$



$ADL_1 = 22.04K - 6K$

$ADL_1 = 16.04K$

$ADL_2 = 6K$

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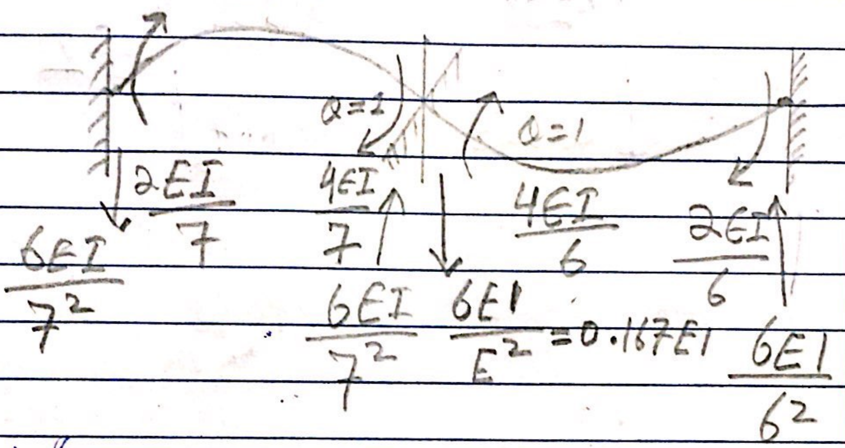
So $[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$

Step #04 compute [S] Matrix

(i)

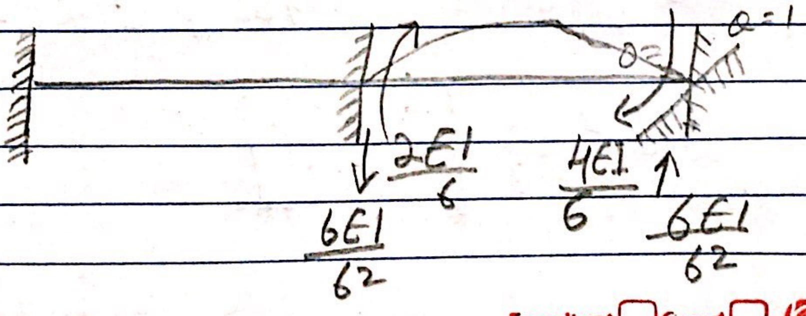


When $D_1 = 1,\ D_2 = 0$

$$S_{11} = \frac{4EI}{7} + \frac{4EI}{6} = 12.38EI$$

$$S_{21} = \frac{2EI}{6} = 0.333EI$$

(ii) When $D_1 = 0,\ D_2 = 1$



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$$S_{12} = \frac{2EI}{6} = 0.333EI$$

$$S_{22} = \frac{4EI}{6} = 0.667EI$$

Stiffness matrix $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$$S = \begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI$$

Step# 05: Compute the values of D_1 and D_2.

$$[AD] = [ADL] + [S][D]$$

$$[D] = [S]^{-1}[AD - ADL]$$

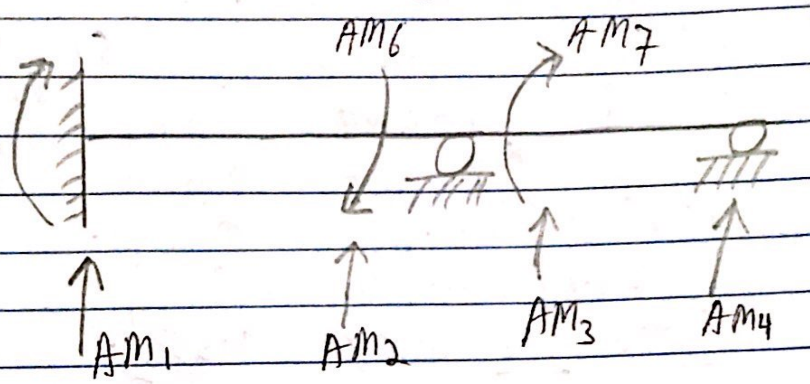
$$[D] = \left(\begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI \right)^{-1} \begin{bmatrix} 0 - (16.04) \\ 4 - (6) \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.933 & -0.466 \\ -0.466 & 1.732 \end{bmatrix} \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$[D] = \frac{1}{EI} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

Step # 06:

compute members and Actions.



