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Section = "A"

Paper = Calculas.

The function $g(t)$ is defined by

$$g(t) = 0 \qquad t < 0$$
$$t^2 \qquad " \qquad 0 \le t \le 3$$
$$2t+3 \qquad 3 < t \le 4$$
$$12 \qquad t > 4.$$

(a) State any point of discontinuity

(b) Find, if they exist

(i) $\lim_{t \to 3} g$

Sol: To check possibility of the dis-continuity of the function it at $t = 0$ & 4.

→ first at $t = 0$

$$g(t) = t^2$$
$$g(0) = 0^2 = 0$$

for R.H.L

$$\text{Lim } g(1+h) = \lim_{h \to 0} (1+h)^2.$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply Limits

$$\Rightarrow 1 + 0^2 + (2(0))$$
$$= 1.$$

for L.H.L

$$\lim_{h\to 0} g(1+h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3$$

$$= \lim_{h\to 0} 2-2h+3$$

Apply Limit

$$2-2(0)+3$$
$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

⇒ Now at $t = 4$

$$g(4) = 2(4)+3$$
$$= 8+3$$
$$= \boxed{11}$$

For R.H.L

$$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$$

$$\lim_{h\to 0} 2+2h+3$$

Apply Limit

$$2+2(0)+3 \Rightarrow \boxed{5}$$

For L.H.L

$$\lim_{h \to 0} g(1-h) = 12$$

$\cancel{g(t-4}$ $g(4) = R.H.L + L.H.L$

point of discontinuity is at $t = \underline{4}$

B Find, if They exist.

(i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) \Rightarrow \lim_{h \to 3} (1+h)^2$

$$= \lim_{h \to 3} 1 + h^2 + 2h$$

Apply Limits

$$= 1 + 3^2 + 2(3) \Rightarrow 16$$

L.H.L $\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t+3$

$$= \lim_{h \to 3} 2(1-h)+3$$

$$= \lim_{h \to 3} 2-2h+3$$

Apply Limit

$\Rightarrow \quad 2-2(3)+3$

$= \quad 2-6+3$

$= \quad -1$

R.H.L $\neq$ L.H.L

(do not exist since L.H.L is −ve)

Q = 2

Find The maclarin's Series for
$y(x) = x^2 + \sin x$.

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

Putting $x = 0$ in all function

Let $y(x) = x^2 + \sin x$	$y(0) = 0$
$y'(x) = 2x + \cos x$	$y'(0) = 1$
$y''(x) = 2 - \sin x$	$y''(0) = 2$
$y'''(x) = -\cos x$	$y'''(0) = -1$

Putting value in Formula

$$\cancel{= 0 (}$$

$$= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1)$$

$$= 0 + x + x^2 - \frac{x^3}{6}$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} \text{ Ans.}$$

Q = 3

(i) Find y'' given

$$1 + xy = x^2 + y^2$$

Solution:- $1 + xy = x^2 + y^2$ —(1)

diff eq (1) b/s w.r.t "x"

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$\Rightarrow 0 + \left(x \cdot \frac{d}{dx}y + y \cdot \frac{d}{dx}x\right) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot \frac{d}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$= x \cdot y' + y = 2x + 2y \cdot y$$

$$= x \cdot y' - 2y \cdot y' = 2x - y$$

$$= y'(x - 2y) = -(2x - y)$$

$$= \boxed{y' = \frac{2x - y}{x - 2y}}$$

Differ Again w.r.t 'x'

$$y'' = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$= \frac{(x-2y)\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2)\left(-\frac{dy}{dx}\right)-(2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(2x-4y)(-y')-(2x-4x\frac{dy}{dx}-y+2yy')}{(x-2y)^2}$$

$$y''(x-2y)^2 = 2xy'+2yy'-2x+y$$

$$y''(x-2y)^2 = \left(\frac{2x-y}{x-2y}\right)(2x-2y)-2x+y$$

$$\boxed{y'' = \frac{\left(\frac{2x-y}{x-2y}\right)(2x+2y)-2x+y}{(x-2y)^2}}$$

Ans..

Q = 3.

B Parts

Find y' by using logarithmic differentiation

$$y = x^3 (1+x)^9 e^{6x}$$

Solution:- Taking ln on b/s

$$\ln y = \ln (x^3 (1+x)^9 \cdot e^{6x})$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + 6x \ln e$$

Diff w.r.t x.

$$\frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln x + \frac{d}{dx} 9 \ln (1+x) + \frac{d}{dx} 6x \ln e$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}} \cdot 6$$

$$\frac{dy}{dx} = y \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

$$\frac{dy}{dx} = \underline{x^3 (1+x)^9 e^{6x} \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)}$$

Ans.