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Q # 1

a.) Differentiate $\frac{3x^4 - 2x^3 + 5}{x^3 + 1}$ w.r.t "x"

Use the Quotient Rule

$$= \frac{d}{dx}\left(\frac{3x^4 - 2x^3 + 5}{x^3 + 1}\right)$$

$$\frac{(x^3+1)\frac{d}{dx}(3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(12x^3 - 6x^2) - (3x^4 - 2x^3 + 5)(3x^2)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(12x^3 - 6x^2) - (3x^4 - 2x^3 + 5)(3x^2)}{(x^3+1)^2}$$

(2)

$$= \frac{3x^6 + 12x^3 - 21x^3}{(x^3+1)^2}$$

$$= \boxed{\frac{3x^2(x^4 + 4x - 7)}{(x^3+1)^2}}$$ ans

Q#1

b)

Differentiate $\frac{(x^3+1)^2}{(x^3-1)}$ w.r.t "x"

By Using Quotient Rule

$$= \frac{d}{dx}\left(\frac{(x^3+1)^2}{x^3-1}\right)$$

$$= \frac{(x^3-1)\left(\frac{d}{dx}(x^3+1)^2\right) - (x^3+1)^2 \frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)(2(x^3+1)(3x^2)) - (x^3+1)^2(3x^2)}{(x^3-1)^2}$$

$$= \frac{3x^8 - 6x^5 - 9x^2}{(x^3-1)^2}$$

$$= \frac{3x^2(x^6 - 2x^3 - 3)}{(x^3-1)^2}$$

Hence

$$= \boxed{\frac{3x^2(x^6 - 2x^3 - 3)}{(x^3-1)^2}} \text{ Ans}$$

(5)

(2 11:02

a)

Integration of $\int \frac{1}{\sqrt{x^5}}\, dx$

Soln:

$$\int \frac{1}{\sqrt{x^5}}\, dx$$

$$= \int \frac{1}{x^{5/2}}\, dx$$

$$= \int x^{-5/2}\, dx$$

$$= \frac{x^{-5/2+1}}{-5/2+1}$$

$$= \frac{x^{-3/2}}{-3/2}$$

(6)

$$\int \frac{-2}{3x^{3/2}} + C$$

Ans

Q#02

b) Integration of $\int \frac{1}{(8x+7)^8}\, dx$

Soln:

$$\int \frac{1}{(8x+7)^8}\, dx$$

Substitute $u = 8x+7$

$$\boxed{\frac{du}{dx} = 8}$$

$$dx = \frac{1}{8}\, du$$

$$= \frac{1}{8}\int \frac{1}{u^8}\, du$$

$$\int \frac{1}{u^8}\, du$$

Apply power rule

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$\boxed{n = -8}$$

$$= -\frac{1}{7u^7}$$

$$\frac{1}{8}\int \frac{1}{u^8}\, du$$

$$= -\frac{1}{56u^7}$$

Substitution $u = 8x + 7$

$$= \frac{-1}{56(8x+7)^7} + C$$

Q#03

a) $\int \frac{-x+9}{2x^2-8x+6}\,dx$ by Partial fraction

Soln

Denominator

$2x^2-8x+6$

$2x^2-6x-2x+6$

$2x(x-3)-2(x-3)$

$(x-3)(2x-2)$

let

$$\frac{-x+9}{(x-3)(2x-2)} = \frac{A}{(x-3)} + \frac{B}{(2x-2)}$$

x by $(x-3)(2x-2)$ on Both sides

$-x+9 = A(2x-2) + B(x-3)$

Putting $\boxed{x=3}$

$-3+9 = A(2(3)-2 + B(3-3)$

$6 = A(6-2) + B(0)$

$6 = A(4) + B(0)$

$\left[A = \frac{42}{63}\right]$ $\boxed{A = \frac{2}{3}}$

Now

putting $2x-2=0$

$\boxed{x=1}$

$-1+9 = A(2(1)-2 + B(1-3)$

$8 = A(2-2) + B(-2)$

$$8 = A(0) + B(-2)$$

$$B = -\frac{2}{8}$$

$$\boxed{B = -\frac{1}{4}}$$

Thus

$$\int \frac{-x+9}{(x-3)(2x-2)}\,dx = \int \left(\frac{A}{(x-3)} + \frac{B}{(2x-2)} \right)$$

$$= \int \frac{-x+9}{(x-3)(2x-3)}\,dx = \int \left(\frac{2/3}{(x-3)} + \frac{-\frac{1}{4}}{(2x-2)} \right) dx$$

$$= \frac{2}{3}\int \frac{1}{(x-3)}\,dx - \frac{1}{4}\int \frac{1}{(2x-2)}\,dx$$

$$= \boxed{\frac{2}{3}\ln(x-3) - \frac{1}{4}\ln(2x-2) + C} \text{ ANS}$$

Ex # 1

a) Find

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}.$$

Soln:

$$X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (5-3) & 1-(-1) \\ (-3-2) & (1-2) \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} \quad \text{Ans}$$

b) Part

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

Sol:

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} (2+(-4) & 6+(-8 \\ 1+(-2) & 5+0 \end{bmatrix}$$

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$x = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} -2(+1) & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$ Ans

c) Past

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} (2\times1) & (2\times0) \\ (2\times0) & (2\times1) \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} (3-2) & (-1-0) \\ (1-0) & (2-2) \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

ANS

Q No 5

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$ Find $A^2 + BC$

Soln

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times1) + (4\times2) & (1\times4) + (4\times1) \\ (2\times1) + (1\times2) & (2\times4) + (1\times1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (1+2) & (4+4) \\ (2+2) & (8+1) \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

How

$B \times C$

$$B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$B \times C = \begin{bmatrix} (-3 \times 1) + (2 \times 0) & (-3 \times 0) + (2 \times 2) \\ (4 \times 1) + (0 \times 0) & (4 \times 0) + (0 \times 2) \end{bmatrix}$$

$$B \times C = \begin{bmatrix} (-3 + 0) & (4) \\ (4) & (0) \end{bmatrix}$$

$$B \times C = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

Now

$$A^2 + BC$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} (9+(-3) & (8+4) \\ (4+4) & (9+0) \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

Hence

$$A^2 + BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$ Ans