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Assignment of EMF

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Submitted to.

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Q.1: The value of E at P ($\rho = 2$, $\phi = 40^\circ$, $z = 3$) is given as

$E = 100\, a_\rho - 200\, a_\phi + 300\, a_z$ V/m.

Determine the incremental work required to move a $20\,\mu C$ charged a distance of $6\,\mu m$.

Ans:- In the direction of a_ρ: The incremental work is given by $dW = -q\, E \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6 \times 10^{-6}\, a_\rho$. Thus

$$dW = -(20 \times 10^{-6}\, C)(100\, V/M)(6 \times 10^{-6}\, M) = -12 \times 10^{-9}\, J = \underline{-12\, nJ}$$

* in the direction of a_ϕ: in this case $dL = 2\, d\phi\, a_\phi = 6 \times 10^{-6}\, a_\phi$, and so

$$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6})$$
$$= 2.4 \times 10^{-8}\, J = \underline{24\, nJ}$$

* in the direction of a_z:

Here $dL = dz\ \vec{a}_z = 6\times10^{-6}\ \vec{a}_z$,

$dw = -(20\times10^{-6})(300)(6\times10^{-6})$

$= -3.6\times10^{-8}\ J = \underline{-36}\ nJ$

* in the direction of E : Here,

$dL = 6\times10^{-6}\ \vec{a}_E$, where

$$\vec{a}_E = \frac{100\vec{a}_\rho - 200\vec{a}_\phi + 300\vec{a}_z}{(100^2 + 200^2 + 300^2)^{1/2}} =$$

$$= 0.267\vec{a}_\rho - 0.535\vec{a}_\phi + 0.802\vec{a}_z$$

Thus

$dw = -(20\times10^{-6})(100\vec{a}_\rho - 200\vec{a}_\phi + 300\vec{a}_z)$

$\cdot[0.267\vec{a}_\rho - 0.535\vec{a}_\phi + 0.802\vec{a}_z]$

$(6\times10^{-6}) = \underline{-44.9}\ nJ$

⇒ in the direction of $G = 2\vec{a}_x - 3\vec{a}_y + 4\vec{a}_z$: in this case,

$dl = 6\times10^{-6}\ \vec{a}_G$, where

$$\vec{a}_G = \frac{2\vec{a}_x - 3\vec{a}_y + 4\vec{a}_z}{(2^2 + 3^2 + 4^2)^{1/2}} = 0.371\vec{a}_x - 0.557\vec{a}_y + 0.743\vec{a}_z$$

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So now

$dw = -(20\times10^{-6})[100\,a_\rho - 200\,a_\phi + 300\,a_z]\cdot[0.3718\,a_x - 0.557\,a_y + 0.743\,a_z]\,(6\times10^{-6})$.

$= -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + 111.4(a_\phi\cdot a_y) + 222.9]\,(6\times10^{-6})$

where, at P, $(a_\rho\cdot a_x) = (a_\phi\cdot a_y) = \cos(40^\circ) = 0.766$, $(a_\rho\cdot a_y) = \sin(40^\circ) = 0.643$, and $(a_\phi\cdot a_x) = -\sin(40^\circ) = -0.643$.

Subsituting there result in

$dw = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9]\,(6\times10^{-6}) = \underline{-41.8\ nJ}$

Q2:- Let $E = 400\,a_x - 300\,a_y + 500\,a_z$ in the neighborhood of point $P(6, 2, -3)$. Find the incremental work done in moving a $4\text{-}C$ charge a distance of 1mm in the direction specified by:

=> $a_x + a_y + a_z$: we write

$$dw = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \cdot \frac{(a_x + a_y + a_z)}{\sqrt{3}}(10^{-3})$$

$$= -\frac{(4\times10^{-3})}{\sqrt{3}}(400 - 300 + 500 = 1.39J$$

=> $-2a_x + 3a_y - 3a_z$: The computation is similar to that of part a, but we change the direction.

$$dw = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \cdot \frac{(-2a_x + 3a_y + a_z)}{\sqrt{14}}(10^{-3})$$

$$= -\frac{(4\times10^{-3})}{\sqrt{14}}(-800 - 900 - 500) = 2.35J$$

Q3:- if $E = 120\, a_\rho$ V/m, Find the incremental amount of work done in moving a $50\,\mu$M charge a distance of 2mm form.

Ans:- P (1, 2, 3) toward Q (2, 1, 4):

The vector along this direction will be Q - P = (1 - 1, 1)

from which $a_{PQ} = \frac{(a_x - a_y + a_z)}{\sqrt{3}}$. we now write

$dw = -qE \cdot dL = -(50\times10^{-6})$

$\left[120\, a_\rho \cdot \left(\frac{a_x - a_y + a_z}{\sqrt{3}}\right)\right](2\times10^{-3})$

$= -(50\times10^{-6})(120)\left[(a_\rho \cdot a_x) - (a_\rho \cdot a_y)\right]$

$\frac{1}{\sqrt{3}}(2\times10^{-3})$

At P, $\phi = \tan^{-1}(2/1) = 63.4°$

Thus $(a_\rho \cdot a_x) = \cos(63.4)$

$= 0.447$ and $(a_\rho \cdot a_y) = \sin(63.4)$

$= 0.894$. Subsituting there,

we obtain $dw = 3.1\,\mu J$.

=> Q (2, 1, 4) toward P (1, 2, 3):

A little thought is

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in order here : Note that the field is only a radial component and does not depend on ϕ or z. Note also that P and Q are at the same radius ($\sqrt{5}$) from the z axis, but have different ϕ and z coordinates. we could just as well position the two points at the same z location and the problem would not change. if this were so, then moving along a streigth line Between P & Q would involve moving along a chord of a circle whose radius $\sqrt{5}$. Half way along this line point of symmetry 14 field (make a jekch to see this).

This means that when Starting from either point the intial force will be same. Thus the answer $dw = 31 \mu j$ as part a.
This is also found by going through the same procedure as part a, but with the direction (role of P & Q) reversed.

Q4:- Compute the value of G .... - - - - - using the path.

Ans:- Striaght line of segments A (1, -1, 2) to B (1, 1, 2) to P (2, 1, 2) in general we have

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx$$

The change of x occurs when moving b/w B and

P during which $Y = 1$ thus

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \boxed{2}$$

(B) Straigh line segment A $(1, -1, 2)$, $(2, -1, 2)$ to P $(2, 1, 2)$ In case the chang in x occur when moving from A to C, during which $Y = -1$ thus

$$\int_A^P G \cdot dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx$$

$$= \boxed{-2}$$

Q5:- For $G = 3xy^3 a_x + 2z a_y$. Now things - - - - - - - - - - in that path does matter.

Ans:- Straight line $y = x - 1$, $z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 \, dx + \int_1^3 2z \, dy = \int_2^4 3x (x-1)^2 \, dx + = \boxed{90}$$

$$\int_1^3 2(1) \, dy$$

B. Parabola $6y = x^2 + 2$, $z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 \, dx + \int_1^3 2z \, dy$$

$$\Rightarrow \int_2^4 \frac{1}{12} x (x^2+2)^2 \, dx + \int_1^3 2(1) \, dy = \boxed{82}$$

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