$$[AM] = [AML] + [AMD][D]$$

$$[AML]; (AML) = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 18.19K \\ 11.81K \\ 6K \\ 6K \\ -29.39'K \\ 22.04'K \\ -6'K \end{bmatrix}$$

$$[AMD] = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{61} \\ AMD_{71} & AMD_{72} \end{bmatrix} = EI \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix}$$

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Now $[AM] = [AML] + [AMD][D]$

$$[AM] = \begin{bmatrix} 18.19 \\ 11.81 \\ 6 \\ 6 \\ -29.39 \\ 22.04 \\ -6 \end{bmatrix} + \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & 0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix} \cancel{EI} \times \frac{1}{\cancel{EI}} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

$$[AM] = \begin{bmatrix} 19.9K \\ 10.1K \\ 7.67K \\ 4.33K \\ -33.4K \\ 14.03K \\ -14.03K \end{bmatrix}$$

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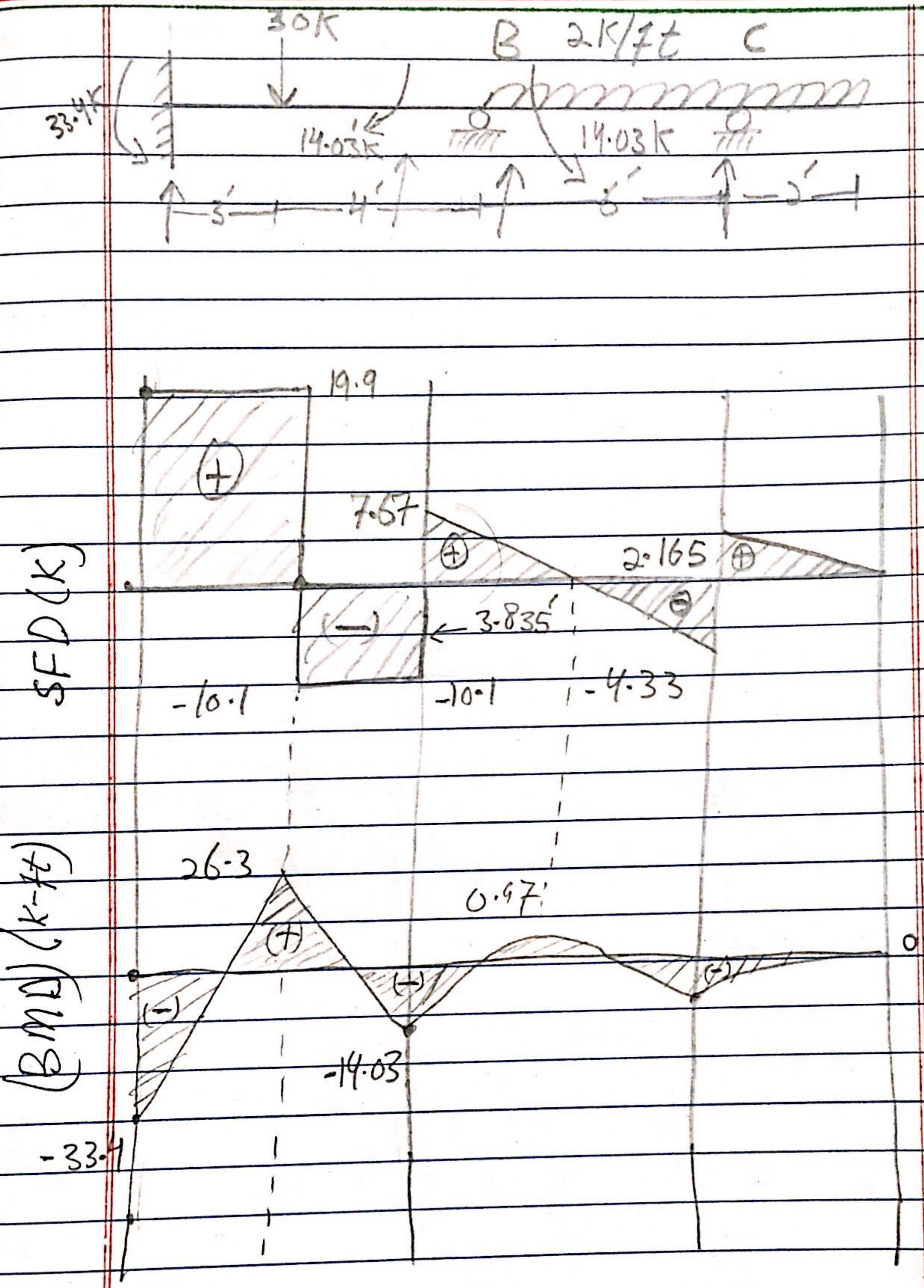

