Differential Equation

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15459

Assignment No 01

Question-No-01 :-

Question-No-01-12 :-

$$x^2y'' - 4xy' + 6y = 0 \qquad y(1) = 0.4$$
$$y'(1) = 0$$

Sol:

Put $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

Put y'' in give D.E equation

$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0$

$x^2 m(m-1)x^m x^{-2} - 4xmx^m \cdot x^{-1} + 6x^m = 0$

Droping common factor (x^m)

$m(m-1) - 4m + 6 = 0$

$m^2 - 5m + 6 = 0$

Now finding root

$m^2 - 5m + 6 = 0$

$$m_{1/2} = \frac{5 \pm \sqrt{(5)^2 - 4 \cdot 6}}{2}$$

$$m_{1/2} = \frac{5 \pm 1}{2}$$

$m = 3$ And $m = 2$

$y_1 = x^{m} = x^3$ And $y_2 = x^{m_2} = x^2$

General Solution is

$$y = C_1 y_1 + C_2 y_2$$

$$C_1 x^3 + C_2 x^2$$

$$y' = 3C_1 x^2 + 2C_2 x$$

Now determine C_1 and C_2.

$$\left\{ \begin{array}{l} 0.4 = y(1) = C_1 \cdot 1^3 + C_2 \cdot 1^2 \\ 0 = y'(1) = 3C_1 \cdot 1^2 + 2C_2 \cdot 1^2 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 0.4 = C_1 + C_2 \\ 0 = 3C_1 + 2C_2 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 0.4 - C_2 = C_1 \\ 1.2 = C_2 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 0.4 - 1.2 = C_1 \\ 1.2 = C_2 \end{array} \right\}$$

$$\left\{ \begin{array}{l} -0.8 = C_1 \\ 1.2 = C_2 \end{array} \right\}$$

$$y = (-0.8x^3 + 1.2x^2)$$

Ans

Question-No-01-13:

$$x^2y''+3xy'+0.75y=0 \quad y(1)=1$$
$$y'(1)=-1.5$$

Put $y=x^m$

$$y'=mx^{m-1}$$

$$y''=m(m-1)x^{m-2}$$

Putting in the given DE

$$x^2m(m-1)x^{m-2}+3xmx^{m-1}+0.75x^m=0$$

$$\cancel{x^2}m(m-1)x^m.x^{-2}+3xmx^m.\cancel{x^{-1}}+0.75m=0$$

Dropping common factor x^m

$$m(m-1)+3m+0.75=0$$

$$m^2+2m+0.75=0$$

Finding root

$$m^2+2m+0.75=0$$

$$m^{1/2}=\frac{-2\pm\sqrt{2^2-(4)(0.75)}}{2}$$

$$m^{1/2}=\frac{-2\pm1}{2}$$

$m_1=-1/2$ And $m_2=-3/2$.

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$y_1 = x^{m_1} = x^{-1/2} = x^{-0.75}$ And $y_2 = x^{m_2} = x^{-3/2}$

The general solution is

$y = C_1 y_1 + C_2 y_2$

$= C_1 x^{-0.5} + C_2 x^{-1.5}$

$y' = -0.5 C_1 x^{-1.5} - 1.5 C_2 x^{-2.5}$

To determine C_1 and C_2.

$$\begin{cases} 1 = y(1) = C_1 \cdot 1^{-0.5} + C_2 \cdot 1^{-1.5} \\ 1.5 = y'(1) = 0.5 C_1 \cdot 1^{-1.5} - 1.5 C_2 \cdot 1^{-2.5} \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 1.5 = -0.5 C_1 - 1.5 C_2 / 0.5 \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 3 = C_1 + 3 C_2 \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 2 = 2 C_2 / \cdot 2 \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 1 = C_2 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$

