Department of Electrical Engineering
Mid term exam
Date: 21/08/2020

Course Details

Course Title:	Complex & Multivariable Calculus	**Module:**	03
Instructor:	Engr. Mujtaba Ihsan Sir	**Total Marks:**	30

Student Details

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Q1.	(a)	**Express** $-3 + 4i$ in polar form and represent it graphically.	Marks 06 +06
	(b)	Given that $u(x, y) = (x^3 + 3xy^2) + i(3x^2y - y^3)$. **Determine** if the function is analytic or not?	CLO 1
Q2.		If $z_1 = 5 + 3i$ and $z_2 = 4 - 2i$, **evaluate** $z_1 z_2$ and z_1/z_2	Marks 06
			CLO 1
Q3.		Given that $u(x, y) = x^3 - 3xy^2 - 5y$. **Determine if the function is harmonic?** If so, **evaluate** the conjugate harmonic function of u.	Marks 08
			CLO 1
Q4.		**Differentiate** the following:	Marks 04
	i.	$f(z) = z^2 / (5z+2)$	
	ii.	$f(z) = 3z^4 - 5z^3 + 2z + 1$	CLO 1

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=> Question No (1)

=> Part (A)

Express $-3+4i$ in Polar form and represent it graphically.

Solution:- ⓐ Express $3+4i$ in polar form

$$\text{Let} \quad z = 3+4i$$

$$|z| = \sqrt{x^2+y^2}$$

$$|z| = \sqrt{(3)^2+(4)^2}$$

$$|z| = \sqrt{9+16} = \sqrt{25}$$

$$\boxed{|z| = 5}$$

$$\text{As} \quad \tan\phi = y/x$$

$$\Rightarrow \quad \phi = \tan^{-1}(y/x)$$

$$\phi = \tan^{-1}(4/3)$$

$$\Rightarrow \quad \phi = 53^\circ$$

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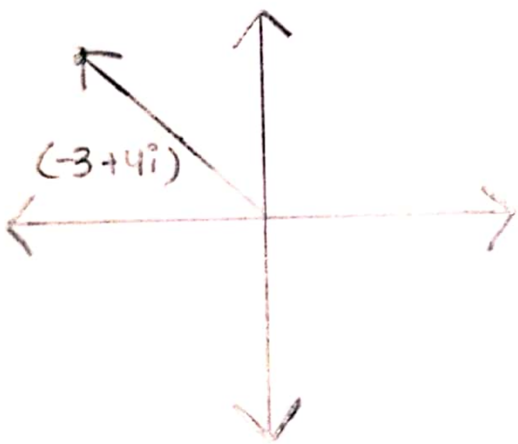
$\Rightarrow$ $z = r(\cos\phi + i\sin\phi)$

$z = 5\left(\cos\left(\frac{53\pi}{180}\right) + i\sin(53\pi/180)\right)$

$\Rightarrow$ Graph



$\longleftrightarrow$

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⇒ Question No (1)

⇒ Part (B)

Given that $u(x,y) = (x^3 + 3xy^2) + i(3x^2y - y^3)$

Determine if the function is analytical or not?

Solution:- Since $u(x,y) = x^3 - 3xy^2$ and

$v(x,y) = 3x^2y - y^3$ we have

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2, \quad \frac{\partial u}{\partial y} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial v} = -6xy, \quad \frac{\partial v}{\partial x} = 6xy$$

Hence the Cauchy-Riemann equation are satisfied and $f(z)$ is analytic The complex deravative of

$f(z)$ is $f'(z) = 3x^2 - 3y^2 + i(6xy)$

$$\boxed{= 3z^2}$$ Answer.

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=> Question No (2)

if $z_1 = 5+3i$ and $z_2 = 4-2i$, evaluate $z_1 z_2$ and z_1 / z_2

Solution :- $z_1 z_2 = (5+3i)(4-2i)$

$$z_1 = (5+3i)$$

$$z_2 = (4-2i)$$

$$z_1 z_2 = (5+3i)(4-2i)$$

$$z_1 z_2 = 20 - 10i + 12i - 6i^2$$

$$= 20 + 2i - 6(-1)$$

$$= 20 + 2i + 6$$

$$= 20 + 6 + 2i$$

$$= \boxed{26 + 2i}$$ Answer.

$$\Rightarrow \frac{z_1}{z_2} = \frac{5+3i}{4-2i}$$

$$= \frac{5+3i}{4-2i} \times \frac{4+2i}{4+2i}$$

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$$\Rightarrow \frac{20+10i+12i+6i^2}{16+4}$$

$$= \frac{20+22i+6(-1)}{20}$$

$$= \frac{20-6+22i}{20}$$

$$= \frac{14+22i}{20}$$

$$= \boxed{\frac{14}{20} + i\frac{22}{20}}$$ Answer.

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⇒ Question No (3)

Given that $u(x,y) = x^3 - 3xy^2 - 5y$. Determine if the function is harmonic?

if so evaluate the conjugate harmonic function of u.

⇒ Solution

ⓐ $\frac{\partial u}{\partial x} = 3x^2 - 3y^2$, $\frac{\partial^2 u}{\partial x^2} = 6x$, $\frac{\partial u}{\partial y} = -6xy - 5$ $\frac{\partial^2 u}{\partial y^2} = -6x$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6x = 0$$

ⓑ $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 3x^2 - 3y^2$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = 6xy + 5$

Integrating the first one, $v(x,y) = 3x^2y - y^3 + h(x)$

and $\frac{\partial v}{\partial x} = 6xy + h'(x)$, $h'(x) = 5$, $h(x) = 5x + C$

Thus $v(x,y) = 3x^2y - y^3 + 5x + C$

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=> Question No (4)

=> Part (A)

Differentiate the following

(i) $f(z) = z^2 / (5z+2)$

Solution :- $f(z) = \dfrac{z^2}{5z+2}$

Take Derivative

$$f'(z) = \frac{(5z+2)\cdot 2z - z^2(5)}{(5z+2)^2}$$

$$f'(z) = \frac{10z^2 + 4z - 5z^2}{(5z+2)^2}$$

$$\boxed{f'(z) = \frac{5z^2 + 4z}{(5z+2)^2}}$$

Answer.

=> Question No (4)

=> Part (B)

$$f(z) = 3z^4 - 5z^3 + 2z + 1$$

Solution :- $F(z) = 3z^4 + 5z^3 + 2z + 1$

Take derivative

$$f'(z) = \frac{d}{dz} 3z^4 + \frac{d}{dz} 5z^3 + \frac{d}{dz} 2z + \frac{d}{dz} 1$$

$$f'(z) = 4 \times 3z^{4-1} + 3 \times 5z^{3-1} + 2 + 0$$

$$F'(z) = 12^{4-1} + 15z^{3-1} + 2$$

$$\boxed{F'(z) = 12z^3 + 15z^2 + 2}$$ Answer.

Thank You.