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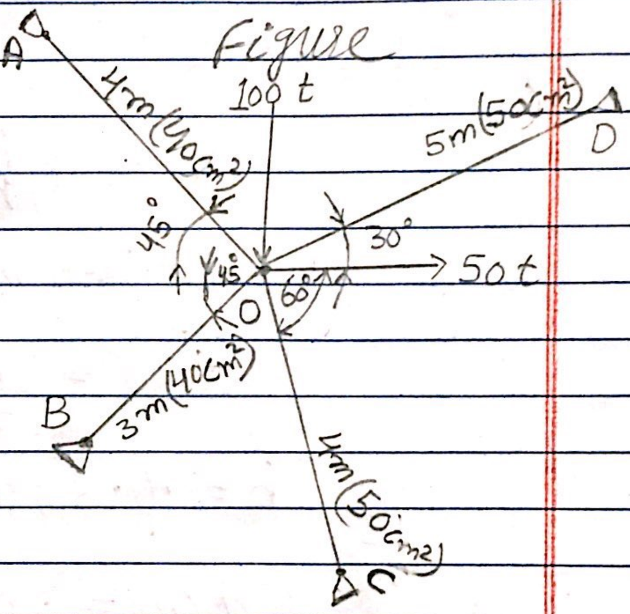
QNo. (02) Analyze the pin-jointed frame shown by stiffness method. Length of the members in 'm' and cross sectional area of the member in cm^2 are shown in FIG-3. Take $E = 2000\ t/cm^2$.

Figure



Solution:

For A:

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828m$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828m$$

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For B:

$$\sin 45^\circ = \frac{P}{3}$$

$$P = 2.12\,m$$

$$\cos 45^\circ = \frac{b}{h}$$

$$\Rightarrow b = 2.12\,m$$

For C:

$$\sin 30^\circ = \frac{P}{h}\; {\scriptstyle =5}$$

$$\Rightarrow P = 2.5\,m$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33\,m$$

NOW $EA(A) = 2000 \times 40 = 80{,}000\,t$

$EA(B) = 2000 \times 40 = 80{,}000\,t$

$EA(C) = 2000 \times 50 = 100{,}000\,t$

$EA(D) = 2000 \times 50 = 100{,}000\,t$

Step # 01:

$$K.I = 2j - 8$$

$$= 2(5) - 8 = 2^\circ$$

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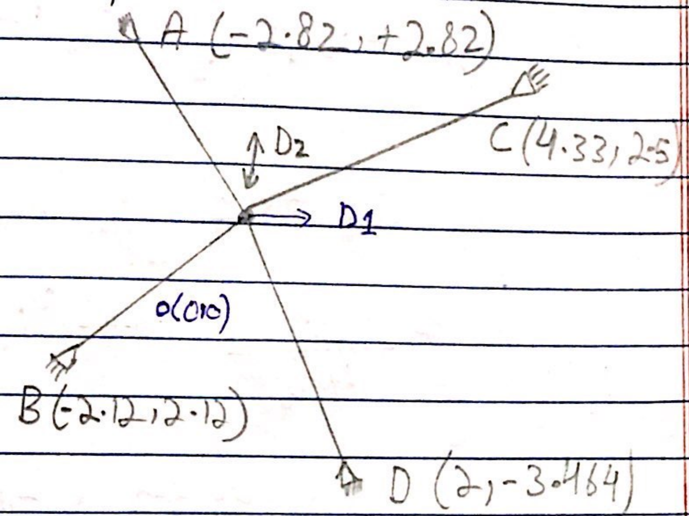
Step # 02 :

Select unknown joint displacement.



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 03 : $[AMD]_{4\times2}$ & $[S]_{2\times2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(x_c - x_i)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0.433) = -173.2$$

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$$AMD_{31} = \frac{100,000}{(500)^2} \times (0.433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

NOW $S_{11} = \sum_{1st}^{M} \frac{EA}{L^3} (X_K - X_j)^2$

$$= \frac{80,000}{(400)^3} \times (282)^2 + \frac{80,000}{(300)^3} \times (282)^2 + \frac{100,000}{(500)^3} \times (-433)^2$$

$$+ \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_K - X_j)(Y_K - Y_i)$$

$$= \frac{80,000}{(4000)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212)$$

$$+ \frac{100,000}{(500)^3} \times (-433)(0-250) + \frac{100,000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