$y = x^{-1.5}$ A

Question-01-14 :

$$x^2y''+xy'+9y=0 \qquad y(1)=0$$
$$y'(1)=2.5$$

Put $y=x^m$ And $y''=m(m-1)x^{-2}$

$$x^2m(m-1)x^{m-2}+mx^m+9x^m=0$$

$$\cancel{x^2}m(m-1)x^m.\cancel{x^{-2}}+mx^m+9x^m=0$$

Dropping common factor x^m

$$m(m-1)+m+9=0$$

$$m^2-\cancel{m}+\cancel{m}+9=0$$

$$m^2+9=0$$

finding root

$$m^2+9=0 \Rightarrow m^2-(3i)^2=0 \Rightarrow (m-3i)(m+3i)=0$$

$m_1=3i$ And $m_2=3i$

$$x^{m_1}=x^{3i}=(e^{\ln x})^{3i}=e^{-3i\ln x}$$

$$x^{m_2}=x^{-3i}=e^{\ln x}=e^{-3i\ln x}$$

$$e^r=e^{a+ib}=e^a(\cos b+i\sin b)$$

$$e^{3i\ln x}=e(\cos(3\ln x)-i\sin(3\ln x))=\cos 3\ln x-i\sin(\sin x)$$

$$x^m=\cos(3\ln x)+i\sin(3\ln x)$$
$$x^{m_2}=\cos(3\ln x)-i\sin(3\ln x).$$

$x^{m_1}+x^{m_2}=\cos(3\ln x)+i\sin(3\ln x)+\cos(3\ln x)$
$-i\sin(3\ln x)=2\cos(3\ln x)$

$$\frac{x^{m_1}+x^{m_2}}{2}=\cos(3\ln x)$$

Substracting 2nd equation from first and dividing "2i"

$x^{m_1}-x^{m_2}=\cos(3\ln x)+i\sin(3\ln x)-\cos(3\ln x)$
$+i\sin(3\ln x)=2i\sin(3\ln x)$

$$\frac{x^{m_1}-x^{m_2}}{2i}=\sin(3\ln x)$$

$y_1=\cos(3\ln x)$ And $y_2=\sin(3\ln x)$

$y=C_1y_1+C_2y_2$

$C_1\cos(3\ln x)+C_2\sin(3\ln x)$

$y_2'=-C\sin(3\ln x)\cdot(3\ln x)'+C_2\cos(3\ln x)\cdot(3\ln x)'$

$$-\frac{3C_1}{x}\sin(3\ln x)+\frac{3C_2}{x}\cos(3\ln x)$$

Now to find C_1 and C_2

$$\begin{cases}0=y(1)=C_1\cos(3\ln x)+C_2(\sin)(3\ln x)\\ 2.5=y'(1)=3C_1\sin(3\ln x)+3C_2\cos(3\ln x)\end{cases}$$

$$\begin{cases}0=C_1\cos(0)+C_2(\sin)(0)\\ 2.5=-3C_1\sin(0)+3C_2\cos(0)\end{cases}$$

$$\begin{cases}0=C_1\\ 5/6=C_2\end{cases}$$

$$y=5/6\sin(3\ln x)\quad \text{Ans}$$

Question-No-15 :-

$$x^2y'' + 3xy' + y = 0 \qquad y(1) = 3.6$$
$$y'(1) = 0.4$$

Put $y = x^m$, $y^x = m(m-1)x^{m-2}$

$$x^m(m-1)x^{m-2} + 3mx^m + x^m = 0$$

$$x^2 m(m-1)x^m \cdot x^{-2} + 3\cancel{xm}\, m x^m + x^m = 0$$

Dropping common factor

$$m(m-1) + 3m + 1 = 0$$

$$m^2 - m + 3m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + 2m + 1 = 0, \quad (m+1)^2 = 0$$

