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I.D :→ "7353"

Paper :→ Differential Equation

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Exam :→ Mid-Term (Spring-2020)

Q NO:→ (01)

Solution of Objective type Question.

(i)→ the order of matrix A is $m \times p$ and the order B is $p \times n$. then the order of matrix AB is ?

Answer

Order of matrix AB = rows of matrix A × Column of matrix B

Order of matrix AB = $m \times n$.

(ii)→ the number of non-zero rows in an Echelon from ?

Answer

The number of non-zero row in an Echelon form are Called rank of the matrix.

P.T.O

(iii)⇒ If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Answer

We know $|B| = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$$

$$\Rightarrow a - 8 = 0$$

$$\boxed{a = 8}$$

(iv)⇒ If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Answer

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = 2i^2 - i^2$$

$$|A| = -2(-1) - (-1) \quad \because \boxed{i^2 = -1}$$

$$\boxed{|A| = 3}$$

(V)⇒ The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

P.T.O

Answer

'A' is a Scalar matrix.

(vi)⇒ Solution of $\frac{dy}{dx} + 2xy = y$?

Answer

$$\frac{dy}{dx} = y - 2xy \Rightarrow \frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{y} = (1-2x)dx$$

$$\int \frac{dy}{y} = \int (1-2x)dx$$

$$\ln y = (x - x^2) + C_1$$

$$e^{\ln y} = e^{(x-x^2)+C_1}$$

$$\boxed{y = e^{x-x^2} \cdot e^{C_1}}$$

(vii)⇒ The Order and degree of differential equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is ?

Answer

Taking Square on Both Sides of differential equation.

$$(dy/dx)^2 = 1 + (dy/dx)^2$$

So Degree = 6 and order = 1

P.T.O

(viii) ⇒ The order and degree of Differential Equation
$d^2y/dx^2 - 4xy = \sin(d^2y/dx^2)$ is ?

Answer

Order = 2
and Degree is Undefined.

(ix) ⇒ The differential equation $2\frac{dy}{dx} + x^2y = 2x+3, y(0) = 5$ is?

Answer

$\Rightarrow y' + (x^2/2)y = 1/2(x^2+3) \Rightarrow u = x^2/2$

$e^{\int x^2/2\,dx} = e^{x^3/6} \Rightarrow e^{x^3/6}y' + e^{x^3/6}(x^2/2)y = 1/2\, e^{x^3/6}(x^2+3)$

$y = (0) = 3/2$

$$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + 3/2$$

(x) ⇒ $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Answer

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 1(bc^2 - b^2c) - 1(ac^2 - a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$$
$$= a^2c - a^2b + ab^2 - b^2c + bc^2 - ac^2$$

P.T.O

Alla-ud-Din 7353 Page:→ (05)

Q NO:→ (02) (i)

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factor which are linear in a, b, c.

Solution

$$A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \text{Adj}(A) = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_3 \text{ ------ } (1)$$

Now

$$A_{11} (-1)^{1+1} M_{11} (-1)^2 \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$A_{12} = (-1)^{1+2} M_{12} (-1)^3 \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$$

$$A_{12} = \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$$

$$A_{13} = (-1)^{1+3} M_{13} (-1)^4 \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

Put the Value in (1)

$$|A| = a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} + b(1) \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

P.T.O

Allaud-Din 7353

$= \cancel{abc^3 - ac^2b^3 - ba^2c^3}$

$= a[b^2c^3 - c^2b^3] - b(a^2c^3 - c^2a^3) + c(a^2b^3 - b^2c)$

$= ab^2c^3 - ac^2b^3 - ba^2c^3 + bc^3 + bc^2a^3 + ca^2b^3 - cb^2a.$

$(x^2+3y^2)dy - 2xy\,dy = 0$ at $x=2, y=6$

$M\,dx + N\,dy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = \cancel{8}\,6y, \quad \frac{\partial N}{\partial x} = -2y.$

$\frac{M_y - N_x}{N} = \frac{6y-(-y)}{-2xy} = \frac{6y+2y}{-2xy} = \frac{8y}{-2xy} \Rightarrow -\frac{4}{x}$

$I.F = e^{\int \frac{4}{x}dx} = e^{-4\int \frac{1}{x}dx} = e^{-4\ln x} = e^{\ln x^{-4}}$

$= x^{-4}$

$x^{-4}(x^2+3y^2)dx - \frac{2xy}{-x^4}dy = 6$

$\left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right)dx - \frac{2y\,dy}{x^3} = 6$

$\frac{\partial M}{\partial y} = \frac{6y}{x^4}, \quad \frac{\partial N}{\partial x} = +\frac{6y}{x^4}$

$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$

General Sol

$\int \frac{6y}{x^2}dx + \int 0\,dy$

$6y\int \frac{1}{x^2}dx \Rightarrow \frac{-6y}{x} = c \Rightarrow -6y = cx$

No at $x=2, y=6$

$-6(6) = c(2) \Rightarrow -12 = 2c \Rightarrow c = -6$

$-6y/x = -6 \Rightarrow -6y = -6x$

$-6y + 6x = 0.$

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Alla-ud-Din 7353 Page:- (06)

Q NO:- (02) (ii)

Find The Eigen Value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$ & 'λ' is the eigen value of A.

