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Assignment :- 2

Subject : differential Equation.

:Q1:-

Part (a)

$x' = \sqrt{x}$

Sol)

Step1:-

$x' = \sqrt{x}$

x is dependent variable
t is independent variable

$$\frac{dx}{dt} = \sqrt{x}$$

Step 2:-

Cosides

$$\frac{dx}{dt} = \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = dt$$

Take integration on bothside

$$\int \frac{dx}{\sqrt{x}} = \int dt$$

$$\int x^{-1/2}\, dx = \int dt$$ c is the constant of integration

$$-\frac{1}{2} x^{-1/2 - 1} = t + c$$

$$\boxed{-\frac{1}{2x^{3/2}} = t + c}$$ → The required solution

Part (B)

$x' = e^{-2x}$

Step 1:-

$$x' = e^{-2x}$$

$$\frac{dx}{dt} = e^{-2x}$$

Multiply b/s by dt

$$dx = e^{-2x}\, dt$$

divide b/s by e^{-2x}

$$\frac{dx}{e^{-2x}} = dt$$

$$e^{2x}\, dx = dt$$

Step 2:-

$$e^{2x}\, dx = dt$$

Take integrated b/s

$$\int e^{2x}\, dx = \int dt$$

$$\frac{e^{2x}}{2} = t + c$$

Multiply b/s by 2

$$e^{2x} = 2(t + c)$$

Take natural logarithm function from b/s

$$\ln(e^{2x}) = \ln(2t + 2c)$$

$$= 2x = \ln(2t + c_1)$$

$$= \boxed{x = \frac{\ln(2t + c_1)}{2}}$$

Part (c)

$$y' = 1 + y^2$$

Step 1 :-

$$y' = 1 + y^2$$

Step 2 :-

$$y' = 1 + y^2$$

$$\frac{dy}{dx} = 1 + y^2 \qquad \because y' \frac{dy}{dx}$$

$$= dy = (1 + y^2)\, dx$$

$$\frac{dy}{1+y^2} = dx$$

integrating bothsides

$$\int \frac{dy}{1+y^2} = \int dx + c \qquad \because \int \frac{dx}{1+y^2} = \tan^{-1}(y)$$

$$\tan^{-1}(y) = x + c$$

$$\boxed{y = \tan(x + c)}$$

Step 3:-

general solution & differential equation

$$y' = 1 + y^2 \text{ is}$$

$$\boxed{y = \tan(x+c)}$$

Part (d)

$$u' = \frac{1}{5-2u}$$

Step 1:-

$$u' = \frac{1}{5-2u}$$

$$\frac{du}{dv} = \frac{1}{5-2u}$$

$$(5-2u)\,du = dv$$

Step 2:-

Now integrate b/s

$$\int (5-2u)\,du = \int dv$$

$$5u - \frac{2u^2}{2} + C = v + c$$

$$5u - u^2 = v + c$$

$$u^2 - 5u = c - v$$

$$\left[u^2 - 5u + \frac{25}{4} - \frac{25}{4}\right] = c - v$$

$$\left[u - 5/2\right]^2 = c - v$$

$$\left[u - 5/2\right] = \sqrt{c - v}$$

$$\boxed{u = 5/2 + \sqrt{c-v}}$$

Step 3:-

general solution is

$u = 5/2 + \sqrt{c-v}$ c is constant.

Part (e)

$x' = au + b$, $a, b > 0$

Step 1:-

$x' = au + b$ --- (1) $a>0, b>0$

Step 2:- $\frac{dx}{du} = au + b$

by variable & separation

$dx = (au + b)\, du$

integration on b/s

$$\int dx = \int (au + b)\, du + c$$

$$\int dx = \int au\, du + \int b\, du + c$$

$$\int dx = a \int u\, du + b \int du + c$$

$$\boxed{x = a \frac{u^2}{2} + bu + c}$$

Part (f)

$$Q' = \frac{Q}{4+Q^2}$$

Step 1:-

$$Q' = \frac{Q}{4+Q^2}$$

independent variable bel t
and we have

$$Q' = \frac{dQ}{dt}$$

So equation becomes

$$\frac{dQ}{dt} = \frac{Q}{4+Q^2}$$

$$\frac{(4+Q^2)}{Q} dQ = dt$$

The left side only one variable Q
& right side only one variable t

So, differential Equation $\left(\frac{4+Q^2}{Q}\right) dQ = dt$.