$$m = 1$$

$$y_1 = x^m = x^{-1} = 1/x$$

$$y'' + 3/x \cdot y' + 1/x^2 \cdot y = 0$$

$$P(x) = 3 \cdot 1/x \Rightarrow \int P dx = 3\ln|x|$$

$$y_2 = uy_1$$

$u = \int u dx$ And $u = 1/y_1^2 \, e^{-\int P dx}$

To find u

$$e^{-\int P dx} = e^{-3\ln|x|} = (e^{\ln|x|})^{-3} = x^{-3}$$

$$U = x^{-3} \cdot 1/x^{2} = x^{-3+2} = x^{-1} = 1/x$$

$$x = \int dx/x = \ln|x|$$

$$y = uy_2 = y_1 \ln x = 1/x \ln x$$

$$y = c_1 y_1 + c_2 y_2$$

$$c_1 \cdot 1/x + c_2\, 1/x \ln(x)$$

$$1/x \cdot c_1 + c_2 \ln x$$

$$y' = (x^{-1})'(c_1 + c_2 \ln(x)) + x^{-1}(c_1 + c_2 \ln x)'$$

$$= -x^{-2}(c_1 + c_2 \ln x) + 1/x\, c_2 \cdot 1/x)$$

$$= 1/x^2(-c_1 - c_2 \ln x + c_2)$$

now find c_1 and c_2

$$\begin{cases} 3.6 = y(1) = 1/1(c_1 + c_2 \ln 1) \\ 0.4 = y'(1) = 1/2(-c_1 - c_2 \ln 1 + c_2) \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 0.4 = -c_1 + c_2 \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 0.4 = -3.6 + c_2 \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 4.0 = c_2 \end{cases} \rightarrow y = (3.6 + 4.0 \cdot 0 \ln x)\, 1/x$$

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Question-16 ι

$$(x^2D^2-3xD+4I)y=0 \qquad y(1)=-\pi, \; y'(1)=2\pi$$

$$x^2D^2y-3xDy+4Iy = x^2D(Dy)-3xDy+4y$$

$$= x^2y''-3xy'+4y$$

$$x^2y''-3xy'+4y=0$$

Put

$$y=x^m \text{ and } y''=m(m-1)x^{m-2}, \; y'=mx^{m-1}$$

$$x^2m(m-1)x^{m-2}-3xmx^{m-1}+4x^m=0$$

$$x^2m(m-1)x^m\cdot x^{-2}-3x\cdot mx^m \, x^{-1}+4x^m=0$$

Dropping x^m

$$m(m-1)-3m+4=0 \; , \; m^2-4m+4=0$$

hence

$y=x^m$ is a solution

$$m^2-4m+4=0 \; , \; (m-2)^2=0$$

$$y_1=x^m=x^2$$

$$y''=\frac{-3}{x}y'+\frac{4}{x^2}y=0$$

$$P(x)=-3\cdot\frac{1}{x} \Rightarrow \int P\,dx=-3\ln(x)$$

$$y_2=uy_1$$

$$u=\int U\,du \quad \Lambda \quad U=\frac{1}{y_1^2}e^{-\int P\,dx}$$

To find u

$$e^{-\int p\,dx} = e^{3\ln(x)} = (e^{\ln(x)})^3 = x^3$$

$$u = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = 1/x$$

$$u = \int \frac{dx}{x} = \ln(x)$$

$$y_2 = u y_1 = y_1 \ln(x) = u \ln(x)$$

$$y_1 = y_2 \in R$$

General factor is

$$y = c_1 y_1 + c_2 y_2$$

$$c_1 x^2 + x^2 \ln x$$

$$x^2 (c_1 + c_2 \ln x)$$

$$y' = (x^2)' (c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$$

$$= 2x (c_1 + c_2 \ln x) + c_2 x^2 = 1/x$$

$$= 2c_1 x + 2c_2 x \ln x + c_2 x$$

$$= 2c_1 x + c_2 x (2\ln x + 1)$$

$$\begin{cases} -\pi = y(1) = 1^2(c_1 + c_2 \ln 1) \\ 2\pi = y'(1) = 2\cdot c_1 + c_2(2\ln 1 + 1) \end{cases}$$

$$\begin{cases} -\pi = c_1 \\ 2\pi = 2c_1 + c_2 \end{cases}$$

$$\begin{cases} -\pi = c_1 \\ 4\pi = c_2 \end{cases}$$

$$y = x^2(-\pi + 4\pi \ln x)$$

Question - 17 ^

$$(x^2D^2 + xD + I)y = 0, \quad y(1) = 1, \quad y'(1) = 1$$

First Applying given operation to fraction

$$x^2D^2y + xDy + Iy = x^2D(Dy) + xDy + y$$

$$= x^2y'' + xy' + y$$

Now

$$x^2y'' + xy' + y = 0$$

$$y = x^m, \quad y' = mx^m, \quad y'' = m(m-1)$$

$$x^2m(m-1)x^{m-2} + xmx^{m-1} + x^m = 0$$

$$x^2m(m-1)x^m \cdot x^{-2} + xmx^m x^{-1} + x^m = 0$$

Dropping common factor

$$m(m-1) + m + 1 = 0 \Rightarrow m^2 - m + m + 1 = 0$$

$$= m^2 + 1 = 0$$

Now finding the root.

