Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks** 7 **CLO** 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks** 7 **CLO** 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks** 6 **CLO** 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad |z| > |a|$$	**Marks** 6 **CLO** 2
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left|H(\frac{\pi}{4})\right|^2 = \frac{1}{2}$.	**Marks** 6 **CLO** 3

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks** 6 **CLO** 3
Q 4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$	**Marks** 6 **CLO** 2
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	**Marks** 6 **CLO** 2

Q No (1) (a)

The charateristics equation is.

Sol:- $\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ Hence

$y_h(n) = c_1 2^n + c_2 n 2^n$

The particular Solution is.

$y_p(n) = K(-1)^n u(n)$

Substituting this Solution into the difference equation, we obtain.

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$

$(-1)^n u(n) - (-1)^{n-1} u(n-1)$

For $n = 2$, $K(1+4+4) = 2$ $K = 2/9$ The total solution is.

$$y(n) = \left[c_1 2^n + c_2 n 2^n + 2/9 (-1)^n\right] u(n)$$

From the initial condition we obtain

$y(0) = 1$ $y(1) = 2$ then

$c_1 = 2/9 = 1$

$c_1 = 7/9$

$2c_1 + 2c_2 - 2/9 = 2$

$c_2 = 1/3$

Q No 1 (b)

The characteristics equation is :

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2 , 1/3$ Hence

$y_h(n) = c_1 1^n/2 + c_2 1^n/5$

With $z(n) = \delta(n)$ we have.

$y(0) = 2$

$y(1) - 0.7 y(0) = 0 \Rightarrow y(1) = 1.4$

Hence $c_1 + c_2 = 2$ and

$1/2\, c_1 + 1/5\, c_2 = 1.4 = 7/5$

$\Rightarrow c_1 + 2/5\, c_2 = 14/5$

These equation yeild.

$c_1 = 10/3 \quad c_2 = 4/3$

$h(n) = \left[10/3\, (1/2)^n - 4/3\, (1/5)^n \right] u(n)$

The step responce is

$$s(n) = \sum_{K=0}^{n} h(n-K)$$

$$= 10/3 \sum_{K=0}^{n} (1/2)^{n-K} - 4/3 \sum_{K=0}^{n} (1/5)^{n-K}$$

$$= 10/3 (1/2)^{n} \sum_{K=0}^{n} 2^{K} - 4/3 (1/5)^{n} \sum_{K=0}^{n} 5^{K}$$

$$= 10/3 (1/2^{n} (2^{n+1} - 1) u(n) - 1/3 (1/5^{n} (5^{n+1} - 1) u(n).$$

$$\Rightarrow 10/3 \left(1/2^{n} \left(2^{n+1} - 1\right) u(n) - 1/3 \left(1/5^{n} \left(5^{n+1} - 1\right) u(n)\right.\right.$$

Q No 2 (a)

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:- $x(z)$

$$= \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4 \quad B = -3 \quad C = -1$

Hence $x(n) = \left[4(2)^n - 3 - n\right] u(n)$.

Q No 2 (b) $X(z) = \frac{1}{1-az^{-1}}$ $|z| > |a|$

Sol:-

We have

$$x(n) = \frac{1}{2\pi j}\oint_c \frac{z^{n-1}}{1-az^{-1}}\,dz = \frac{1}{2\pi j}\oint_c \frac{z^n dz}{z-a}$$

Where C is a circle at radius greater then $|a|$. We shall evaluate this integral

$f(z) = z^n$. we distanguish two cases.

1) If $n \geq 0$, $f(z)$ has only zeros and hence no poles inside c. The only pole inside c is $z = a$

Hence. $x(n) = f(z_0) = a^n$ $n \geq 0$

2) If $n < 0$ $f(z) = z^n$ has an n^{th} order pole at $z = 0$ which is also inside c. Thus there are contributions from both poles. For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}\,dz = \left.\frac{1}{(z-a)}\right|_{z=0} + \left.\frac{1}{z}\right| = 0$$

If $n = -2$ we have.

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}\,dz = \left.\frac{d}{dz}\left(\frac{1}{z-a}\right)\right|_{z=0} + \left.\frac{1}{z^2}\right| = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$ Thus