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(ii) $D_1 = 0$, $D_1 = 1K'$

$$AMD = \frac{EA}{L^2}(Y_K - Y_i)$$

$$AMD_{12} = \frac{80{,}000}{400^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{400^2}(346) = 216.25$$

Now, $$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_K - Y_j)^2$$

$$= \frac{80{,}000}{400^3}(-282)^2 + \frac{80{,}000}{300^3}(212)^2 + \frac{100{,}000}{500^3}(-250)^3$$

$$+ \frac{100{,}000}{(400)^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

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Step #04 $[D] = [S]^{-1}[AD]$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step #06: $[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.113 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 & +30.46 \\ 22.29 & -40.70 \\ -20.49 & +21.6 \\ -14.79 & -46.71 \end{bmatrix}$$

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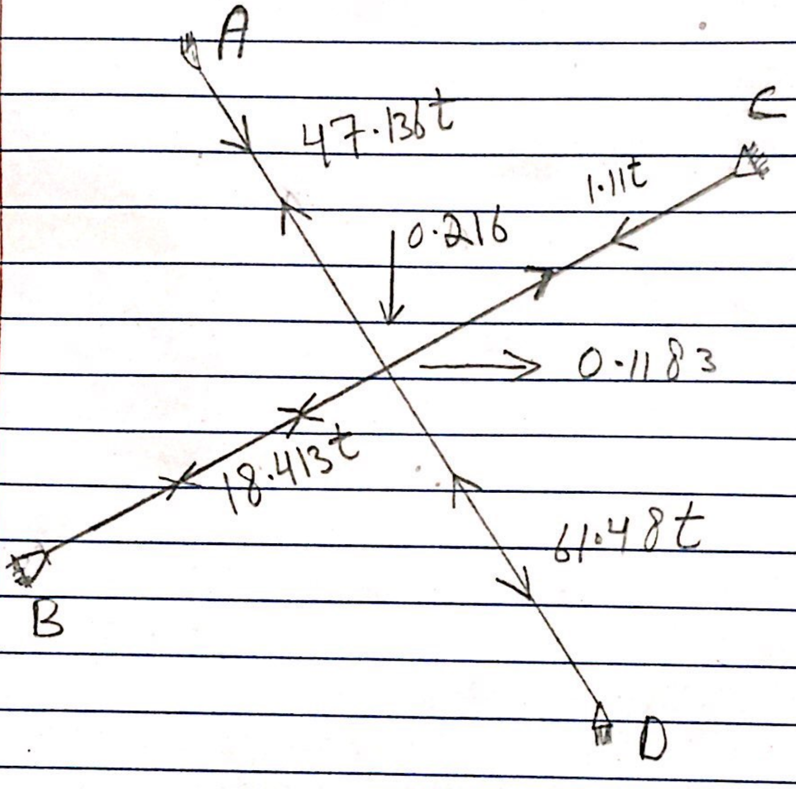
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



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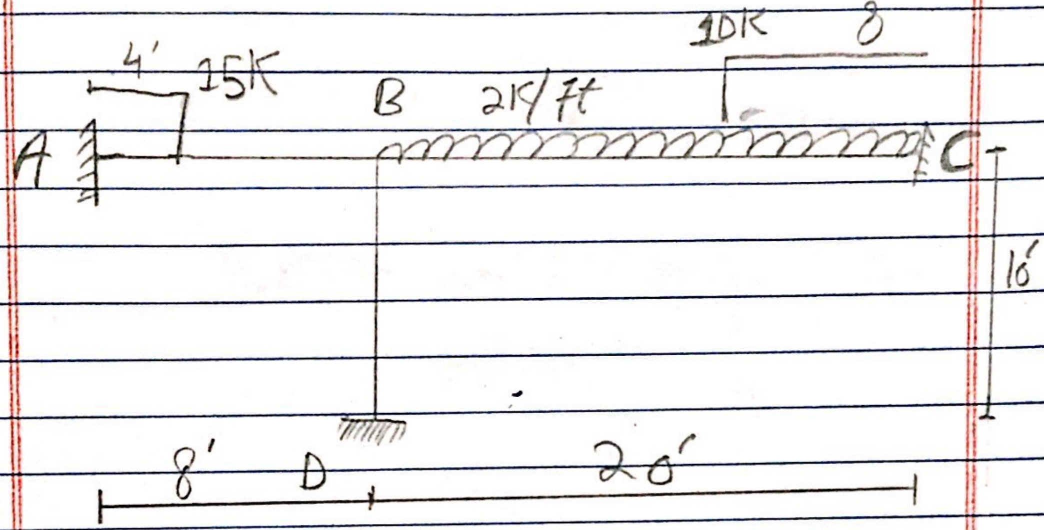
Q No (3):
Analyze the rigid-joint Fram shown FIG-(2) by Stiffness method. Assume EI is constant.