$$m^2 + 1 = 0, \quad m^2 + i^2 = 0, \quad (m-i)(m+i) = 0$$

$$m_1 = i \quad \wedge \quad m_2 = -i$$

$$x = e^{\ln x}$$

$$x^{m_1} = x^1 = (e^{\ln x}) = e^{\ln x}$$

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$$x^{m_2} = x^{-1} = (e^{\ln x})^{mi} = e^{i \ln x}$$

$$e^{m} = e^{a+b} = e^{m}(\cos(\ln x) + i\sin(\ln x))$$

$$= \cos(\ln x) + i\sin(\ln x)$$

$$e^{-3i\ln x} = e^{n}(\cos(\ln x) \pm i\sin(\ln x))$$

$$x^{m_1} = \cos(\ln x) + i\sin(\ln x)$$

$$x^{m_2} = \cos(\ln x) - i\sin(\ln x)$$

$$x^{m_1} + m^2 = \cos(\ln x) + i\sin(\ln x) + \cos(\ln x)$$

$$- i\sin(\ln x)$$

$$= 2\cos(\ln x)$$

$$\frac{x^{m_1} + x^{m_2}}{2} = \cos(\ln(x))$$

$$x^{m_1} - x^{m_2} = \cos(\ln x) + i\sin(\ln x) - \cos(\ln x) + i\sin(\ln x)$$

$$\frac{x^{m_1} - x^{m_2}}{2} = \sin(\ln x)$$

$$y' - \cos(\ln x) \quad \wedge \quad y_2 = \sin(\ln x)$$

$$y' = -C_1 \sin(\ln x) \cdot (\ln x)' + C_2 \cos(\ln x) \cdot (\ln x)$$

$$= \frac{-C_2}{2} \sin(\ln x + \frac{C_2}{x} \cos(\ln x))$$

To determine C_1 and C_2

$$\begin{cases} 1 = y(1) = C_1 \cos(\ln 1) + C_2 \sin(\ln 1) + C_3 \sin(\ln 2) \\ 1 = y'(1) = C_1 \sin(\ln 1) + 3C_2 \cos(\ln 1) \end{cases}$$

$$\begin{cases} 1 = C_1 \cos(0) + C_2 \sin(0) \\ 1 = -C_1 \sin(0) + C_2 \cos(0) \end{cases}$$

$$\begin{cases} 1 = C_1 \\ 1 = C_2 \end{cases}$$

$$y = \sin(\ln x + \cos(\ln x) \;/\!/$$

Question - 18 ^

$$(9x^2D^2 + 3xD + I)y = 0 \qquad y(1) = 1$$
$$y'(1) = 0$$

Apply given operation to the equation.

$$9x^2D^2y + 3xDy + Iy = 9x^2D(Dy) + 3xD1$$

$$= 9x^2y'' + 3xy' + y$$

$$9x^2y'' + 3xy' + y = 0$$

Let. $y = x^m$, $y' = mx^{m-1}$; $y'' = m(m-1)x^{m-2}$

$$9x^2m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$$

$$9\cancel{x^2}m(m-1)x^m + \cancel{x^{-2}} + 3\cancel{x}mx^m\cancel{x^{-1}} + x^m = 0$$

$$9m(m-1) + 3m + 1 = 0, \quad 9m^2 - 9m + 3m + 1 = 0$$

$$= 9m^2 - 6m + 1 = 0$$

finding the root of equation.