So

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \det\left\{ \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \right\} = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} + 1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$$

P.T.O

$$\Rightarrow (2-\lambda)\left[(3-\lambda)\{(3-\lambda)(2-\lambda)-1\}+1\{(-1)(2-\lambda)-1\}-1\{1-(-1)(3-\lambda)\}\right]$$

$$+\left[-1\{(3-\lambda)(2-\lambda)-1\}+1\{(-1)(2-\lambda)-0\}-1\{(-1)(-1)-0\}\right]$$

$$-\left[-1\{(-1)(2-\lambda)-1\}-(3-\lambda)\{(-1)(2-\lambda)-0\}-1\{(-1)(-1)-0\}\right]=0$$

$$\Rightarrow (2-\lambda)\{(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(\lambda-3)-(4-\lambda)\}+\{-1(6-3\lambda$$

$$-3\lambda-2\lambda+\lambda^2)+(\lambda-2)-1\}-1\{-1(\lambda-3)-(3-\lambda)$$

$$(\lambda-2)-1\}=0$$

$$\Rightarrow (2-\lambda)(3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda+2\lambda-7)+(2\lambda^2+6\lambda-8)-$$

$$(3\lambda-3+6-\lambda^2-2\lambda)=0$$

$$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow -2\lambda^3+16\lambda^2+36\lambda+\cancel{16}+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-2\lambda^2+12\lambda-\cancel{16}=0$$

$$\Rightarrow \lambda^4+10\lambda^3+32\lambda^2-32\lambda=0$$

$$\lambda(\lambda^3-10\lambda^2+32\lambda-32)=0$$

$$\Rightarrow \lambda=0,\quad \lambda^3-10\lambda^2+32\lambda-32=0$$

Now $\lambda^3-10\lambda^2+32\lambda-32=0$

at $\lambda = 2$

$$(2)^3-10(2)^2+32(2)-32=0$$

$$8-40+64-32=0$$

P.T.O

$\Rightarrow 7/2 - 7/2 = 0$

Using Synthetic Division

$$\begin{array}{c|cccc} 2 & 1 & -10 & 32 & -32 \\ & \downarrow & 2 & -16 & 32 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

So,

$$\lambda^2 - 8\lambda + 16 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$$

$$\Rightarrow (\lambda - 4) = 0, (\lambda - 4) = 0$$

$$\lambda = 4, \quad \lambda = 4$$

So the Required Eigen Values are 0, 2, 4, 4.

Q NO:- (03)

The Rate of Change in the form of differential equation is given by

$(x^2 + 3y^2)dx - 2xy\,dy = 0$. Find the general Solution at $x = 2$ and $y = 6$

Solution

$$(x^2 + 3y^2)dx = 2xy\,dy$$

$$\Rightarrow dy/dx = x^2 + 3y^2/2xy$$

Let

$$u = y/x$$

$$\Rightarrow y = ux$$

P.T.O

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} ux$$

$$\frac{dy}{dx} = u(1) + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{x^2 + 3(ux)^2}{2x(ux)}$$

$$u + x\frac{du}{dx} = \frac{x^2 + 3u^2x^2}{2ux^2}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x^2}(1-3u^2)}{2u\cancel{x^2}}$$

$$u + x\frac{du}{dx} = \frac{1+3u^2}{2u}$$

$$x\frac{dy}{dx} = \frac{1+3u^2}{2u} - u$$

$$x\frac{dy}{dx} = \frac{1+3u^2-2u^2}{2u}$$

$$x\frac{dy}{dx} = \frac{1+u^2}{2u}$$

$$x\,du = \frac{1+u^2}{2u}dx$$

$$\Rightarrow \frac{2u}{1+u^2}du = \frac{dx}{x}$$

P.T.O

Integrating $\Rightarrow \int \frac{2u}{1-u^2}\,du = \int \frac{dx}{x}$

$\Rightarrow \ln(1+u^2) = \ln x + \ln c$

$1+u^2 = Cx$

$\Rightarrow \frac{x^2+y^2}{x^2} = Cx$

Now putting $x=2$ & $y=6$ in the above equation.

$\frac{4+36}{4} = 2c$

$\Rightarrow 2c = 40/4$

The general solution is;

$\frac{x^2+y^2}{x^2} = 5x$

$\Rightarrow \boxed{x^2+y^2 = 5x^3}$