

Name = Fawad Ahmad

ID = 7783.

Section = "A"

Paper = Calculas -

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq t \leq 3 \end{cases}$$

$$\begin{cases} 2t + 3 & 3 < t \leq 4 \\ 12 & t > 4. \end{cases}$$

(a) State any point of discontinuity

(b) Find, if they exist

(i) $\lim_{t \rightarrow 3} g$

Sol: To check possibility of the discontinuity of the function it at $t = 0$ & 4.

→ first at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

for R.H.L

$$\lim_{h \rightarrow 0} g(1+h) = \lim_{h \rightarrow 0} (1+h)^2$$

$$= \lim_{h \rightarrow 0} 1 + h^2 + 2h$$

Apply Limits

$$\Rightarrow 1 + 0^2 + (2 \cdot 0) = 1$$

for L.H.L

(2)

$$\lim_{h \rightarrow 0} g(1+h) = 2t+3$$

$$= \lim_{h \rightarrow 0} 2(1-h) + 3$$

$$= \lim_{h \rightarrow 0} 2 - 2h + 3$$

Apply Limit

$$= \cancel{2} - 2(0) + 3$$

$$= (5)$$

$$R.H.L \neq L.H.L = g(t) = 5$$

⇒ Now at $t = 4$

$$g(4) = 2(4) + 3$$

$$= 8 + 3$$

$$= \boxed{11}$$

For R.H.L

$$\lim_{h \rightarrow 0} g(1+h) = \lim_{h \rightarrow 0} 2(1+h) + 3$$

$$\lim_{h \rightarrow 0} 2 + 2h + 3$$

Apply Limit

$$2 + 2(0) + 3 \Rightarrow \boxed{5}$$

(3)

For L.H.L

$$\lim_{h \rightarrow 0} g(1-h) = 12$$

$$\cancel{g(1-4)} \quad g(4) = \text{R.H.L} + \text{L.H.L}$$

point of discontinuity is at $t = \underline{4}$

(B) Find, if they exist.

(i) $\lim_{t \rightarrow 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \rightarrow 3} g(1+h) \Rightarrow \lim_{h \rightarrow 3} (1+h)^2$

$$= \lim_{h \rightarrow 3} 1 + h^2 + 2h$$

Apply limits

$$= 1 + 3^2 + 2(3) \Rightarrow 16$$

L.H.L $\lim_{h \rightarrow 3} g(1-h) = \lim_{h \rightarrow 3} 2t + 3$

$$= \lim_{h \rightarrow 3} 2(1-h) + 3$$

$$= \lim_{h \rightarrow 3} 2 - 2h + 3$$

Apply limit

(4)

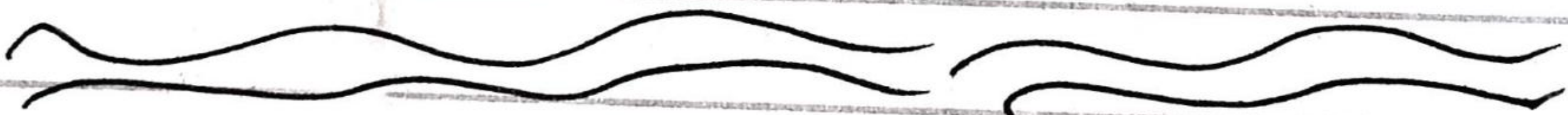
$$\Rightarrow 2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -1$$

R.H.L \neq L.H.L

(do not exist
since L.H.L is -ve)



$$C = 2$$

Find the Maclaurin's Series for
 $y(x) = x^2 + \sin x$.

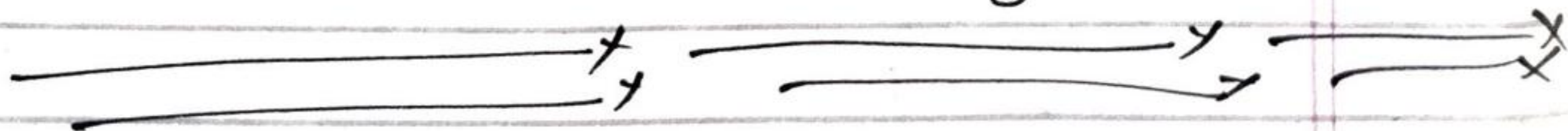
$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

Let	$y(x) = x^2 + \sin x$	Putting $x=0$ in all function
	$y'(x) = 2x + \cos x$	
	$y''(x) = 2 - \sin x$	
	$y'''(x) = -\cos x$	
		$y(0) = 0$ $y'(0) = 1$ $y''(0) = 2$ $y'''(0) = -1$

Putting value in formula

$$\begin{aligned}
 &= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1) \\
 &= 0 + x + x^2 - \frac{x^3}{6}
 \end{aligned}$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} \text{ Ans.}$$



(i) Find y'' given

$$1 + xy = x^2 + y^2$$

Solution:

$$1 + xy = x^2 + y^2 \quad \text{--- (1)}$$

diff eq (1) b/s w.r.t "x"

$$\frac{d}{dx} (1 + xy) = \frac{d}{dx} (x^2 + y^2)$$

$$\frac{d}{dx} (1) + \frac{d}{dx} (xy) = \frac{d}{dx} x^2 + \frac{d}{dx} y^2$$

$$\Rightarrow 0 + (x \cdot \frac{d}{dx} y + y \cdot \frac{d}{dx} x) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot \frac{d}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$= x \cdot y' - 2y \cdot y' = 2x - y$$

$$= y' (x - 2y) = (2x - y)$$

$$= \boxed{y' = \frac{2x - y}{x - 2y}}$$

(7)

Differ Again w.r.t ('n')

$$y'' = d/dn \left(\frac{2n-y}{n-2y} \right)$$

$$= \frac{(n-2y) d/dn (2n-y) - (2n-y) d/dn (n-2y)}{(n-2y)^2}$$

$$= \frac{(n-2y)(2)(-dy/dn) - (2n-y)(1-2dy/dn)}{(n-2y)^2}$$

$$= \frac{(2n-4y)(-y') - (2n-y)(1-2y y')}{(n-2y)^2}$$

$$y''(n-2y)^2 = 2ny' + 2yy' - 2n + y$$

$$y''(n-2y)^2 = \left(\frac{2n-y}{n-2y} \right) (2n-2y) - 2n + y$$

$$y'' = \frac{\left(\frac{2n-y}{n-2y} \right) (2n+2y) - 2n + y}{(n-2y)^2}$$

Ans..

⑧

Q=3.

B Parts

Find y' by using logarithmic differentiation

$$y = x^3 (1+x)^9 e^{bx}$$

Solution:- Taking \ln on b/s

$$\ln y = \ln (x^3 (1+x)^9 \cdot e^{bx})$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{bx}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + bx \ln e$$

Diff w.r.t x .

$$\frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln x + \frac{d}{dx} 9 \ln (1+x) + \frac{d}{dx} bx \ln e$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{bx}} \cdot b$$

$$\frac{dy}{dx} = y \left(\frac{3}{x} + \frac{9}{1+x} + \frac{b}{e^{bx}} \right)$$

$$\frac{dy}{dx} = \underline{x^3 (1+x)^9 e^{bx} \left(\frac{3}{x} + \frac{9}{1+x} + \frac{b}{e^{bx}} \right)}$$

Ans.