$$m^2 - 4m + 4 = 0, \quad (m-2)^2 = 0$$
$$9m^2 - 6m + 1 = 0 \qquad m_{1/2} = \frac{6 \pm \sqrt{6^2 - 4.}}{18}$$

$$m_{1/2} = 6/18$$

$$m_{1/2} = 1/3$$

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$m = \frac{1}{3}$

$y_1 = x^m = x^{1/3}$

$y'' = \frac{1}{3x} \cdot y' + \frac{1}{9x^2} \cdot y = 0$

$p(x) = \frac{1}{3} \cdot \frac{1}{x} = \int p\,dx = \frac{1}{3} \ln(x)$

$y^2 = u y_1$

$u = \int U\,dx \quad \wedge \quad U = \frac{1}{y} e^{\int p\,dx}$

finding U

$Ue^{\int p\,dx} = e^{-1/3 \ln(x)} = \left(e^{\ln(x)}\right)^{-\frac{1}{3}} = x^{-1/3}$

$U = x^{-1/3} \frac{1}{(x^{1/3})^2} = x^{-1/3 - 2/3} = x^{-1} = 1/x$

$u = \int \frac{dx}{x} = \ln|x|$

$y = xy = y \cdot \ln|u| = u^{1/3} \ln x$

hence

$y = c_1 y_1 + c_2 y_2$

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$c_1 x^{1/3} + x^{1/3} \ln x$

$\cancel{x}\; x^{1/3} (c_1 + c_2 \ln x)$

$y' = (x^{1/3})' (c_1 + c_2 \ln x) + x^{1/3} (c + c_2 \ln x)'$

$= \frac{1}{3} \cdot x^{-2/3} (c_1 + c_2 \ln x) + x^{\frac{1}{3}} \cdot c_2 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}}$

$= \frac{1}{3} \cdot x^{-2/3} (c_1 + c_2 \ln x) + x^{-\frac{2}{3}} c_2$

$(1 = y(1) = 1^{1/3} (c_1 + c_2 \ln 1)$

$(0 = y'(1) = 1/3 \cdot 1^{-2/3} (c + c_2 \ln 1) + 1^{-2/3} c_2 \}$

$\begin{cases} 1 = c_1 \\ 0 = \frac{c_1}{3} + c_2 \end{cases}$

$\begin{cases} 1 = c_1 \\ -\frac{1}{3} = c_2 \end{cases}$

$y = x^{1/3} (1 - 1/3 \ln x)$ Ans.

Q19

$(x^2 D^2 - xD - 15I)y = 0$

$y(1) = 1$
$y'(1) = 4.5$

Sol.

Applying given operator on the equation.

$x^2 D^2 y - xDy - 15Iy = x^2 D(Dy) - xDy - 15y$

$= x^2 y'' - xy' - 15y$

let $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} - xmx^{m-1} - 15x^m = 0$

$\cancel{x^2} m(m-1) x^m \cdot \cancel{x^{-2}} - \cancel{x} m \cdot x^m \cancel{x^{-1}} - 15x^m = 0$

Dropping x^m

$= m(m-1) - m - 15 = 0$

$= m^2 - 2m - 15 = 0$

finding the roots

$m^2 - 2m - 15 = 0$,

$m_{1}/_{2} = \dfrac{2 \pm \sqrt{-2^2 + 4 \cdot 15}}{2}$

$m_1/_2 = \dfrac{2 \pm 8}{2}$

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The two ~~root~~ real solution are

$y_1 = x^{m_1} = x^5 \quad \wedge \quad y_2 = x^{m_2} = x^{-3}$

Now the general solution is

$y = C_1 y_1 + C_2 y_2$

$= C_1 x^5 + C_2 x^{-3}$

$\Rightarrow y' = 5C_1 x^4 - 3C_2 x^{-4}$

Now to Determine the C_1 & C_2

$\begin{cases} 0.1 = y(1) = C_1 \cdot 1^5 + C_2 \cdot 1^{-3} \\ 4.5 = (y')(1) = 5C_1 \cdot 1^4 - 3C_2 \cdot 1^{-4} \end{cases}$

$\begin{cases} 0.1 = C_1 + C_2 \\ 4.5 = 5C_1 + (C_2 3C_2) \end{cases}$

$\begin{cases} 0.1 - C_2 = C_1 \\ 5 = 8C_2/8 \end{cases}$

$\begin{cases} 0.1 - 0.625 = C_1 \\ 0.625 = C_2 \end{cases} \Rightarrow \begin{cases} -0.525 = C_1 \\ 0.625 = C_2 \end{cases}$

Dertica Solution

$y = -0.525 x^5 + 0.625 x^{-3}$ Ans.