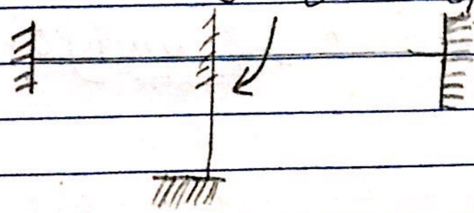


Solution:

Step # 01:
Determine Kinematic Indeterminacy.

K.I = 1°

Step # 02:
Determine Unknown joint Displacement.



$[D] = [?]$

$[AD] = [0]$

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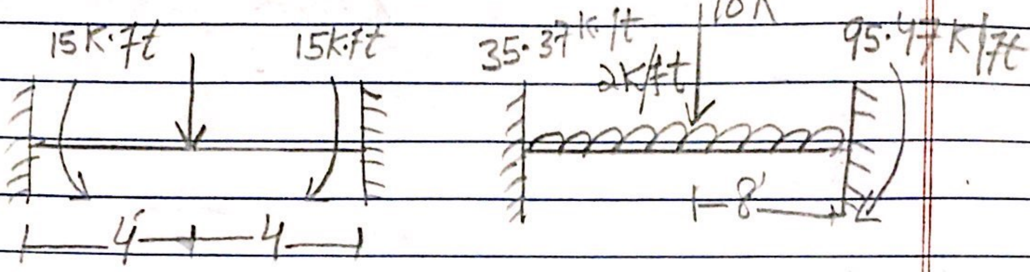
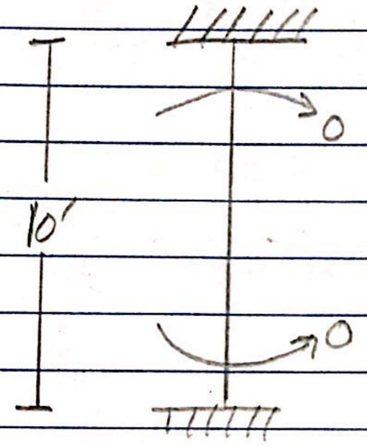
Step # 03:

Compute (ADL) matrix





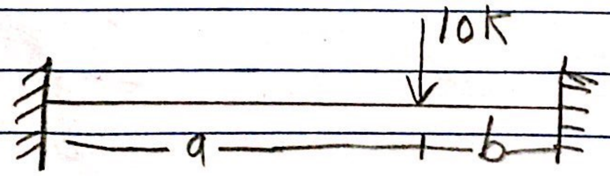
⇒ Point Load at center:

$$\frac{PL}{8} = \frac{(15)(8)}{8} = \boxed{15 \text{ Kip·ft}}$$

⇒ Uniformly Distributed Load:

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ K·ft}$$

⇒ Point Load (not at mid):



For Left End:

$$\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2\,k\cdot ft$$

For Right End:

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8\,k\cdot ft$$

So total moment at left End.

$$19.2 + 66.7 = 85.87\,k\cdot ft$$

Similarly at right End:

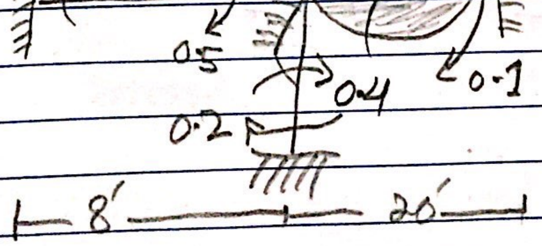
$$28.8 + 66.67 = 95.47\,k\cdot ft$$

So $[ADL] = -85.87 + 15 \Rightarrow -70.87\,k\cdot ft$

Step #04: Determine $[S]$ Matrix

$$S = [S_{11}]$$

Now $D = 1K$



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$$\Rightarrow \frac{4EI}{8} = 0.5 \quad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \quad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \quad \frac{2EI}{10} = 0.2$$

$$[S] = (0.5 + 0.4 + 0.2)\,EI$$

$$= 1.1\,EI$$

$$[S] = 1.1\,EI$$

Step # 05: Compute [D] Matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42]\,\frac{1}{EI}$$