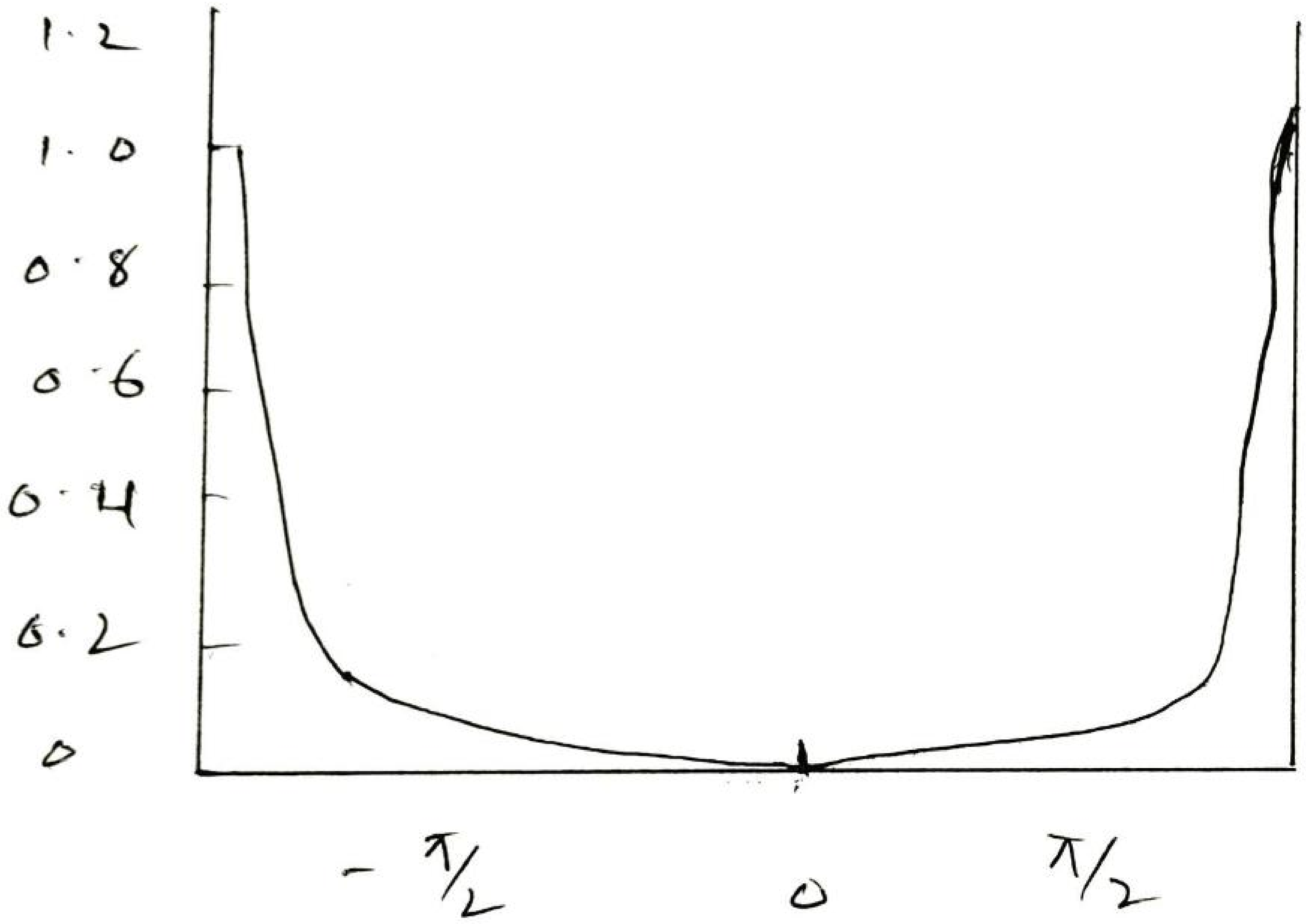
$$x(n) = a^n u(n).$$

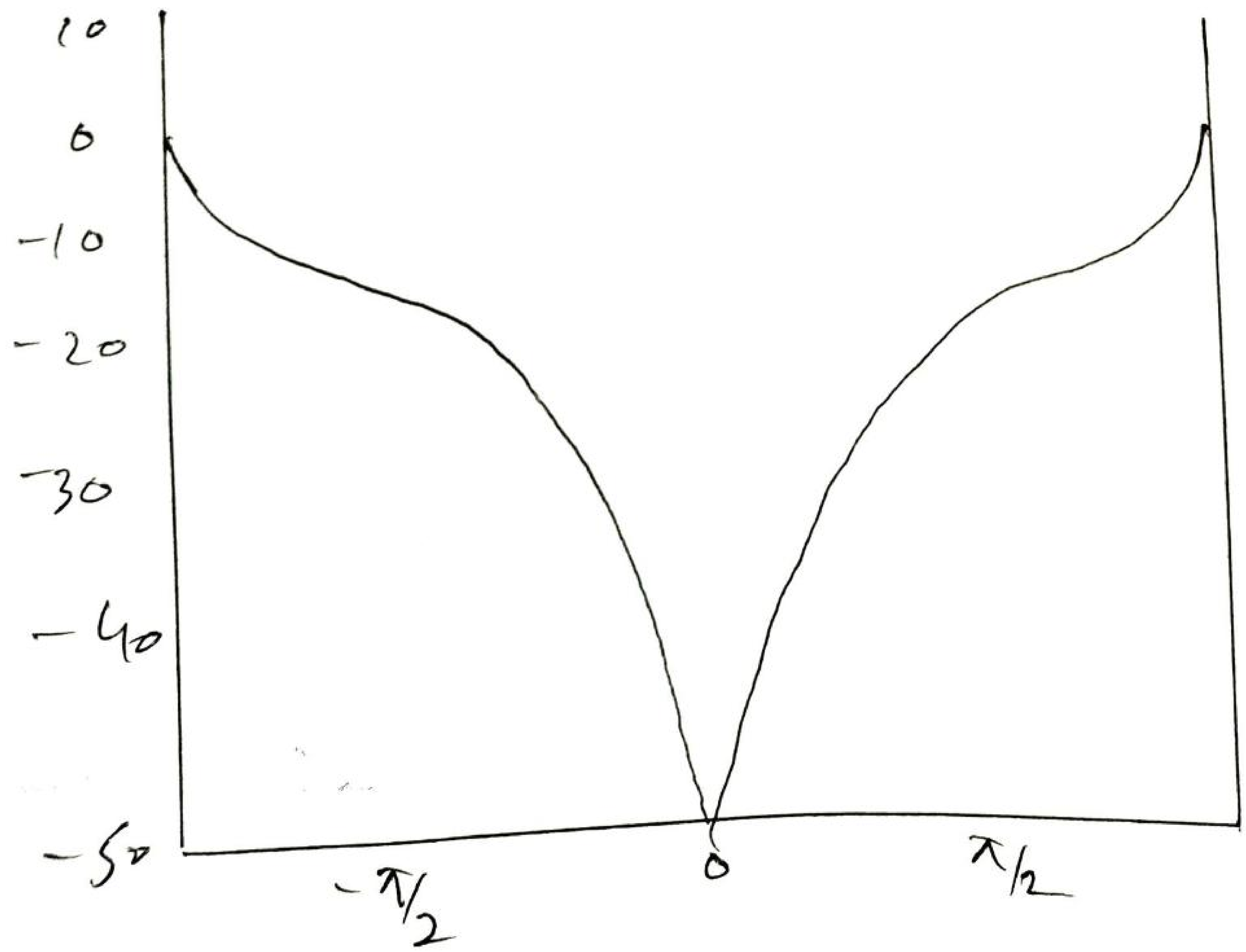
Q. no 3(a)

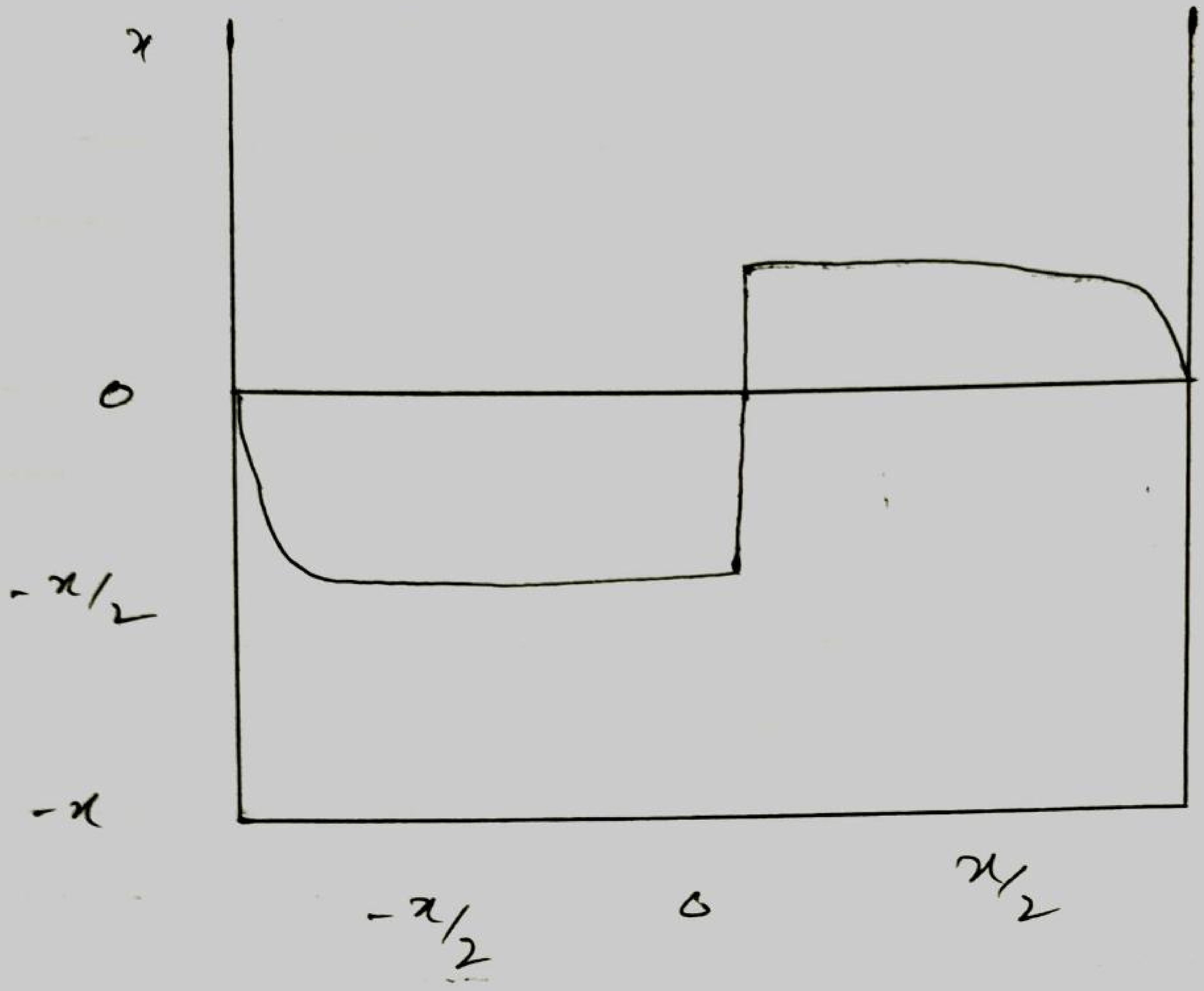
Sol:- A $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{1-P\cos(\pi/4)+jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}+jP/\sqrt{2})^2}$$

$$= \frac{(1-P)^4}{[(1-P/\sqrt{2})^2+P^2\sqrt{2}]^2} = 1/2$$

or equalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

$$p = 0.32$$

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}.$$

Q no 3(b)

Sol:- clearly the filter must have poles

$$P_{1.2} = re^{x\pi/2}$$

and zeros at $Z=1$ and $Z=-1$

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determine by evaluating the frequency.

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response $H(\omega)$ of the filter. at $\omega = \pi/2$

thus we have.

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4} \frac{2 - 2\cos(8\pi/9)}{1+r^4+2r^2\cos 8\pi/9)} = \frac{1}{2}$$