Question - 02 :-

Question - 01 - a - Part :-

(a) $x' = \sqrt{x}$ $\qquad y' = \frac{dy}{dx}$

Sol:-

$x' = \sqrt{x}$ $\qquad Mdx + Ndy = 0$

$\frac{dx}{dy} = \sqrt{x}$

$dx = \sqrt{x}\, dy$ $\qquad \sqrt{x} = x^{1/2}$

$\Rightarrow \frac{dx}{\sqrt{x}} = dy$ $\qquad \int x^n dx = \frac{x^{n+1}}{n+1}$

$\Rightarrow \int x^{-1/2} dx = \int dy$ $\qquad -\frac{1}{2} + 1$

$\frac{-1+2}{2} = 1/2$

$\Rightarrow \frac{x^{1/2}}{1/2} = y + c$

$\Rightarrow 2\sqrt{x} = y + c$ Ans

(e) $x' = au + b$ $\quad a, b > 0$ $\qquad a, b\ (0, \infty)$

$\frac{dx}{du} = au + b$

$dx = au\,du + b\,du$

$\int dx = a \int u\,du + b \int du$

$\Rightarrow x = a\frac{u^2}{2} + bu + c_1$

$\Rightarrow 2x = au^2 + 2bu + 2c_1$

Put $2c_1 = c$

$\Rightarrow 2x = (au + 2b)u + c$ Ans

(b) $x' = e^{-2x}$

$\int e^{ax}\,dx$

$\frac{dx}{dy} = e^{-2x}$

$\Rightarrow \frac{e^{ax}}{a}$

$\Rightarrow dx = e^{-2x}\,dy$

$\Rightarrow e^{2x}\,dx = dy$

$\Rightarrow \int e^{2x}\,dx = \int dy$

$\Rightarrow \frac{e^{2x}}{2} = y + c_1$

$\Rightarrow e^{2x} = 2y + 2c_1$

put $2c_1 = c$

$\Rightarrow e^{2x} = 2y + c$

(c) $y' = 1 + y^2$

$\frac{dy}{dx} = 1 + y^2$

$\Rightarrow dy = (1+y^2)\,dx$

$\Rightarrow \frac{dy}{1+y^2} = dx$

$\Rightarrow \int \frac{dy}{1+y^2} = \int dx$

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$$\int \frac{1}{a^2+x^2}\,dx$$

$\Rightarrow \tan^{-1} y = x + c$ **Ans** $\qquad = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

