Q No 1:-

$$\frac{dy}{dx} = 2x \quad ; \quad y(0) = 1$$

$$h = 0.1$$

Solution:-

$$f(x, y) = 2x$$

$$x_0 = 0 \quad , \quad y_0 = 1$$

$$h = 0.1$$

$$x_{n=1} = x_n + h$$

put $n = 0$

$$x_1 = x_0 + h$$
$$= 0 + 0.1$$
$$= 0.1$$

$$x_2 = x_1 + h$$
$$= 0.1 + 0.1$$
$$= 0.2$$

$$x_3 = 0.3$$
$$x_4 = 0.4$$
$$x_5 = 0.5$$

1st Iteration:-

Euler's formula:- $y_{n+1} = y_n + hf(x_n + y_n)$

$n = 0$

$y_1 = y_1 + hf(x_0, y_0)$

$= 1 + 0.1 [(0)(1)]$

$= 1.1$

Modified Euler's method:-

$y_{n+1} = y_n \ h/2 \ [F(x_n, y_n) + f(x_{n+1}, y_{n+1})]$

$y_1 = y_0 + h/2 \ [f(x_0, y_0) + (x_1, y_2)]$

$y_1 = 1 + \frac{0.1}{2} [(0)(1) + (0.1)(1.1)]$

$y_1 = 1 + 0.05 [1 + 1.2]$

$= 1 + 0.05 (.2.2)$

$\boxed{= 1.11}$

2nd Iteration:-

Euler's formula

$y_2^* = y_1 + hf(x_1, y_1)$

$y_2^* = 1.11 + 0.1\left[(0.1)+(1.11)\right]$

$y_2^* = 1.11 + 0.1\,(1.21)$

$= 1.11 + 0.121$

$= 1.231$

Modified Euler's method

$y_2 = y_1 + 0.1/2\left[x_1, y_1\right] + \left[x_2 +, y_2^*\right]$

$= 1.11 + 0.05\left[(0.1)+(1.11)+(0.2)+(1.231)\right]$

$= 1.11 + 0.05\left[1.21 + 1.431\right]$

$= 1.11 + 0.05\,(2.641)$

$\boxed{= 1.242}$

3rd Iteration:-

$y_3^* = y_2 + hf(x_2 + y_2)$

$y_3^* = 1.242 + 0.1(0.2) + (1.242)$

$= 1.247 + 0.1442$

$= 1.3862$

Modified euler's method

$y_3 = y_2 + 0.1/2 \left[x_2 + y_2 \right] + \left[x_3 + y_3^* \right]$

$= 1.242 + 0.05 \left[(0.2) + (1.242) + (0.3) + (1.3862) \right]$

$= 1.242 + 0.05 \left[1.442 + 1.862 \right]$

$\boxed{= 1.398}$

4th Iteration:-

$y_4 = y_3 + hf(x_3, y_3)$

$y_4 = 1.398 + 0.05(0.3 + 1.398)$

$\boxed{y_4 = 1.483}$

Modified Euler's method:-

$y_4 = y_3 + 0.05\,(x_3, y_3) + (x_4, y_4)$

$= 1.398 + 0.05\,[0.3 + 1.398]$

$+ [0.4 + 1.483]$

$= 1.398 + 0.05\,(1.698 + 1.883)$

$\boxed{= 1.577}$

5th Iteration:-

$y_5 = y_4 + hf\,(x_4, y_4)$

$= 1.577 + 0.05\,(0.4 + 1.577)$

$= 1.675$

Modified Euler's Method:-

$y_5 = y_4 + 0.05\,(x_4, y_4) + (x_5, y_5)$

$= 1.577 + 0.05\,(0.4 + 1.577)$

$+ (0.5 + 1.675)$

$= 1.577 + 0.05\,[4.152]$

$\boxed{= 1.785}$ Ans.

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QNO2:- Use the fourth order Range Kutta method to obtain a sol of $\frac{dy}{dx} = x^2 + x - y$ subject to $y = 0$ when $x = 0$, for $0 \leq x \leq 0.6$ with $h = 0.2$ - work throughout to four decimal places.

Given data:- $y = 0$, $x = 0$, $h = 0.2$

$0 \leq x \leq 0.6$ $\quad y_{n+1} = y_n + K$

"Ist Iteration:-"

$y_1 = y_0 + K$, $K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = hf(x_n, y_n)$

$K_1 = h(x_0^2 - x_0 - y_0)$

$K_1 = 0.2(0^2 - 0 - 0)$

$K_1 = 0$

$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$

$= 0.2f(x_0 + h/2, y_0\ h/2)$

$$= 0.2f\left(0+\frac{0.2}{2},\ 0+\frac{0.2}{2}\right)$$

$$= 0.2f(0.1,\ 0.1)$$

$$= 0.2\,(0.1^2+0.1-0.1)$$

$$\boxed{K_2 = 0.0020}$$

$$K_3 = hf\left(x_n+\frac{h}{2},\ y_n+\frac{K_2}{2}\right)$$

$$= 0.2f\left(0+\frac{0.2}{2},\ 0+\frac{0.002}{2}\right)$$

$$= 0.2f(0.1,\ 0.001)$$

$$= 0.2\,(0.1^2+0.1-0.001)$$

$$\boxed{K_3 = 0.0218}$$

$$K_4 = hf(x_n+h,\ y_n+K_3)$$

$$= 0.2f(0+0.2,\ 0+0.0218)$$

$$= 0.2f(0.2,\ 0.0218)$$

$$= 0.2\,(0.2^2+0.2-0.0218)$$

$$\boxed{K_4 = 0.0436}$$

$$K = \frac{1}{6}\left(0+2(0.002)+2(0.218)+0.0436\right)$$

$$K = 0.0152$$

$$y_1 = 0+0.0152$$

$$\boxed{y_1 = 0.0152}$$

Q No 03:-

Given data:-

$a = 0$, $b = 10$, $n = 10$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution:-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

using formula:-

$$f(x)\,dx = \frac{h}{2}\Big(f(x_0)+2(f(x_1)+f(x_2)+f(x_3) \ldots \ldots f(x_9)) + f(x_{10})\Big)$$

$$= \frac{1}{2}\Big[10.1+2(17.2+24.4+29.2+34.6+41.2+50.9+57.8+60.3+61.2)+62.1\Big]$$

$$\boxed{= 412.9} \quad \text{Ans}$$

Qn 104:- (9)

$$\int_2^3 \ln(x^3+1)\,dx$$

use 10 Strips

Solution:- $n = 10$

$$h = \frac{3-2}{10} = 0.1$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\Big[f(x_0) + 4\big(f(x_1) + f(x_3) \cdots\big) + 2\big(f(x_2 \cdots) + f(x_n)\big)\Big]$$

$$= \frac{0.1}{3}\Big(0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + 2(1.003 + 1.320 + 1.628 + 1.922) + 2.062\Big)$$

$$\boxed{= 1.184} \text{ Ans}$$