or equalvating

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

$$r^2 = 0.7$$

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

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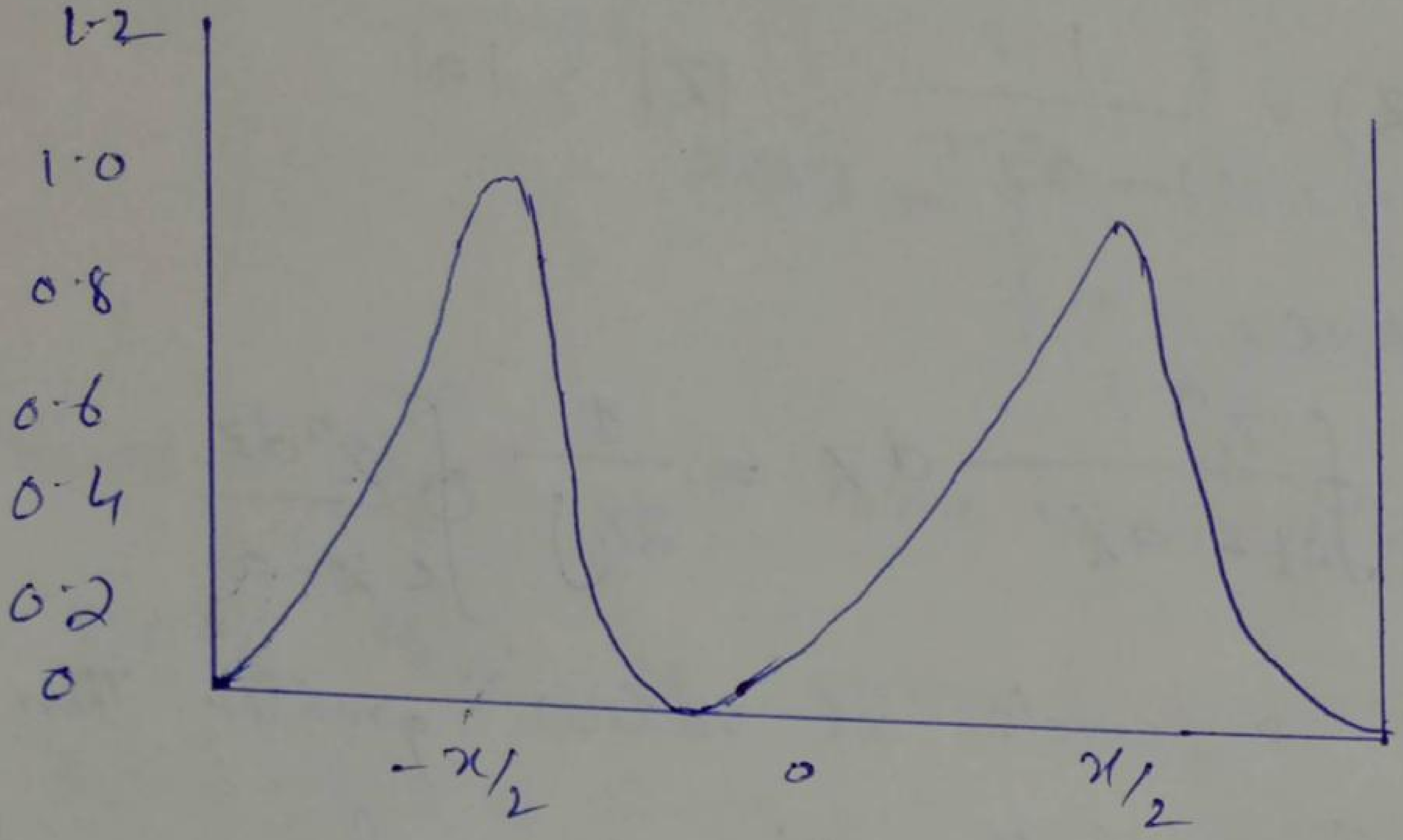
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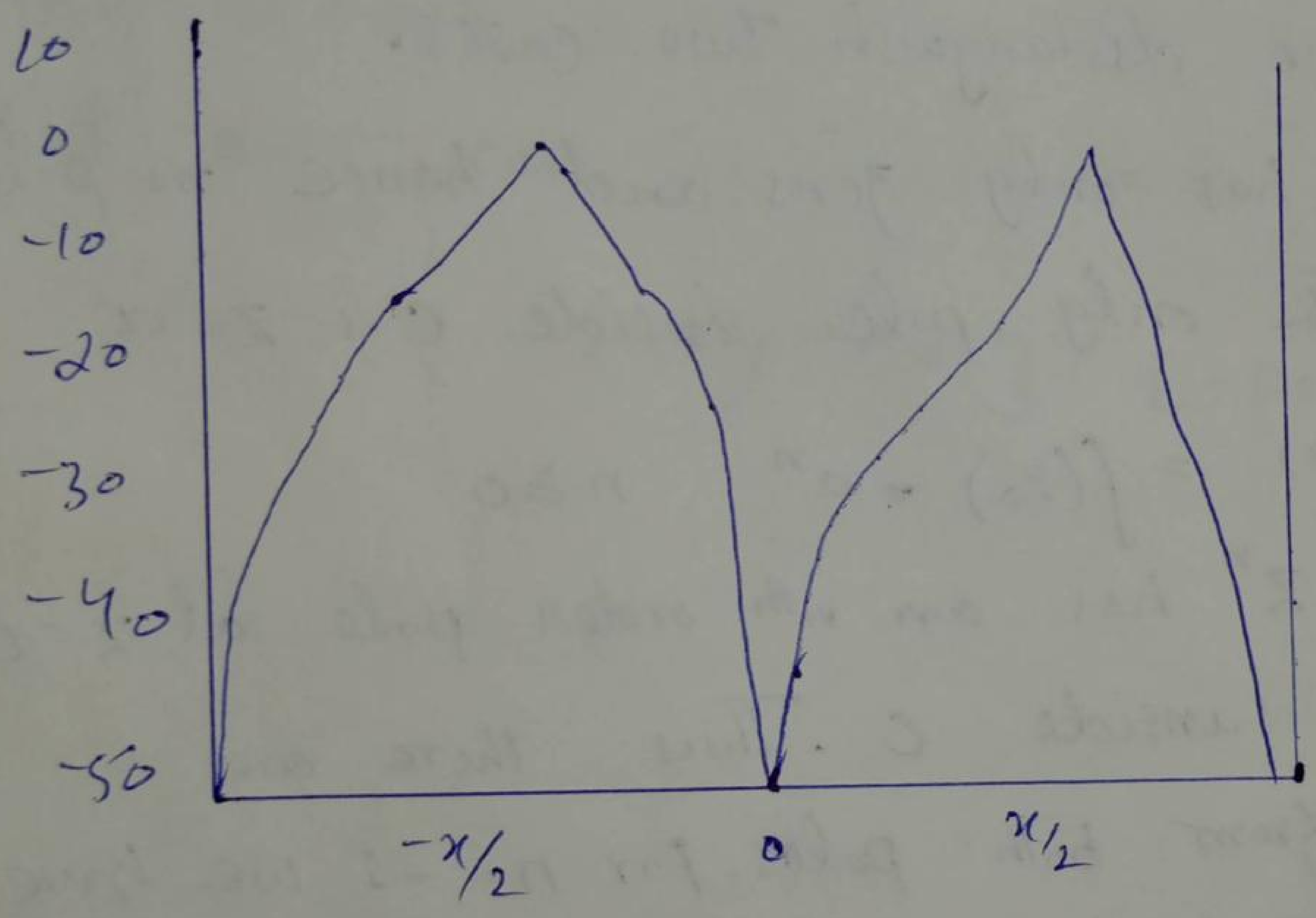
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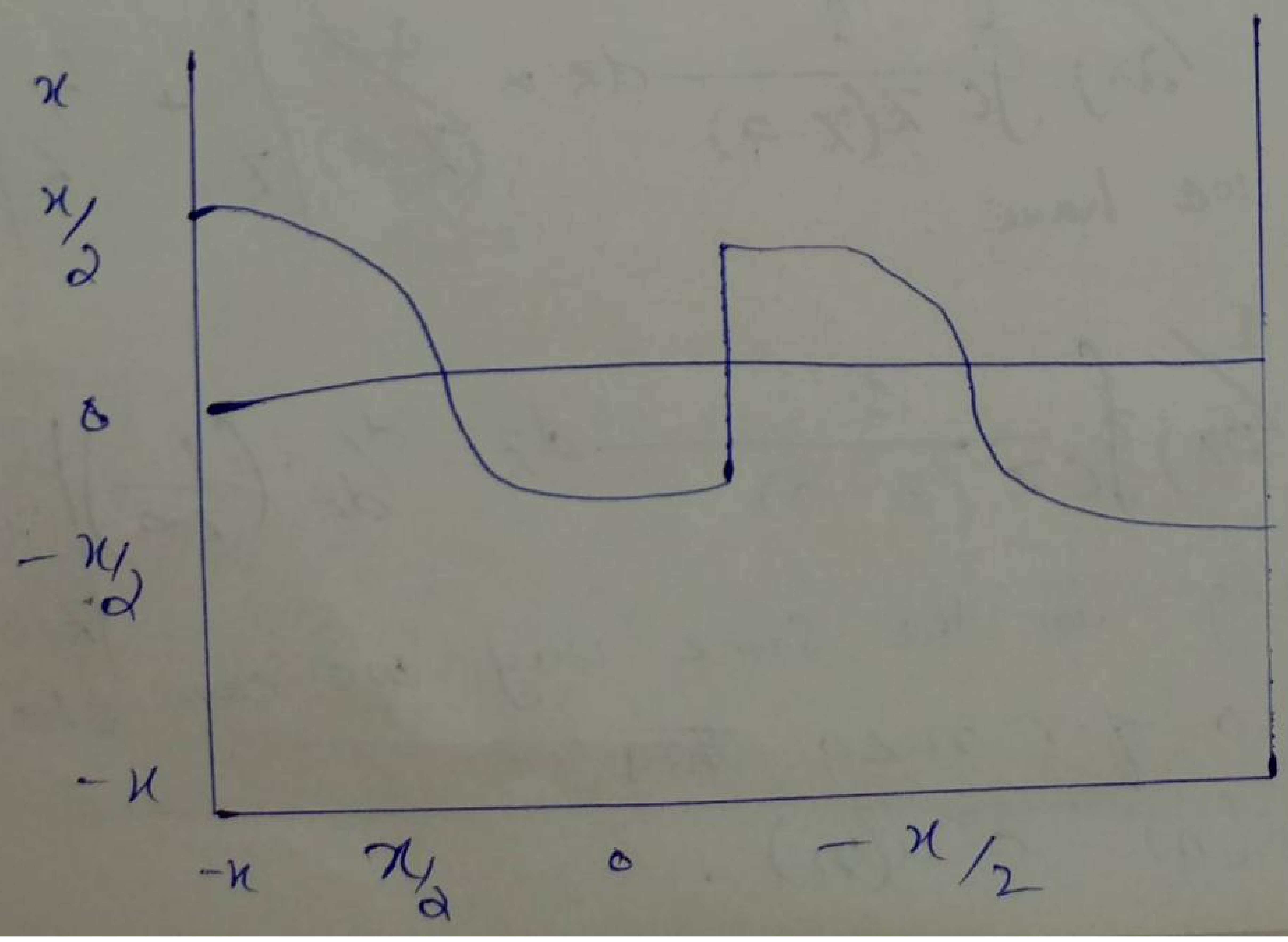
π
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0
$-\pi/2$