(d) $u' = \dfrac{1}{5-2u}$ $\qquad \dfrac{du}{dy} \qquad \dfrac{du}{dx}$

sol =

$$\frac{du}{dx} = \frac{1}{5-2u}$$

$$\Rightarrow du = \frac{1}{5-2u}\,dx$$

$$\Rightarrow (5-2u)\,du = dx$$

$$\Rightarrow \int (5-2u)\,du = \int dx$$

$$\Rightarrow \int 5\,du - 2\int u\,du = \int dx$$

$$\Rightarrow 5u - 2\frac{u^2}{2} = x + c_1$$

$$\Rightarrow 5u - u^2 = x + c_1$$

$$\Rightarrow -5u + u^2 = -x - c_1$$

put $-c_1 = c$

$$\Rightarrow u^2 - 5u = -x + c$$

$$\Rightarrow u^2 - 5u + x = c$$ **Ans**

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(f) $Q' = \frac{Q}{4+Q^2}$

Sol

$$\frac{dQ}{dP} = \frac{Q}{4+Q^2}$$

$$\Rightarrow dQ = \frac{Q}{4+Q^2} dP$$

$$\Rightarrow \frac{4+Q^2}{Q} dQ = dP$$

$$\Rightarrow \left(\frac{4}{Q} + Q\right) dQ = dP$$

$$\Rightarrow \int \frac{4}{Q} dQ + \int Q\,dQ = \int dP$$

$$\Rightarrow 4 \ln Q + \frac{Q^2}{2} = P + C_1$$

$$\Rightarrow 8 \ln Q + Q^2 = 2P + 2C_1$$

put $2C_1 = C$

$$\Rightarrow 8 \ln Q + Q^2 = 2P + C$$

$$\Rightarrow Q^2 + 8 \ln Q = 2P + C$$

Ans

(h) $y' = r(a-y)$

$\Rightarrow \frac{dy}{dx} = ra - ry$

$\Rightarrow dy = (ra - ry)\,dx$

$\Rightarrow \frac{dy}{(ra-ry)} = dx$

$\Rightarrow \frac{1}{r}\frac{dy}{(a-y)} = dx$

$\Rightarrow \int \frac{1}{r}\frac{dy}{(a-y)} = \int dx$

$\Rightarrow \frac{1}{r}\int \frac{dy}{a-y} = \int dx$

$\Rightarrow -\frac{1}{r}\ln(a-y) = x + c_1$

$\Rightarrow \frac{1}{r}\ln(a-y) = -x - c_1$

put $-c_1 = c$

$\Rightarrow \frac{1}{r}\ln(a-y) = -x + c$

$\Rightarrow \frac{1}{r}\ln(a-y) + x = c$ Ans

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Q2-2

Solve $y' = r(a-y)$, where r and a are constants

Sol

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)} \cdot dy = \int dt$$

$$a - y = k(r)^{1/2}$$

Q3:- (a) $x' = \sqrt{x}$ when $x(0) = 1$

$$\frac{dx}{dy} = \sqrt{x}$$

$$\Rightarrow dx = \sqrt{x}\, dy$$

$$\Rightarrow \frac{dx}{\sqrt{x}} = dy$$

$$\Rightarrow x^{-1/2} dx = dy$$

$$\Rightarrow \int x^{-1/2} dx = \int dy$$

$$-\frac{1}{2}+1$$

$$\Rightarrow \frac{x^{1/2}}{1/2} = y + c$$

$$\frac{-1+2}{2}$$

$$1/2$$

$$\Rightarrow 2\sqrt{x} = y + c \rightarrow (1)$$

Now $x(0) = 1$

$x = 1 \quad y = 0$

put in eq (1)

$$2\sqrt{1} = 0 + c$$

$$\Rightarrow 2 = c$$

$\Rightarrow c = 2$ put in eq (1)

we have

$$2\sqrt{x} = y + 2$$

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(b) $x' = e^{-2x}$ when $x(0) = 1$

$x = 1 \quad y = 0$

sol :-

$$\frac{dx}{dy} = e^{-2x}$$

$$\Rightarrow dx = e^{-2x} dy$$

$$\Rightarrow e^{2x} dx = dy$$

$$\Rightarrow \int e^{2x} dx = \int dy$$

$$\Rightarrow \frac{e^{2x}}{2} = y + c_1$$

$$\Rightarrow e^{2x} = 2y + 2c_1$$

put $2c_1 = c$

$$e^{2x} = 2y + c \rightarrow (1)$$

Now

$x = 1 \quad y = 0$

put in eq (1)

$$e^{2(1)} = 2(0) + c$$

$$\Rightarrow e^{2} = c$$

$\Rightarrow c = 7.39$ put in eq (1)

$$\boxed{e^{2x} = 2y + 7.39}$$ Ans

Q4: Find the general solution

(a) $x' = \frac{2x}{t+1}$

sol:

