Q#01 (12900)

$$\frac{(x+yi)}{i} = 7+9i \rightarrow eq\ (1)$$

What is the value of

$(x+yi)(x-yi)$

Sol:

$eq\ (1) \Rightarrow \quad x+yi = 7i+9i^2$

$x+yi = 7i-9 \quad \because i^2=-1$

$x+yi = -9+7i$

By comparin we can see that

$x=-9 \quad y=7$

So value of

$(x+yi)(x-yi) = (-9+7i)(-9-7i)$

Q#02

$(x+iy)(2+i) = 3-i$

To find x and y.

Sol:-

$(x+iy)(2+i) = 3-i$

(12900)

$2x + ix + 2yi + yi^2 = 3 - i$

$2x + ix + 2yi - y = 3 - i$

$(2x - y) + i(2y + x) = 3 - i$

Comparing the coefficients

$2x - y = 3 \rightarrow eq\ 1$

$2y + x = -1 \rightarrow eq\ 2$

$2x - y = 3$
$x + 2y = -1$

multiply eq 1 with 2 and ~~sub~~ add it from eq (ii)

$4x - 2y = 6$
$x + 2y = -1$

$5x = 5$

$\boxed{x = 1}$

Putting $x=1$ in eq 1

$2(1) - y = 3$

$2 - 3 = y$

$\boxed{y = -1}$

Q#3

$2z^2 - 2iz - 5 = 0 \quad z \in C$

Sol:

$$2z^2 - 2iz - 5 = 0$$

$$2z^2 - 2iz = 5$$

$$z^2 - iz = \frac{5}{2}$$

Add $\left(\frac{i}{2}\right)^2$ on both sides

$$z^2 - iz + \left(\frac{i}{2}\right)^2 = \frac{5}{2} + \left(\frac{i}{2}\right)^2$$

$$\left(z - \frac{i}{2}\right)^2 = \frac{5}{2} + \frac{i^2}{4}$$

$$= \frac{5}{2} + \frac{(-1)}{4}$$

$$= \frac{5 \times 2}{2 \times 2} - \frac{1}{4}$$

$$\left(z - \frac{i}{2}\right)^2 = \frac{10-1}{4}$$

$$\left(z - \frac{i}{2}\right)^2 = \frac{9}{4}$$

$$\sqrt{\left(z-\frac{i}{2}\right)^2} = \sqrt{\frac{9}{4}}$$

$$z-\frac{i}{2} = \pm\frac{3}{2}$$

Hence

$$z_1 = \frac{i}{2}+\frac{3}{2}$$

$$z_2 = \frac{i}{2}-\frac{3}{2}$$

Sol set $\Rightarrow \left\{\frac{i}{2}+\frac{3}{2}, \frac{i}{2}-\frac{3}{2}\right\}$

Q # 04

$4-\sqrt{5}i$ in polar form

Sol:

polar form $= r(\cos\theta + i\sin\theta)$

$$r = \sqrt{x^2+y^2}$$

$\because x = 4$

$y = -\sqrt{5}$

$$r = \sqrt{4^2+(-\sqrt{5})^2}$$

$$= \sqrt{16+5}$$

$$= \sqrt{21}$$

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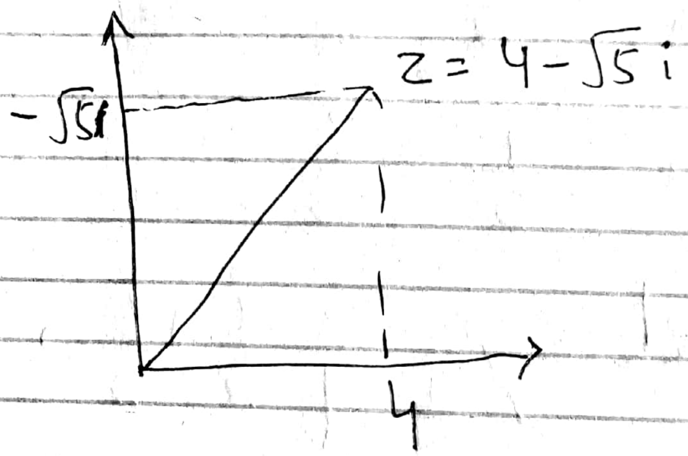
$$\tan\theta = \frac{4}{-\sqrt{5}}$$

$$\theta = -60.79^\circ \quad \text{or} \quad 29.205^\circ$$

so

polar form

$$z = \sqrt{21}\left(\cos(29.205) + i\sin(29.205)\right)$$



Q#5

$$\lim_{z\to 8} \frac{2z^2-17z+8}{8-z}$$

Sol:

$$\lim_{z\to 8} \frac{2z^2-17z+8}{8-z} = \frac{2(8)^2-17(8)+8}{8-8}$$

$$= \frac{0}{0}$$

As it is indeterminate form, so first we will solve the equation

$$\lim_{z\to 8} \frac{2z^2-17z+8}{8-z} = \lim_{z\to 8} \frac{2(z^2-17/2\,z+4)}{8-z}$$

$$= \lim_{x\to 8} \frac{2\cancel{(8-z)}(0.5-z)}{\cancel{8-z}}$$

$$= \lim_{x\to 8} 2(0.5-z)$$

$$= 2(0.5-8)$$

$$\lim_{z\to 8} \frac{2z^2-17z+8}{8-z} = -15$$

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Q #6

① $f(x) = (\ln x)^4$

$\frac{d}{dx} f(x) = \frac{d}{dx} (\ln x)^4$

$f'(x) = 4(\ln x)^{4-3} \frac{d}{dx} \ln x$

$= 4(\ln x)^3 \cdot \frac{1}{x}$

$= \frac{4}{x} (\ln x)^3$

(ii) $g(x) = x^2 \ln x$

$\frac{d}{dx} g(x) = \frac{d}{dx} x^2 \ln x$

$= x^2 \frac{d}{dx} \ln x + \ln x \frac{d}{dx} x^2$

$= x^2 \cdot \frac{1}{x} + \ln x \cdot (2x)$

$= x + 2x \ln x$

$g'(x) = x(1 + 2\ln x)$