Q No 4 (a) $x(n) = \begin{cases} 1. & 0 \le n L - 1, \\ 0. & \text{otherwise} \end{cases}$

Sol:- The fourier transform of this sequence is

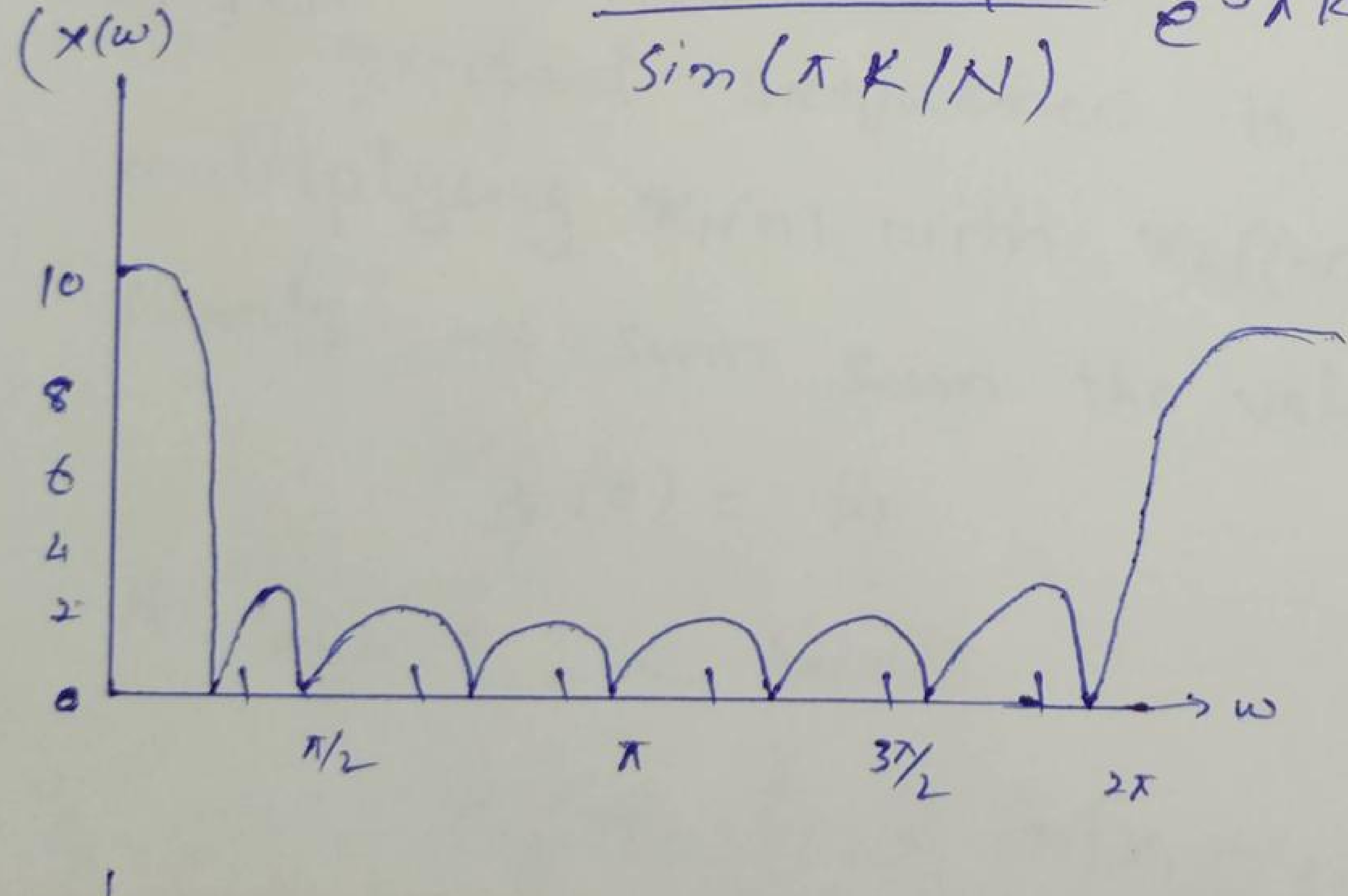
$$x(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

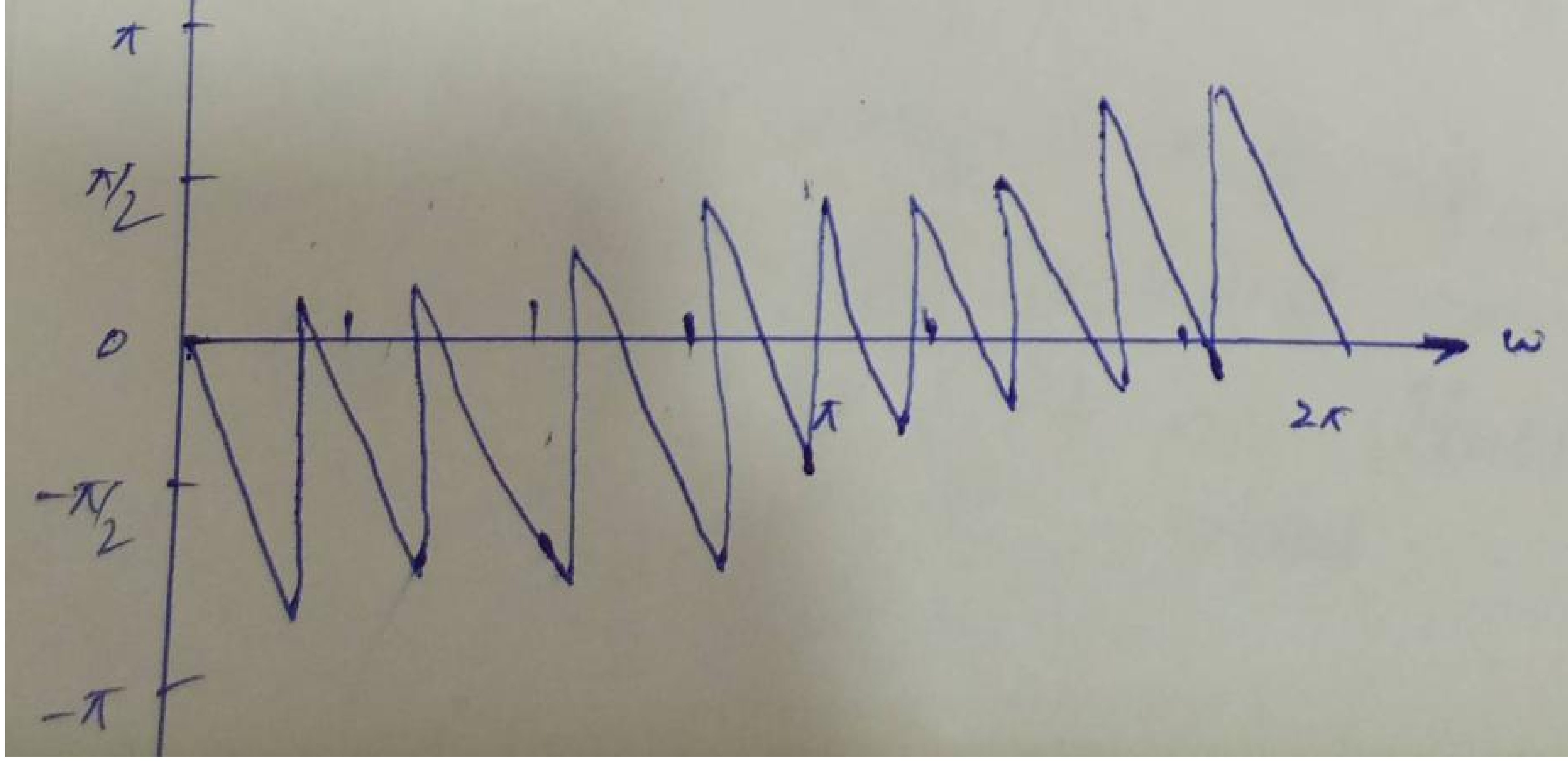
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $(x(\omega)$ are illustrated for $L = 10$. The N point DFT of $x(n)$ is simply $x(\omega)$ evaluated at the ~~stet~~ set of N equally spaced frequencies $\omega_K = 2\pi K/N$ $K = 0, 1 \dots N-1$. Hence

$$x(K) = \frac{1 - e^{-j2\pi K L/N}}{1 - e^{-j2\pi K/N}} \quad K = 0, 1, N-1$$

$$= \frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





If N is selected such that N = L then the DFT becomes

$$x(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2, \ldots L-1 \end{cases}$$

Thus there is one nonzero value in DFT this is apparent from observation of $X(\omega)$. Since $X(\omega) = 0$ at the frequency.

