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Paper Linear Algebra.

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Q 1 (a)

Sol: Calculate the (3, 2) Entry of AB.

Say  $AB = C$ , (3, 2) entry  
as  $C_{32}$  as  $\text{Row}_3(A) \cdot \text{Col}_2(B)$

$$\text{Row}_3(A) \cdot \text{Col}_2(B) = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

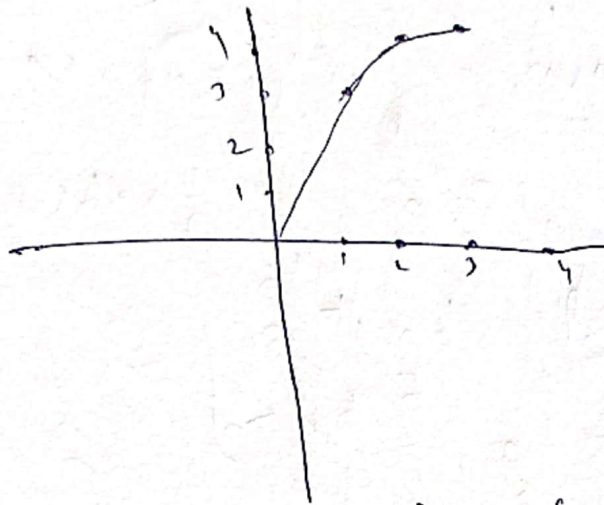
$$= -5$$

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(2)

Q 1 (b)

Sol: Label The Quadratic Polynomial.



Suppose 
$$P(x) = f(1) \frac{(x-2)(x-3)}{(1-2)(1-3)} + f(2) \frac{(x-1)(x-3)}{(2-1)(2-3)} + f(3) \frac{(x-1)(x-2)}{(3-1)(3-2)}$$

(a)  $P(x) = ax^2 + bx + c$ .

o Use the Quadratic formula to

(b) find to find the roots of  $P(x)$ .

(c) Using the above calculation find  $P_3$  of Muller's Method on  $f(x)$  when  $P_0 = 1, P_1 = 2, P_2 = 3$

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$$\begin{bmatrix} 1 & 5 \\ -1 & -7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \quad \textcircled{3} \quad \textcircled{11}$$

Q2 (a)

If  $|A|$  and  $|B|$  are  $n \times n$  Matrix  
 This Mean they are equal  
 by order-

$|A| = 2$  and  $|B| = -3$

So  $A = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$   $B = \begin{bmatrix} 2 & 7 \\ 1 & 2 \end{bmatrix}$

Calculate  $|A^{-1} B^T|$

$A^{-1} = \frac{\text{Adj } A}{|A|}$

$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -2 & 2 \end{bmatrix}$   $|A| = 2$

$A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0.5 \\ -1 & 1 \end{bmatrix}$

$B^T = \begin{bmatrix} 2 & 1 \\ 7 & 2 \end{bmatrix}$

$$A^{-1} B^T = \begin{bmatrix} 1 & 0.5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 7 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 0.5 \times 7 & 1 + 0.5 \times 2 \\ -2 + 7 & -1 + 2 \end{bmatrix}$$

$$A^{-1} B^T = \begin{bmatrix} 5.5 & 2 \\ 5 & 1 \end{bmatrix}$$

$$|A^{-1} B^T| = \begin{vmatrix} 5.5 & 2 \\ 5 & 1 \end{vmatrix}$$

$$= (5.5 \times 1) - (5 \times 2)$$

$$= (5.5 - 10) = -5.5$$

$$|A^{-1} B^T| = -5.5$$

X → X

5

Q 2 (b)

$$x + y + 2z = 1$$

$$x - 2y + z = -5$$

$$3x + y + z = 3$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

Now

$$A^{-1} = ?$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = 14$$

Adj A = ?

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = -2 - 1 = -3$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -(-3) = 3$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = 1 + 2 = 3$$

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$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -5$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = +2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = +1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \begin{vmatrix} -3 & 2 & 7 \\ 1 & -5 & 2 \\ 5 & 1 & -3 \end{vmatrix}$$

$$X = A^{-1} B$$

$$X = \frac{1}{14} \begin{bmatrix} -3 & 2 & 7 \\ 1 & -5 & 2 \\ 5 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$$

$$X = \frac{1}{14} \begin{bmatrix} -3 & -10 & +21 \\ 1 & +25 & +6 \\ 5 & -5 & -9 \end{bmatrix} \Rightarrow \frac{1}{14} \begin{bmatrix} 8 \\ 32 \\ -9 \end{bmatrix}$$

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$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{8}{14} \\ \frac{32}{14} \\ -\frac{9}{14} \end{bmatrix}$$

$$x = \frac{8}{14} = \frac{4}{7}$$

$$y = \frac{32}{14} = \frac{16}{7}$$

$$z = -\frac{9}{14}$$



Q:3

Find  $A^{-1}$  where  $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix}$

Sol:  $|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & 3 \end{vmatrix}$

$= 3 \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} + 2 \begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$

$= 3(-4-6) + 2(-15-2) + 1(0-6)$

$= |A| = -94$

Now  $A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -6$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 7 & 1 \\ 5 & 2 \end{vmatrix} = -1$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$



(9)

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{Adj } A)$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 2 & -28 & -28 \end{bmatrix}$$

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The end