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Paper = Calculs.

Department = Civil Engineering.

Exam = Mid.

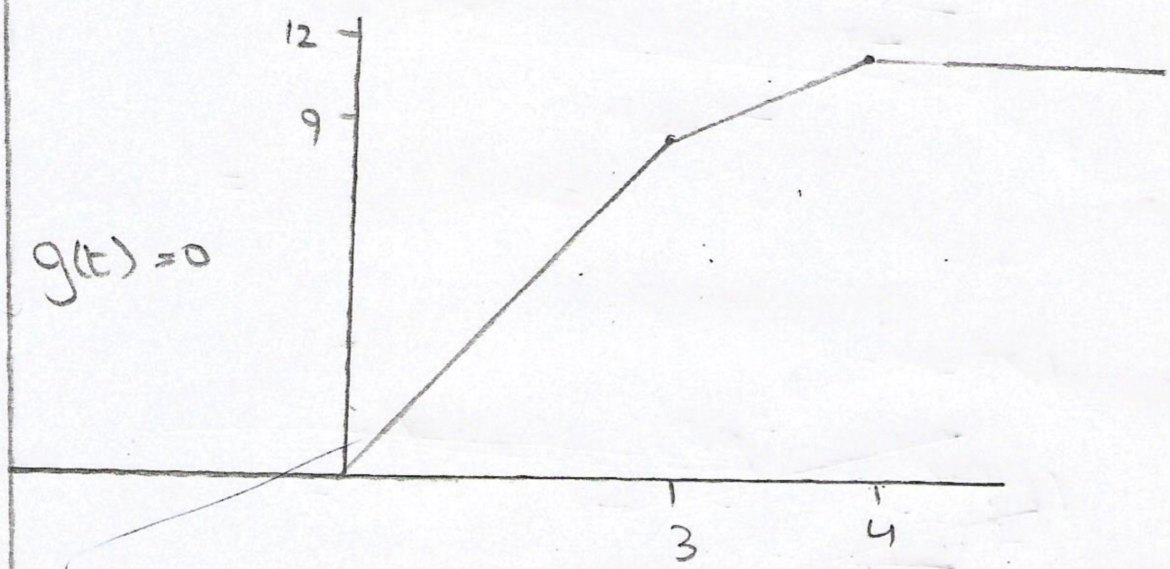
Q. 1) The function g(t).

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

Solution:

a) There is no part of discontinuity.

so these is cotinuity



b) Now to find the g(t) as $t \to 3$

For R. H. L

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} 2t + 3$$

$= 2(3)+3$

$= 6+3$

$= 9$

for L.H.L

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} t^2$$

$= (3)^2$

$= 9$

L.H.L = R.H.L

Q.2) Find the Maclaurin's series for

$y(x) = x^2 + \sin x$

Solution:

$f(x) = x^2 + \sin x$

$f'(x) = 2x + \cos x$

$f''(x) = 2 - \sin x$

$f'''(x) = -\cos x$

$f^{iv}(x) = \sin x$

$f(0) = 0$

$f'(0) = 2(0) + \cos 0$

$= 0 + 1$

$= 1$

$f''(0) = 2 - \sin 0$

$= 2 - 0$

$= 2$

$f'''(0) = -\cos 0$

$= -1$

$f^{iv}(0) = \sin 0$

$= 0$

using Maclaurin's series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0)$$

$$x^2 + \sin x = 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(1) + \frac{x^4}{4!}(0)$$

$$x^2 + \sin x = x + \frac{x^2}{\cancel{2}}(\cancel{2}) + \frac{x^3}{3!} + 0$$

$$x^2 + \sin x = x + x^2 + \frac{x^3}{3!} + 0$$

Q.3a) Find y'' (5)

$$1 + xy = x^2 + y^2$$

Solution:

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$x\frac{dy}{dx} + y \cdot 1 = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$y' \Rightarrow \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$y'' \Rightarrow \frac{d^2y}{dx^2} = \frac{(x-2y)\left(2 - \frac{dy}{dx}\right) - (2x-y)\left(1 - 2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)\left(2 - \left(\frac{2x-y}{x-2y}\right)\right) - \left((2x-y)(1-2)\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\left(\frac{2x-y}{x-2y}\right)\right) - (2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)}{(x+2y)^2}$$

$$= \frac{(x-\cancel{2y})\left(\frac{2(x-2y)-(2xy)}{\cancel{x-2y}}\right) - (2x-y)\left(\frac{x-2y-2(2x-y)}{x-2y}\right)}{(x+2y)^2}$$

$$= \frac{(2\cancel{x}-4y-\cancel{2}x+y) - (2x-y)\left(\frac{x-\cancel{2y}-4x+\cancel{2y}}{x-2y}\right)}{(x+2y)^2}$$

$$= \frac{(-3y) - (2x-y)\left(\frac{-3x}{x-2y}\right)}{(x+2y)^2}$$

$$\cancel{= (-3y)(x-2y) - (}$$

$$= \frac{(-3y) - \frac{(-6x^2+3xy)}{x+2y}}{(x-2y)^2}$$

$$= \frac{\frac{(-3y)(x-2y) - (-6x^2+3xy)}{(x-2y)}}{(x-2y)^2}$$

$$= \frac{\dfrac{-3xy + 6y^2 + 6x^2 - 3xy}{(x-2y)}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-6xy + 6y^2 + 6x^2}{(x-2y)} \times \frac{1}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-6xy + 6y^2 + 6x^2}{(x-2y)^3}$$

Q3b) Find y' by using logarithmic differentiation. (8)

$$y = x^3(1+x)^9 e^{6x}$$

Solution:

$$y = x^3(1+x)^9 e^{6x}$$

applying ln on b/s

$$\ln y = \ln(x^3)(1+x)^9 e^{6x}$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\frac{1}{y}\frac{dy}{dx} = 3\ln x + 9\ln(1+x) + \ln e^{6x}$$

$$\frac{1}{y}\frac{dy}{dx} = 3\cdot\frac{1}{x} + \frac{9}{1+x}\frac{d}{dx}(1+x) + \frac{1}{e^{6x}}\frac{d}{dx}(e^{6x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{(1+x)}(1) + \frac{1}{\cancel{e^{6x}}}\cdot \cancel{e^{6x}}\cdot 6$$

$$\frac{dy}{dx} = y\left[\frac{3}{x} + \frac{9}{(1+x)} + 6\right]$$

Put value of y

$$\frac{dy}{dx} = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$