$\omega_K = 2\pi K / L$ $K \neq 0$. The reader should verif[y] [t]hat $x(n)$ can be recovered from $X(K)$ by performing L-point IDFT.

Q no 4(b) $x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$ $x_2(n) \{\underset{\uparrow}{1}, 2, 3, 4\}$

Sol:- Each sequence consist of four nonzero points. For the purpose of illustrating the operation involved in circulalar convolution.

Now $x_3(m)$ obtained by circularly convolving $x_1(n)$ with ~~as specified by~~. $x_2(n)$.

With $m = 0$ we have.

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$x_2((-n))$ is simply the sequence $x_2(n)$

The folded sequence is simply $x_2(n)$

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$ point by point.

Finaly we sum sum the values.

$$x_3(0) = 14$$

For $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

It is easily verified $x_2((1-n))_4$ is the simple sequence $x_2((-n))_4$ rotated counter clock wise by one unit in time as illustrated.

Finally we sum the values.

$$x_3(1) = 16.$$

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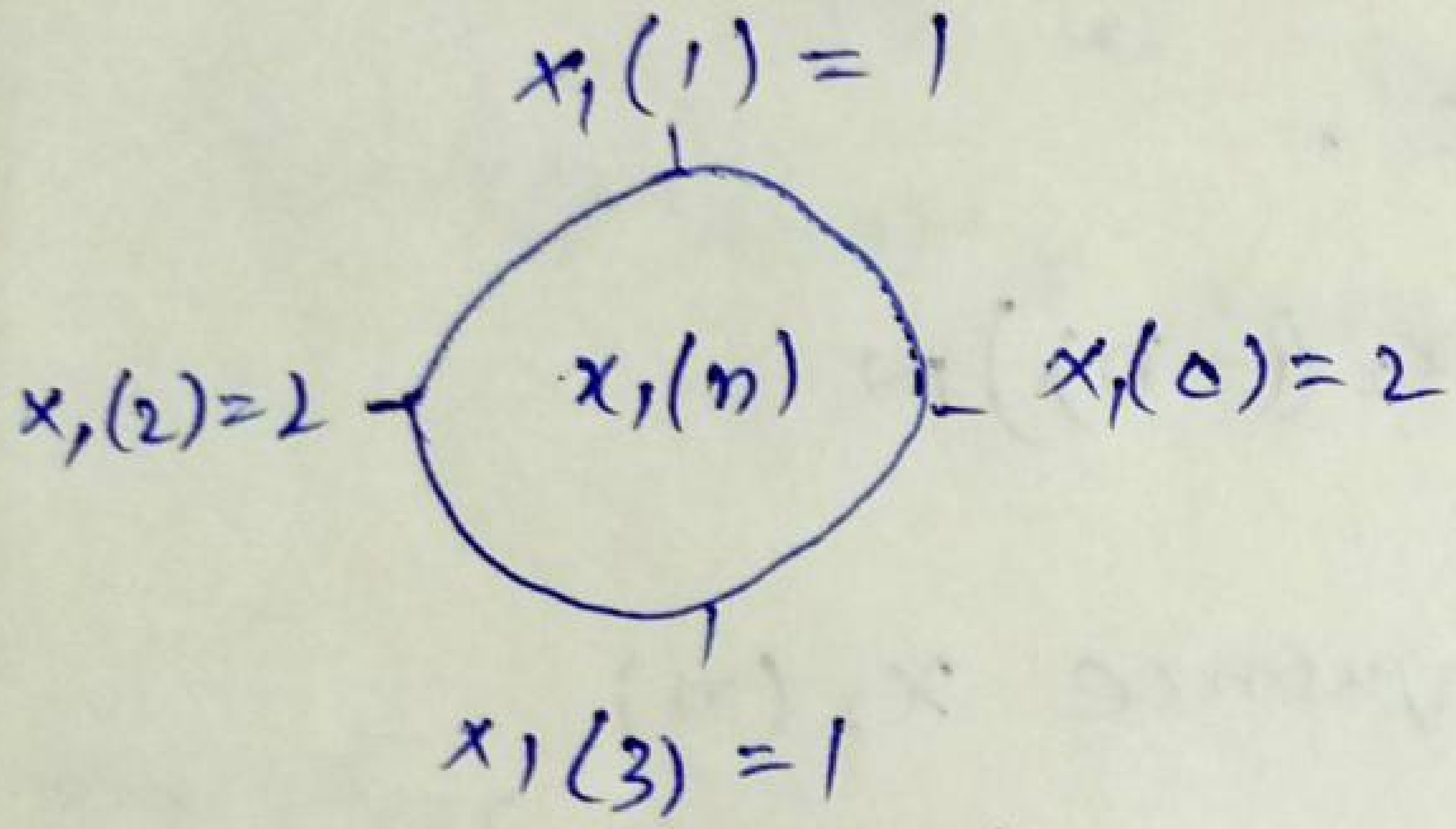
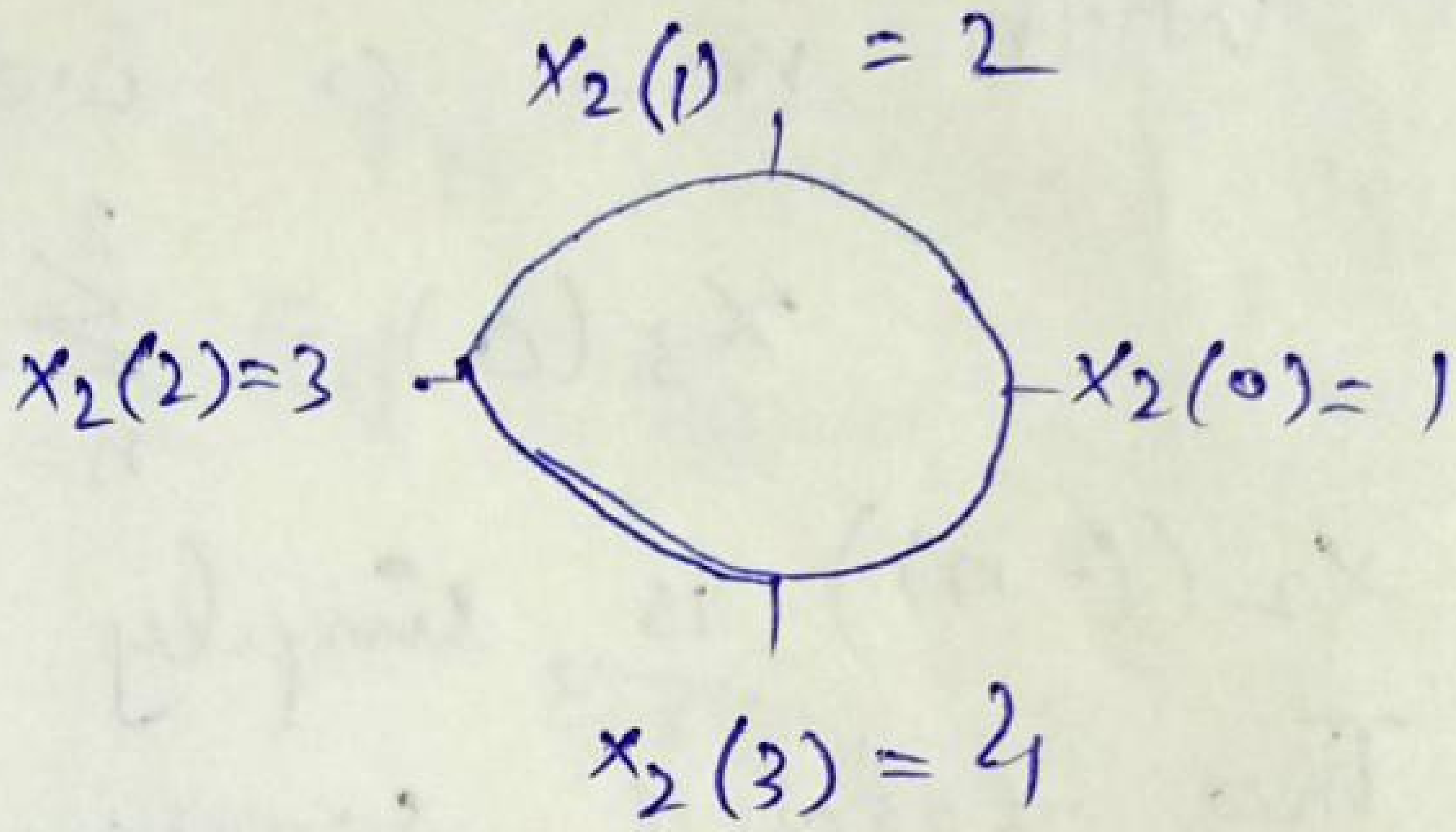
$m = 2$ we have.

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is folded sequence. rotated two units of Time in the counter clock wise direction.





