Saheeb ul Hassan

ID #16318

subject calculus

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Final term paper

Submited to Himayatullah

Q (1)
(A) Estimate $\int \phi^4 \sqrt{1-\phi^2}\, d\phi$

Solution:- Given that

$$\int \phi^4 \sqrt{1-\phi^2}\, d\phi$$

Let

$$1-\phi^2 = u$$

$$\frac{d}{d\phi}(1-\phi^2) = \frac{d}{d\phi} u$$

$$-2\phi = \frac{du}{d\phi}$$

$$\phi\, d\phi = -\frac{1}{2}\, du$$

Now

$$= \int (u)^{1/4} \cdot \left(-\frac{1}{2}\right) du$$

$$= \frac{-1}{2} \int u^{1/4} \cdot (-1/2)\, du$$

$$= -\frac{1}{2} \int u^{1/4}\, du \qquad \therefore \frac{1}{4}+1 = \frac{5}{4}$$

$$= \frac{-1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{5}\, u^{5/4} + c$$

$$= \frac{-2}{5}\, u^{5/4} + c$$

By back substitution

$$= \frac{-2}{5} (1-\phi^2)^{5/4} + c$$

Q (1) (b) Estimate $\int_0^1 x^3 (1+x^4)^3 \, dx$ using substitution method.

Soloution:- Given

$$\int_0^1 x^3 (1+x^4)^3 \, dx \quad \text{--- (i)}$$

By substitution method

let $1+x^4 = u$

$$d/dx (1+x^4) = \frac{du}{dx}$$

$$0+4x^3 = du/dx$$

$x^3 dx = 1/4 \cdot du$ Put in eq (i)

$$\int_0^1 (u)^3 \cdot 1/4 \, du$$

$$1/4 \left(\int_0^1 u^3 \, du \right)$$

$$1/4 \int_0^1 \frac{u^{3+1}}{3+1}$$

$$1/4 \int_0^1 \frac{u^4}{4} \quad \text{--- (ii)}$$

Put $u = 1+x^4$

$$1/4 \int_0^1 \frac{1+x^4}{4}$$

Apply limits

$$1/4 \left(\frac{1+(1)^4}{4} - \frac{1+0}{4} \int_0^1 \right)$$

$$1/4 \left(\frac{2}{4} - \frac{1}{4} \right)$$

$$1/4 \left(\frac{2-1}{4} \right)$$

$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

$$\boxed{\int_0^1 x^3 (1+x^4)^3 \, du = \frac{1}{16}}$$

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Q(2) Illustrate the centre and radius
(a) of the sphere $x^2+y^2+z^2+3x-4x+1$.

Soloution:- $x^2+y^2+z^2+3x-4x+1=0$

$$(x^2+3y)+y^2+z^2-4z+1=0$$

$$(x^2+3x+(1/3)^2)+(y+0)^2+(z^2-4z+(-4/2)^2)$$

$$=-1+(3/2)^2+(-4/2)^2$$

$$(x+3/2)^2+(y+0)^2+(z-2)^2=\frac{21}{4}$$

So $(x_0, y_0, z_0) \Rightarrow$ centre

$$=(3/2, 0, 2)$$

Find radius $a=\sqrt{\frac{21}{4}}$

Q 2 The region between the curve
(b) $y=\sqrt{x}$, $0 \le x \le 4$, and the x-axis
is revolved about the x-axis to
generate a solid. Apply the Integretion
find the volume of solid

Soloution:-

Given

$$y=\sqrt{x}$$

$$0 \le x \le 4 \Rightarrow a \le x < b.$$

as $v=\int_a^b \pi y^2 \, dx$

$$v=\int_a^b \pi (\sqrt{x})^2 \, dx$$

$$v=\pi \int_a^b x \, dx = \pi \frac{x^2}{2} \Big|_a^4$$

$$v=\frac{\pi}{2}\left((4)^2-0\right)$$

$$=8\pi$$

Q(3) IF $A = 2i - 4j + \sqrt{5}k$, and $B = -2i + 4j - \sqrt{5}k$

then illustrate the vector $Proje_A B$

Soloution:. Given

$A = 2i - 4j + \sqrt{5}K$

$B = -2i + 4j - \sqrt{5}K$

By dot product

$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k$

$= ((-2)(2) + (4)(-4) + (-\sqrt{5})(\sqrt{5}))$

$= -4 - 16 + \sqrt{-5 \times 5}$

$= -4 - 16 - \sqrt{25}$

$= -4 - 16 - 5$

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$\Rightarrow B \cdot A = -25$

Now $A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= ((2)(2) + (-4)(-4) + (\sqrt{5})(\sqrt{5}))$

$= 4 + 16 + \sqrt{5 \times 5}$

$= 4 + 16 + \sqrt{25}$

$= 4 + 16 + 5$

$\boxed{A \cdot A = 25}$

so $\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

Putting values

$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k$

$= (-1)(2i - 4j + \sqrt{5}k)$

$\boxed{\text{Proj}_A B = -2i + 4j - \sqrt{5}k}$ Ans.

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Q(4) Find the area of region between the graph and the x-axis. Where $y = -x^2 + 5x - 4$, $[0, 2]$

Soloution:- Given

$y = Fx = x^2 + 5x - 4$

and

$[a, b] = [0, 2]$

As $a = 0$

$b = 2$

So area under graph will

$A = \int_a^b f(x)\,dx$ by putting the value

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

$$\frac{-x^3}{3} + \frac{5x^2}{2} - 4 \Big]_0^2$$

$$\left(\frac{(-2)^2}{3} + \frac{5(2)^2}{2} - 4(2)\right) - \frac{(0)^3}{3} + \frac{5(0)^2}{2} - 4(0)$$

$$= \left(\frac{-4}{3} + \frac{20}{2} - 8\right) - 0 + 0 - 0$$

$$= \left(\frac{-4}{3} + 10 - 8\right)$$

$$= \frac{-4}{3} + 2 \Rightarrow \frac{-4+6}{3}$$

$$\boxed{A = \frac{2}{3}}$$

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Q5 Estimate the angle between
(a) $A = i - 2j - 2K$ & $B = 6i + 3j + 2K$

Solocution:-

$$A = i - 2j - 2K$$
$$|A| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2K$$
$$|B| = \sqrt{36+9+4} = \sqrt{49} = 7$$

$$\phi = \cos^{-1}\left(\frac{A \cdot B}{|A| \cdot |B|}\right)$$

$$\phi = \cos^{-1}\left(\frac{(i-2j-2K)\cdot(6i+3j+2K)}{3 \times 7}\right)$$

$$\phi = \cos^{-1}\left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21}\right)$$

$$\phi = \cos^{-1}\left(\frac{-4}{21}\right)$$

Q(5) Find a spherical coordinate
(b) equation for the sphere
$x^2+y^2+(z-1)^2=1$

Solution :- Given
$$x^2+y^2+(z-1)^2=1$$

$$(\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2+(\rho\cos\phi-1)^2=1$$
$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$
$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$
$$\rho^2(\sin^2\phi)+\rho^2\cos^2\phi-2\rho\cos\phi=1-1$$
$$\rho^2.(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$
$$\rho^2=2\rho\cos\phi$$

$$\boxed{\rho=2\cos\phi}$$ Ans