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Section :- C.

Q.NO :- 1.

Solution:-

$$k = \frac{3EI}{L^3}$$

$$k = \frac{3 \times 29000 \frac{k}{in^2} \times 150 in^4}{(10 \times 12 in)^2}$$

$$k = 7.55 \, k/in \quad \boxed{= 90625 \, lb/ft}$$

$$m = \frac{7746 \, lb sec^2}{32.2 \, ft}$$

$$\boxed{m = 240.55 \text{ slug}}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\omega_n = \sqrt{\frac{90625}{240.55}}$$

$$\boxed{\omega_n = 19.97 \text{ rad/sec}}$$

$$T_n = \frac{2\pi}{\omega_n}$$

$$T_n = \frac{2(3.14)}{19.97}$$

$$\boxed{T_n = 0.314 \text{ sec}}$$

Substituting the Corresponding values eq-1:-

$$90625u + 240.55\ddot{u} = 0$$

where "k" is in lb/ft & m is in lb sec/ft

General solution of EOM is :-

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{48} \text{ ft} \quad \text{and} \quad \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.97t) + 0 = \frac{1}{24} \times \cos(19.97)$$

Equivalent static force at any time "t" is :-

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.97t)}{24}$$

$$\boxed{f_s(t) = 3776 \cos(19.97t)}$$

Amplitude of dynamic displacement, U_0 for undamped free vibration is :-

$$U_0 = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]} = \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right)} = \frac{1}{24}\ ft$$

Amplitude of equivalent satic force, f_{so}

$$kU_0 = 90625 \times \frac{1}{24}$$

$$\boxed{kU_0 = 3776\ lb}$$

Q.NO:-2:

Solution:-

EOM for damped free vibration is:-

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{---} (1)$$

It is known in Prob: 1:-

$k = 90625$ lb/ft , $m = 240.55$ lb.sec²/ft

$$C = \xi \times 2 m \omega_n$$

$$C = 2 \times 240.55 \times 19.97 \times 0.013$$

$$C = 124.89 \text{ lb.sec/ft}$$

put k, c, m in equ. 1:-

$$90625u + 124.89\dot{u} + 240.55\ddot{u} = 0$$

Solution to EOM for damping free vibration:-

$$u(t) = e^{-\xi\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\xi\omega_n\right]\sin(\omega_D t)\right]$$

$$W_D = 19.97 \text{ rad/sec}$$

$$u(t) = e^{-0.013 \times 19.97t}\left[\frac{1}{24} \times \cos(19.97t) + \frac{1}{19.97} \times \left[0 + \frac{1}{24} \times 0.013 \times 19.97 \times \sin(19.97t)\right]\right]$$

$$u(t) = e^{-0.259t}[0.0416 \times \cos(19.97t) + 0.0005 \sin(19.97t)]$$

$$fs(t) = k \cdot u(t) = 90625 \times u(t)$$

$$fs(t) = e^{-0.259t}[3770 \cos(19.97t) + 45.31 \sin(19.97t)]$$

Q.NO:-3

Solution:-

$$U_1 = 7.74''$$

After $j = 7$, $u_{j+1} = U_6 = 0.9''$

a :- Damping:-

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{U_1}{u_{j+1}}\right]$$

$$\Rightarrow 7 = \frac{1}{2\pi\xi} \ln\left(\frac{7.74}{0.9}\right)$$

$$\xi = \frac{2.15}{2\pi(7)}$$

$$\boxed{\xi = 0.0488 = 4.88\%}$$

b :- Tn :-

7 cycles vibration are completed in 3.57 secs.

Now,

Time required for 1 cycle

$$T_D = \frac{3.57}{7}$$

$$\boxed{T_D = 0.51 \text{ sec}}$$

Now,

$$\omega_D = \omega_n\sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = 2\pi / \omega_n\sqrt{1-\zeta^2}$$

$$T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

$$T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$\boxed{T_n = 0.5093 = 0.51 \text{sec}}$$

C :- K = ?

$$K = \frac{60 \times \cos 60^\circ}{2} = 15 \text{ k/in}$$

$$\boxed{K = 18000 \text{ lb/ft}}$$

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d:- weight of tank, $w = ?$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\left(\frac{W}{g}\right)}} = \sqrt{\frac{k \times g}{W}}$$

$$\omega^2_n = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{\omega^2_n}$$

Also,

$$\omega_n = \frac{2\pi}{T_n}$$

$$W = \frac{k \times g}{\frac{4\pi^2}{T_n^2}}$$

$$W = k \times g \times \frac{T_n^2}{4\pi^2}$$

$$W = 18000 \times 32.2 \times \frac{(0.51)^2}{4\pi^2}$$

$$\boxed{W = 3818.64 \text{ lb} = 3.81 K}$$

e :- $C = ?$

It is known that $\zeta = \frac{C}{2m\omega_n}$

$$C = \zeta \times 2m\omega_n$$

$$C = \zeta \times 2m(2\pi/T_n)$$

$$C = 0.0488 \times 2 \times \left(\frac{3818.64}{32.2}\right)\left(\frac{2\pi}{0.51}\right)$$

$$\boxed{C = 142.59 \text{ lb}\cdot\text{sec/ft}}$$

f :- NO of cycle to reduce displacement amplitude from 7.74" to 0.5", $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$j = \frac{1}{2\pi \times 0.0488} \ln\left(\frac{7.74''}{0.5''}\right)$$

$$\boxed{j = 8.93 \text{ or } 9 \text{ cycles}}$$