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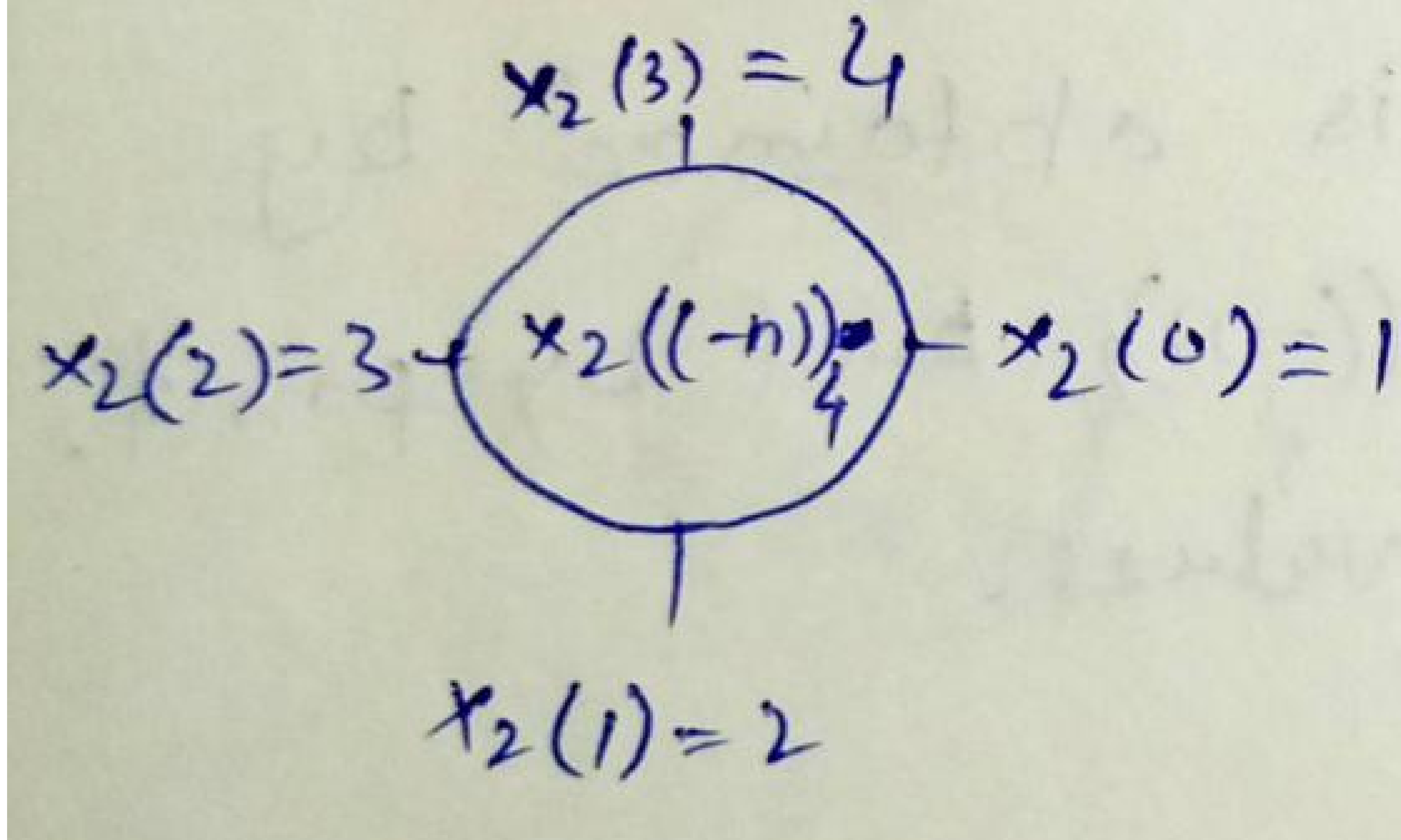
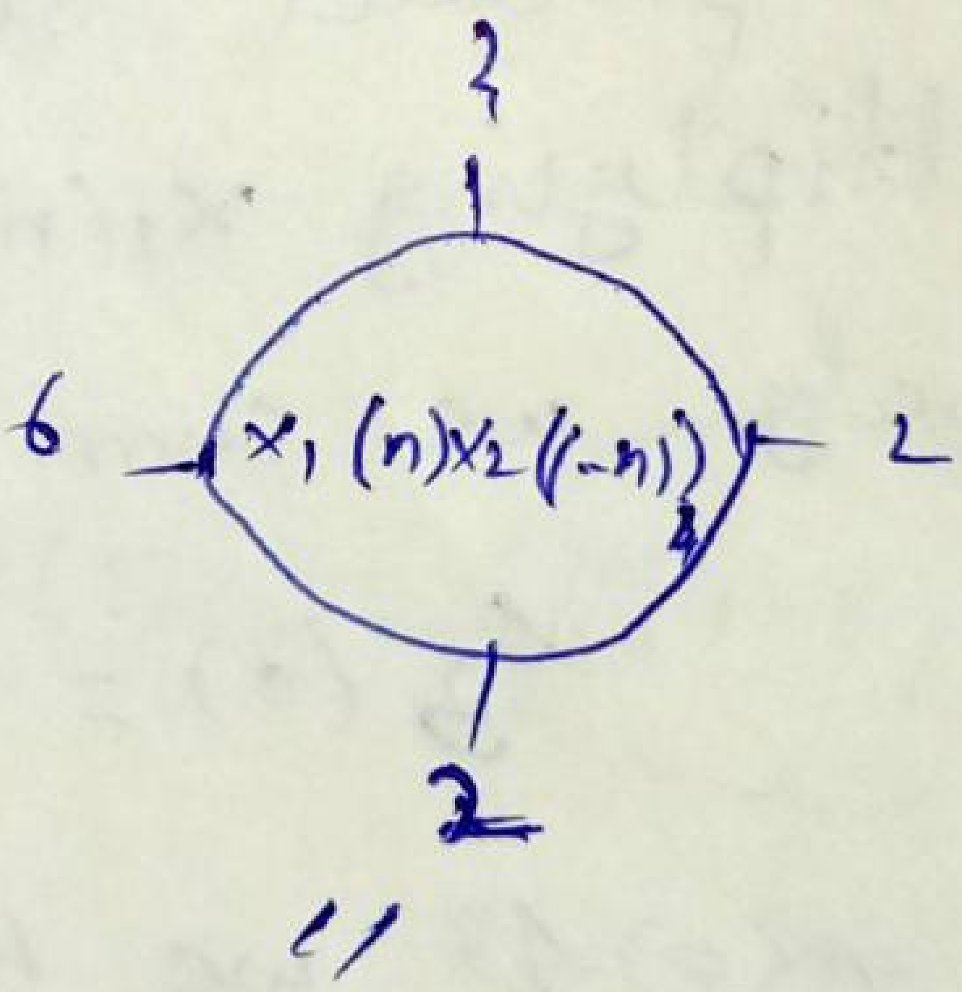


