M T W T F S Date: ....

M. UMAIR

ID: 5902

Electrical.

Signal and System.

25/6/2020.

Question No 1

As we know that

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Differentiating both sides with r.t

$$\frac{dX}{d\omega}(j\omega) = \int_{-\infty}^{\infty} jt\, x(t)\, e^{-j\omega t} \cdot dt$$

$$\frac{dX}{d\omega}(j\omega) = -jt \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\frac{dX}{d\omega}(j\omega) = -jt\, F\{x(t)\}$$

$$-jt\, x(t) \xleftrightarrow{f} \frac{d}{dt} X(j\omega).$$

Question 1(b)

=> Solution

$X(z) = 2 - 4z^{-2} + 2z^{-3}$

$h(z) = 3 + z^{-1} + 2z^{-2}$

Now

$= H(z) . X(z)$

$= (2 - 4z^{-2} + 2z^{-3})(3 + z^{-1} + 2z^{-2})$

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} + 2z^{-5}$

$= 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$

To find y(x) use the delay property

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 9\delta[n-3]$

$- \delta[n-4] + 4\delta[n-5]$

Question 2.

Solution: →

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x).\,dx\right]$$

$$= \frac{1}{2\pi}\int_{\pi}^{0} -\pi/2\,dx + \pi/2\int_{0}^{\pi/2} dx$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}x\Big|_{-\pi}^{0}\; x\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi/2(\pi) + \pi/2(\pi)\right]$$

$$= \frac{1}{2\pi}\left[-\cancel{\pi/2} + \cancel{\pi/2}\right]$$

$$= \frac{1}{2\pi}\left[0/2\right] \Rightarrow a = 0$$

Date: ....

Now coefficent.

$$a_x = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_{-x}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \cos nx\,dx + \int_{0}^{\pi} \pi/2 \cos nx\,dx$$

$$= \frac{1}{\pi}\left[\pi/2 \frac{\sin nx}{n}\Big|_{-\pi}^{0} + \pi/2 \frac{\sin nx}{n}\Big|_{0}^{n}\right.$$

$$= \frac{1}{n\pi}\left[-\pi/2 \sin n(0) - \sin n(-\pi) \right]$$

$$+ \pi/2 \left[\sin n(\pi) - \sin n(0) \right]$$

$$= \frac{1}{n\pi}\left[-\pi/2 (0) + \pi/2 (0) \right]$$

$$= \frac{1}{n\pi}(0)$$

$$a_n = 0$$

Now

$$bn = \frac{1}{\pi}\int_{\pi}^{\pi} \sin dx.$$

(5)

Date:

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi/_2 \sin nx\,dx + \int_{0}^{\pi} \pi/_2 \sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\pi/_2\int_{-\pi}^{0}\sin nx\,dx + \pi/_2\int_{0}^{\pi}\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\pi/_2 \; -\frac{\cos nx}{n}\Big|_{-\pi}^{0} + \pi/_2 \; -\frac{\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\pi/_2\left[-1+\cos n.(\pi)\right] + \pi/2\left[-\cos n\pi + 1\right.\right.$$

$$\pi/_2 = \frac{1}{n\pi}\left[-1\left[1+\cos n(-\pi)\right] + 1\left[-\cos nx + 1\right]\right.$$

$$= 1/_{2n}\left[1.\cos n\pi - \cos n\pi + 1\right]$$

$$= 1/_2\left[2 - 2\cos n\pi\right)$$

Now

$$bn = \begin{cases} 0 & \text{if } n \text{ is even} \\ 4/2n & \text{if } n \text{ is odd} \end{cases}$$

$$bn = 4/2n$$

$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$

$\cdots \; b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots)$

$f(x) = (0) + (0) \cos x + 0 \cos(2x) + 0 \cos 3x + \cdots$

$= 4/_2 \sin x + (0) \sin^2 x + 4/_{3(2)} \sin 3x + \cdots$

$\{ 4/_2 \sin x + 4/_6 \sin 3x + \cdots \}$

Question 3

$$x(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$x(z) = \frac{2z(z+1)}{z^2+2z-3}$$

$$\frac{x(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{A}{z+3} + \frac{B}{z-1} \quad \text{— (1)}$$

$$2(z+1) = A(z-1) + B(z+3) \quad \text{— (2)}$$

put $z = 1$

$$2(1+1) = A(1-1) + B(1+4)$$

$$4 = 0 + 4$$

$$B = 1$$

Now put $z = -3$ in eq (ii)

$z(-3+1) = A(-3-1) + B(-3+3)$

$g(-2) = A(-4) + 0$

$-4 = 4A$

$A = 1$ put in (1)

$$\frac{g(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{z1}$$

Inverse z - transform.

$x(n) = u[3] + 1[-1]$.

8

QUESTION 4

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$D = [0]$$

$$G(s) = C(SI - A)^{-1} B + 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ 0-1 & s-0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}.$$

Question NO 5

$$x(t) = e^{-at}\, u(t) \quad ; \quad a > 0$$

$$x(j\omega) = ?$$

Solution:-

(CT) and (FT) of a given signal $x(t)$ is given by.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

As given signal is multiplied by a step function, hence limit $0 \le t \le \infty$

$$x(j\omega) = \int_{0}^{\infty} e^{-at} (1)\, e^{-j\omega t}\, dt.$$

$$\int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= \frac{-1}{a+j\omega} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{-1}{a+j\omega} \left[0 - 1 \right]$$

$$x(j\omega) = \frac{1}{a+j\omega}$$

Since the fourier transform is a complex valued, so to plot it as a function of 'ω' we express $x(j\omega)$ in term of it magnitude and phase.

$$|x(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \Rightarrow \text{Magnitude.}$$

$$\angle x(j\omega) = -\tan^{-1}(\omega/a).$$