is avariable seperable equation

Step 2:-

$$\frac{(4+Q^2)}{Q}\, dQ = dt$$

$$\int \frac{(4+Q^2)}{Q}\, dQ = \int dt$$

$$\int \left(\frac{4}{Q} + \frac{Q^2}{Q}\right) dQ = \int dt$$

$$\int \left(\frac{4}{Q} + Q\right) dQ = \int dt$$

$$\int \frac{4}{Q}\, dQ + \int Q\, dQ = \int dt$$

$$4\int \frac{1}{Q}\, dQ + \int Q\, dQ = \int dt$$

$$\boxed{4 \ln|Q| + \frac{Q^2}{2} = t + C}$$

Part - (g)

$$x' = e^{x^2}$$

Step 1:-

$$x' = e^{x^2}$$

$$\frac{1}{e^{x^2}} \frac{dx}{dt} = 1$$

$$\frac{1}{e^{x^2}} dx = dt$$

Integrating bothsides

$$\int \frac{1}{e^{x^2}} dx = \int 1 \, dt$$

$$\int e^{-x^2} dx = \int 1 \, dt$$

Step 2:-

$$\int \frac{1}{e^{x^2}} dx = \int 1 \, dt$$

$$\int e^{-x^2} dx = \int 1 \, dt$$

multiply & divided by $\sqrt{\pi}$

$$\int \frac{1}{e^{x^2}} dx = \int 1 \, dt$$

$$\sqrt{\pi} \int \frac{1}{\sqrt{\pi}} e^{-x^2} dx = \int 1 \, dt$$

Multiply & divided by 2

$$\frac{\sqrt{\pi}}{2} \int 2 \frac{1}{\sqrt{x}} e^{-x^2} dx = \int 1 \, dt \text{ ------- } 1$$

integer $\int \frac{2}{\sqrt{x}} e^{-x^2} dx$ is erf(x)

therefore 1 redues to $\frac{\sqrt{x}}{2}$ erf(x) = t+C

where C is constant Integration.

$$\boxed{\frac{\sqrt{x}}{2} \operatorname{erf}(x) = t+C}$$

Part (h)

$y' = r(a-y)$

Step 1:-

$$y' = r(a-y)$$

Step 2:-

$$y' = r(a-y)$$

$$y' = r(a-y)$$

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{(a-y)} = r\,dt$$

Integrating

$$\int \frac{dy}{(a-y)} = \int r\,dt$$

$$-\ln(a-y) = rt + C$$

$$\ln(a-y) = -rt - C$$

Step 3:-

$\ln(a-y) = -\gamma t - c$ for y

$\ln(a-y) = -\gamma t - c$

$e^{\ln(a-y)} = e^{-\gamma t - c}$

$a-y = e^{-\gamma t - c}$

$y = -e^{-\gamma t - e}$

Therefore

general solution of differential equation is

$y' = \gamma(a-y)$ is $\boxed{y = -e^{-\gamma t - c} + a}$

-: Q2 :-

Solve $y' = r(a-y)$, where r & a are constant.

Sol

Step 1:-

$$y' = r(a-y)$$

$$\frac{dy}{dx} = r(a-y)$$

$$\frac{1}{a-y}\,dy = r\,dx \longrightarrow (1)$$

Step 2:-

Integrate equation (1)

$$\int \frac{1}{a-y}\,dy = \int r\,dx$$

$$-\ln(a-y) = rx + c$$

$$a-y = e^{-rx-c}$$

$$\boxed{y = -e^{-(rx+c)} + a}$$

Step 3:

$$\boxed{y = -e^{-(rx+c)} + a}$$

-: Q3 :-

Sol! :-

Step 1 :- 1 (a)