b

product sequence



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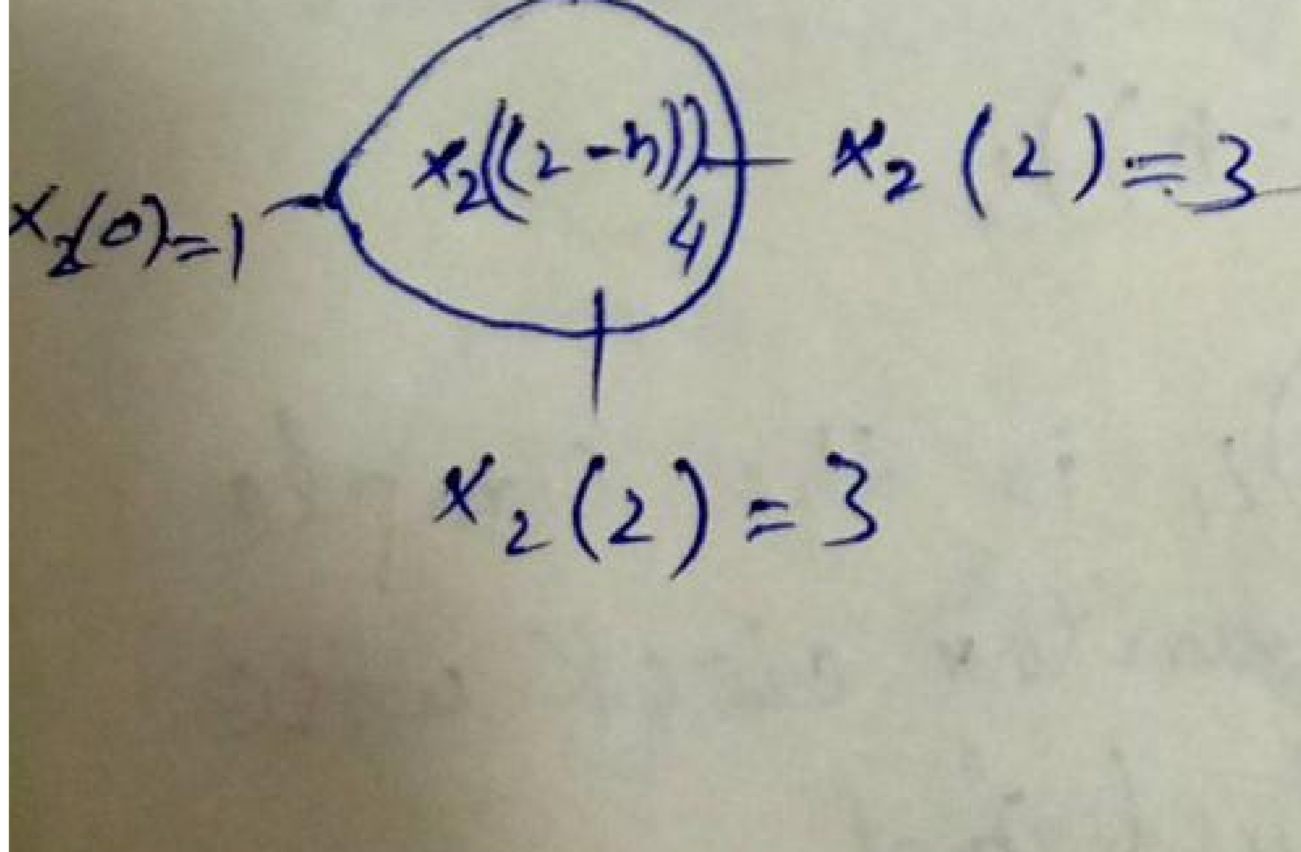
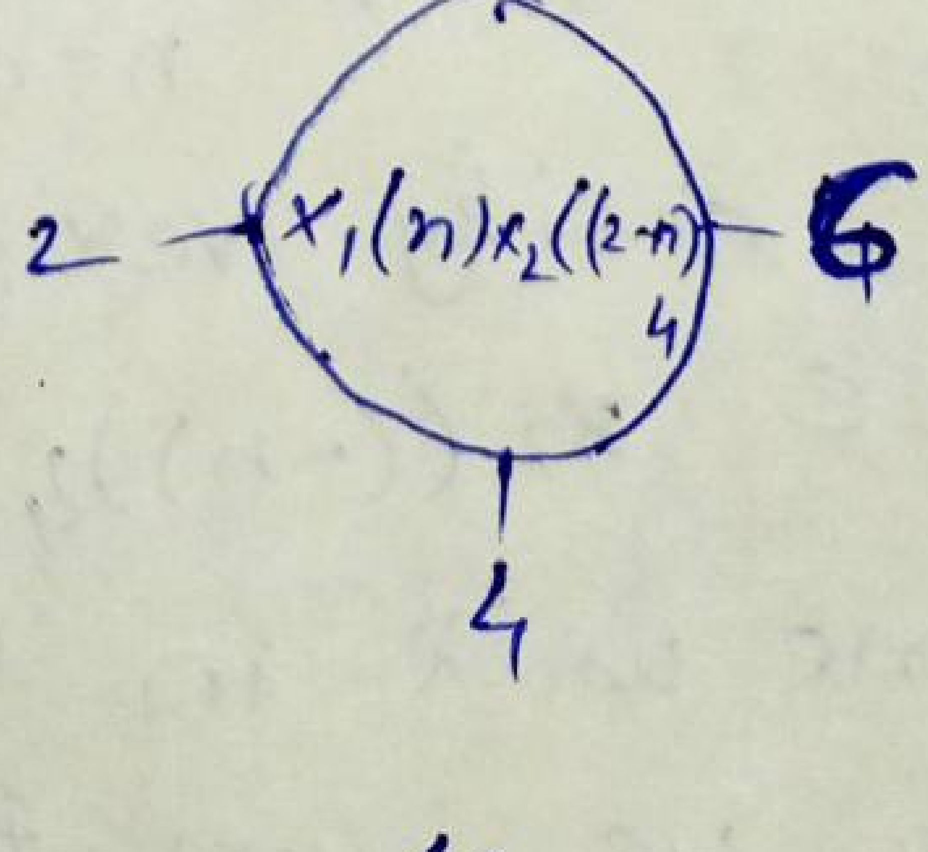


(d)



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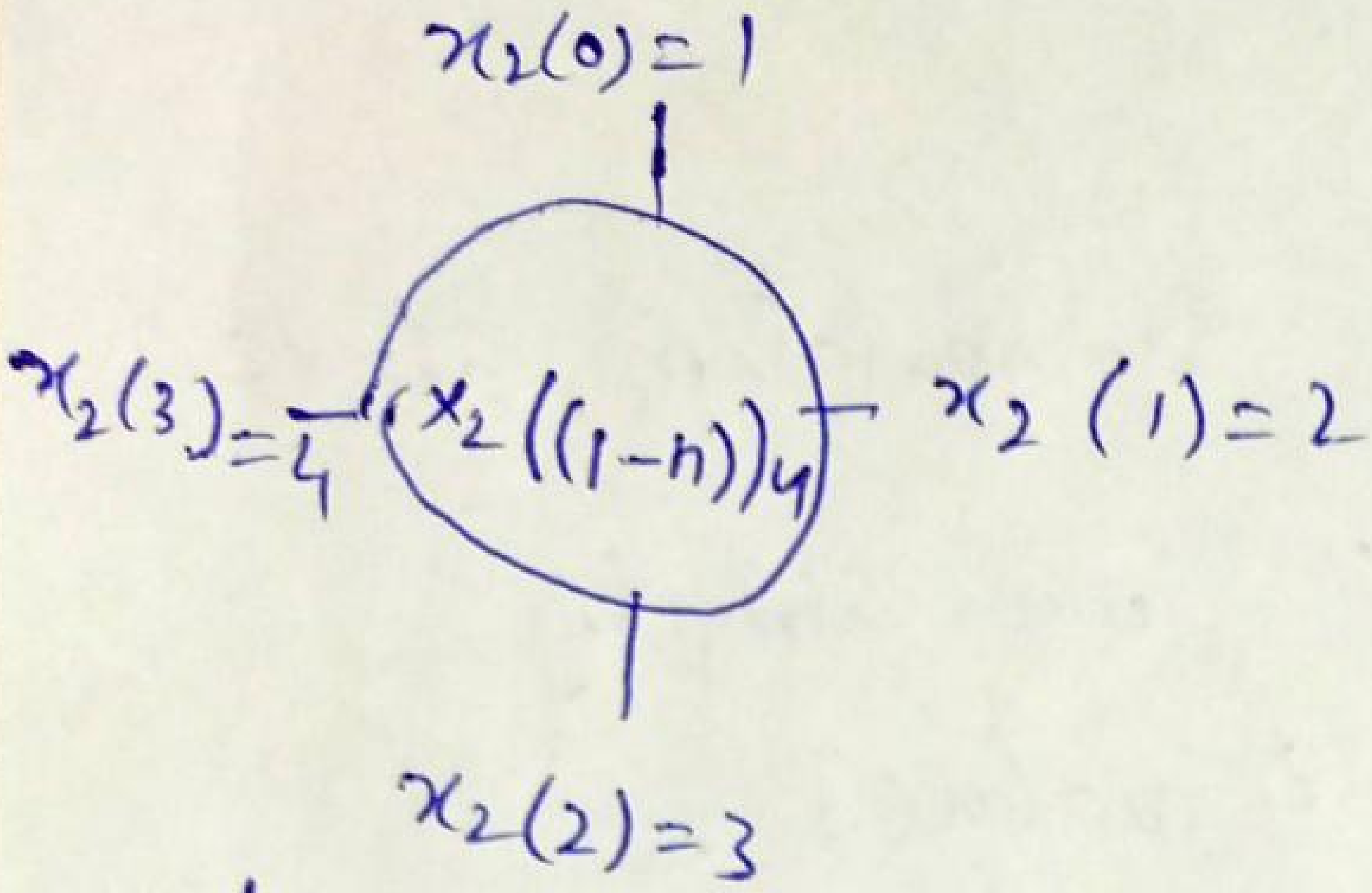