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\Rightarrow \frac{dx}{2x} = \frac{1}{t+1} dt$$

$$\Rightarrow \int \frac{dx}{2x} = \int \frac{1}{t+1} dt$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{x} dx = \int \frac{1}{t+1} dt$$

$$\Rightarrow \frac{1}{2} \ln x = \ln(t+1) + c$$ Ans

(b) $\theta' = t\sqrt{t^2+1} \sec\theta$

sol:

$$\frac{d\theta}{dt} = t\sqrt{t^2+1} \sec\theta$$

$$\Rightarrow \frac{d\theta}{\sec\theta} = t\sqrt{t^2+1}\, dt$$

$$\Rightarrow \cos\theta\, d\theta = t\sqrt{t^2+1}\, dt$$

$$\Rightarrow \int \cos\theta \, d\theta = \int t\sqrt{t^2+1}\, dt$$

$$\Rightarrow \int \cos\theta \, d\theta = \frac{1}{2}\int 2t\,(t^2+1)^{1/2}\, dt$$

$\frac{1}{2}+1$

$\frac{1+2}{2} = 3/2$

$$\Rightarrow \sin\theta = \frac{1}{2}\,\frac{(t^2+1)^{3/2}}{3/2} + C$$

$$\Rightarrow \sin\theta = \frac{2}{2}\,\frac{(t^2+1)^{3/2}}{3} + C$$

$$\Rightarrow \sin\theta = \frac{1}{3}(t^2+1)^{3/2} + C \quad \text{Ans}$$

(c) $(2u+1)u' - (t+1) = 0$

Sol:

$$(2u+1)\frac{du}{dt} = t+1$$

$$\Rightarrow (2u+1)\,du = (t+1)\,dt$$

Taking integration

$$\int (2u+1)\,du = \int (t+1)\,dt$$

$$\Rightarrow 2\int u\,du + \int du = \int t\,dt + \int dt$$

$$\Rightarrow 2\frac{u^2}{2} + u = \frac{t^2}{2} + t + c_1$$

$$\Rightarrow u^2 + u = \frac{t^2}{2} + t + c_1$$

$$\Rightarrow 2u^2 + 2u = t^2 + 2t + 2c_1$$

put $2c_1 = c$

$2u(u+1) = t(t+2) + c$ Ans

(d) $R' = (t+1)(R^2+1)$

Sol:

$$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\Rightarrow \frac{dR}{1+R^2} = (t+1)\,dt$$

Taking integration

$$\int \frac{1}{1+R^2}\,dR = \int (t+1)\,dt$$

$$\Rightarrow \int \frac{1}{1+R^2}\,dR = \int t\,dt + \int dt$$

$$\Rightarrow \tan^{-1} R = \frac{t^2}{2} + t + c$$ Ans

(e) $y' + y + \frac{1}{y} = 0$

Sol:

$$\frac{dy}{dx} + \frac{y^2+1}{y} = 0$$

$$\Rightarrow \frac{dy}{dx} + \frac{y^2+1}{y} = 0$$

$$\Rightarrow y\frac{dy}{dx} + (y^2+1) = 0$$

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$$\Rightarrow \frac{y\,dy}{dx} = -(1+y^2)$$

$$\Rightarrow \frac{y\,dy}{1+y^2} = -dx$$

Taking integration

$$\int \left(\frac{y}{1+y^2}\right) dy = -\int dx$$

$$\Rightarrow \frac{1}{2}\int \frac{2y}{1+y^2}\,dy = -\int dx$$

$$\Rightarrow \frac{1}{2}\ln(1+y^2) = -x + c$$

$$\Rightarrow x + \frac{1}{2}\ln(1+y^2) = c \quad \text{Ans}$$

(f) $(t+1)x' + x^2 = 0$

$$\Rightarrow (t+1)\frac{dx}{dt} + x^2 = 0$$

$$\Rightarrow (t+1)\frac{dx}{dt} = -x^2$$

$$\Rightarrow (t+1)\,dx = -x^2\,dt$$

$$\Rightarrow \frac{dx}{x^2} = -\frac{dt}{t+1}$$

$$\Rightarrow \int x^{-2}\,dx = -\int \frac{dt}{1+t}$$

$$\Rightarrow \frac{x^{-1}}{-1} = -\ln(1+t) + c_1$$

$$\Rightarrow x^{-1} = \ln(1+t) - c_1$$

Put $-c_1 = c$

$$\Rightarrow \frac{1}{x} = \ln(1+t) + C$$ Ans

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