initial value $x' = \sqrt{x}$, $x(0) = 1$

$$x' = \sqrt{x}$$

$$\frac{dx}{dt} = \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = dt$$

$$x^{-1/2}\,dx = dt$$

Step 2 :- Integrating

$$\int x^{-1/2}\,dx = \int dt$$

$$\frac{x^{-1/2+1}}{-1/2+1} = t + c$$

$$\frac{x^{1/2}}{1/2} = t + c$$

$$2\sqrt{x} = t + c$$

Step 3:-

evalute constant c using initial condition $x(0)=1$

$$2\sqrt{x} = t + c$$

$$2\sqrt{1} = 0 + c$$

$$2 = 0 + c$$

$$c = 2$$

Therefore

$$2\sqrt{x} = t + 2$$

Solve the eq^n for x

$$2\sqrt{x} = t + 2$$

$$(2\sqrt{x})^2 = (t+2)^2$$

$$4x = t^2 + 4t + 4$$

$$x = \frac{t^2}{4} + t + 1$$

Hence, solution IVP $x' = \sqrt{x}$, $x(0) = 1$ is

$$x = \frac{t^2}{4} + t + 1$$

Step 4:-

1 (b)

intial value problem $x' = e^{-2x}$, $x(0)=1$

$x' = e^{-2x}$ as follow

$$x' = e^{-2x}$$

$$\frac{dx}{dt} = e^{-2x}$$

$$\frac{dx}{e^{-2x}} = dt$$

$$e^{2x}\,dx = dt$$

Step 5:- Integerate

$$\int e^{2x}\,dx = \int dt$$

$$\frac{e^{2x}}{2} = t + c$$

evaluate the constant c
intial condition $x(0) = 1$

$$\frac{e^{2x}}{2} = t + c$$

$$\frac{e^{2(1)}}{2} = (0) + c$$

$$c = \frac{e^{2}}{2}$$

Step 6:- $$\frac{e^{2x}}{2} = t + \frac{e^{2}}{2}$$

Solve for x

$$\frac{e^{2x}}{2} = t + \frac{e^2}{2}$$

$$e^{2x} = 2t + e^2$$

$$\ln(e^{2x}) = \ln(2t + e^2)$$

$$2x = \ln(2t + e^2)$$

$$x = \frac{\ln(2t + e^2)}{2}$$

Hence solution of the IVP

$x' = e^{-2x}$, $x(0) = 1$ is $x = \dfrac{\ln(2t + e^2)}{2}$

-: Q4 :-

Find general solution.

(a) $x' = \dfrac{2x}{t+1}$

Step 1 :- $x' = \dfrac{2x}{t+1}$

i.e $\dfrac{dx}{dt} = \dfrac{2x}{t+1}$

$$\frac{dx}{2x} = \frac{dt}{t+1}$$

which is variable separable form

Integrate b/s

$$\int \frac{dx}{2x} = \int \frac{dt}{t+1}$$

$\frac{1}{2} \log x = \log (t+1) + \log c$

$\log x^{1/2} = \log [(t+1) \cdot c]$

$x^{1/2} = c(t+1)$

$x = c^2 (t+1)^2$

$\boxed{x(t) - c(t+1)^2}$ $\therefore c = c^2$

(b) $\theta' = t\sqrt{t^2+1} \sec\theta$

Step 1:-

$\theta' = t\sqrt{t^2+1} \sec\theta$

$\frac{d\theta}{dt} = t\sqrt{t^2+1} \times \sec\theta$

$\frac{d\theta}{\sec\theta} = t\sqrt{t^2+1}\, dt$

$\because \cos\theta = \frac{1}{\sec\theta}$

$\cos\theta\, d\theta = t\sqrt{t^2+1}\, dt$:.

which is variable separable

$\int \cos\theta\, d\theta = \int t\sqrt{t^2+1}\, dt + c$

Substitute

$t^2 = x$

$2t\, dt = dx$

$t\, dt = \frac{dx}{2}$

$\int \cos\theta\, d\theta = \int \frac{1}{2} \sqrt{x+1} \times dx + c$

$\int \cos\theta\, d\theta = \frac{1}{2} \int (x+1)^{1/2}\, dx + c$

$\sin\theta = \frac{1}{2} \times \frac{(x+1)^{3/2}}{3/2} + c$

$\boxed{\sin\theta = \frac{1}{3} (x+1)^{3/2} + c}$