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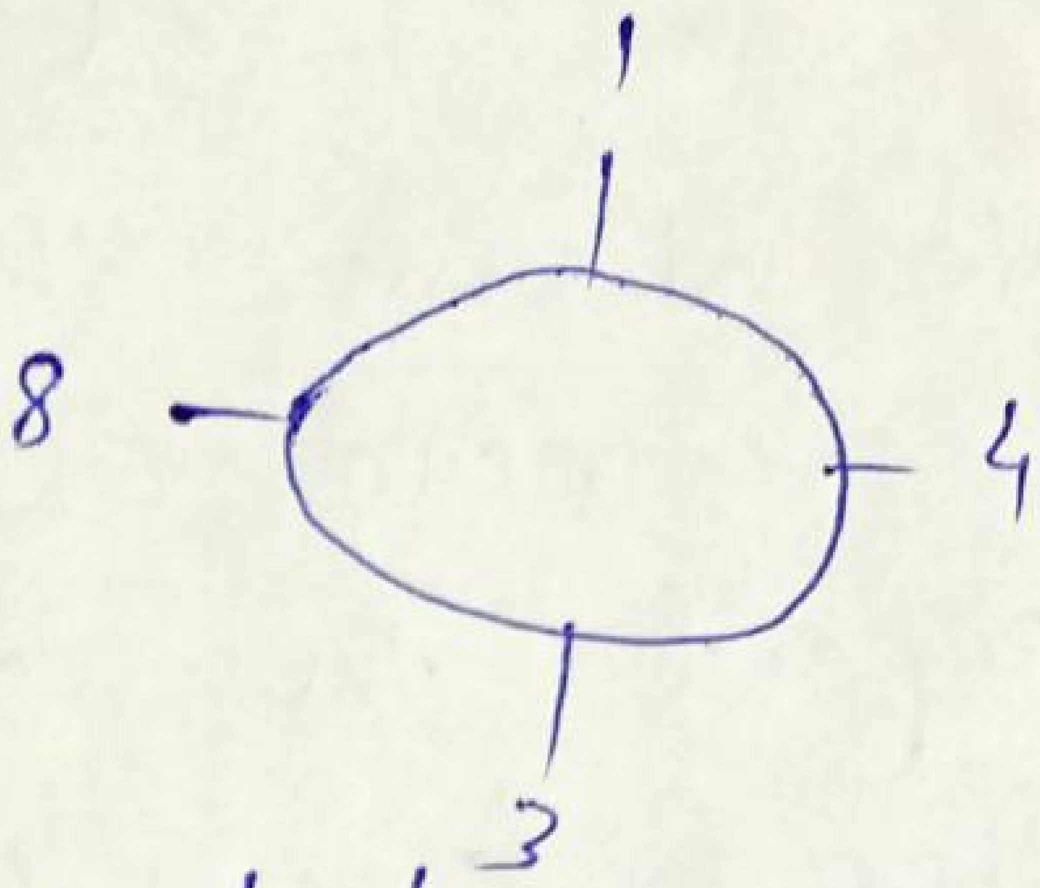
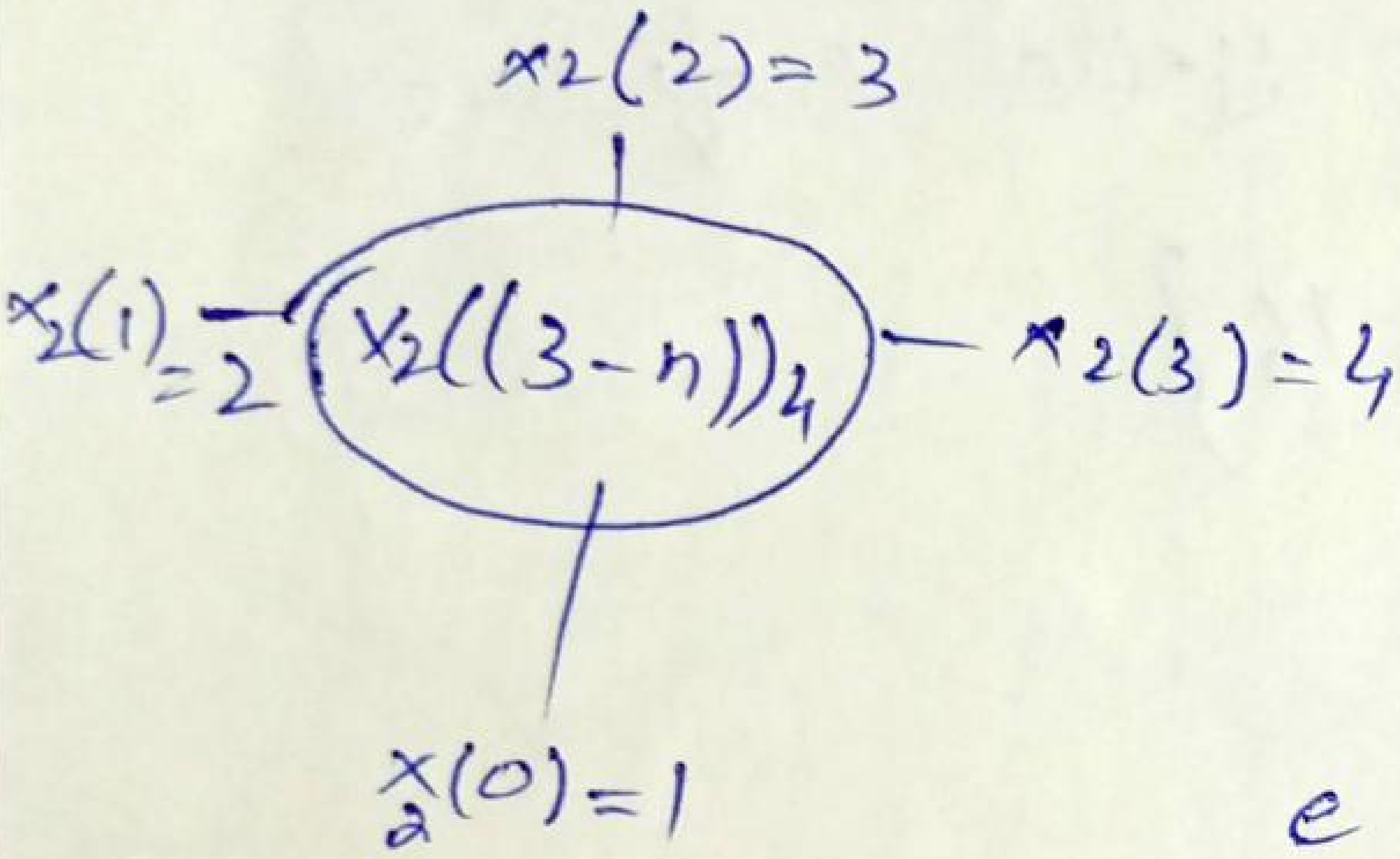


by one unit

product sequence



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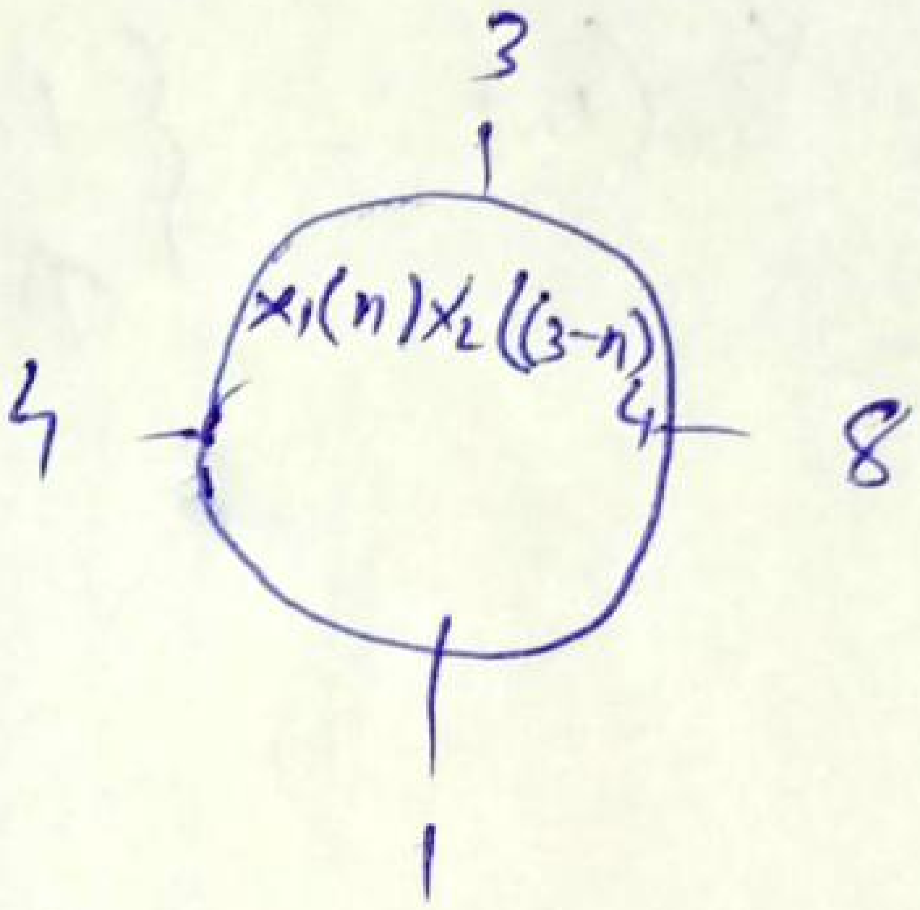

Folde sequence rotated by three units

Product sequence

along with product sequence $x_1(n)x_2((2-n))_4$ By suming the four terms. in the product Sequence.

$$x_3(2) = 14$$

$m=3$ we have.

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The folded sequence $x_2((-n))_4$ is now three units in time to yeild $x_2((3-n))_4$ is multiplied by $x_1(n)$ to yeild the product sequence.

$$x_3(3) = 16.$$

We observe that if the computation above is contineud beyond $m=3$ we simply repeat the sequence of 4 values. Therefore circular convolution of two sequencies $x_1(n)$ and $x_2(n)$ yeild sequence.

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$