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Section: A

Subject: MOS 2

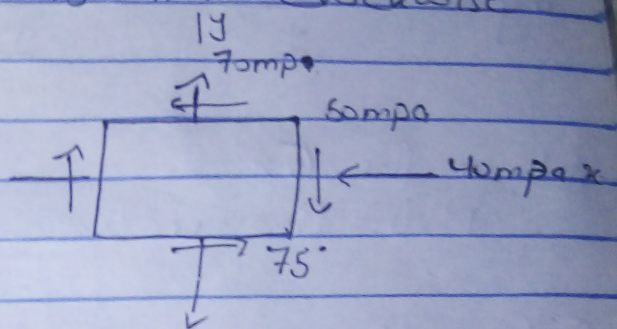
Submitted to: Sir Usama

Date : 22 August 20

Question

The state of stress at point is shown by element of a beam shown in fig below.

Determine state of stress at point rotated 15° in clockwise direction



Sol:-

Data:-

$$\sigma_x = -40 \text{ mpa}$$

$$\sigma_y = 70 \text{ mpa}$$

$$\tau_{xy} = 50 \text{ mpa}$$

Find Stress at point of rotated 15° in clockwise direction.

$$\sigma'_x = ?$$

$$\sigma'_y = ?$$

$$\tau'_{xy} = ?$$

Now we derived the following equation

Equations =

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad (2)$$

$$\sigma_{y'} = -\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Now for $\sigma_{x'}$

put the value in equation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{x'} = \frac{-40 + 70}{2} + \frac{(-40 - 70)}{2} \cos 2(-15^\circ) + (-50) \sin 2(-15^\circ)$$

$$\sin 2(-15^\circ)$$

$$\sigma_{x'} = -7.63 \text{ mpa}$$

Now for $\sigma_{y'}$

putting value in equation

$$\sigma_{y'} = -\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = 37.63 \text{ mpa}$$

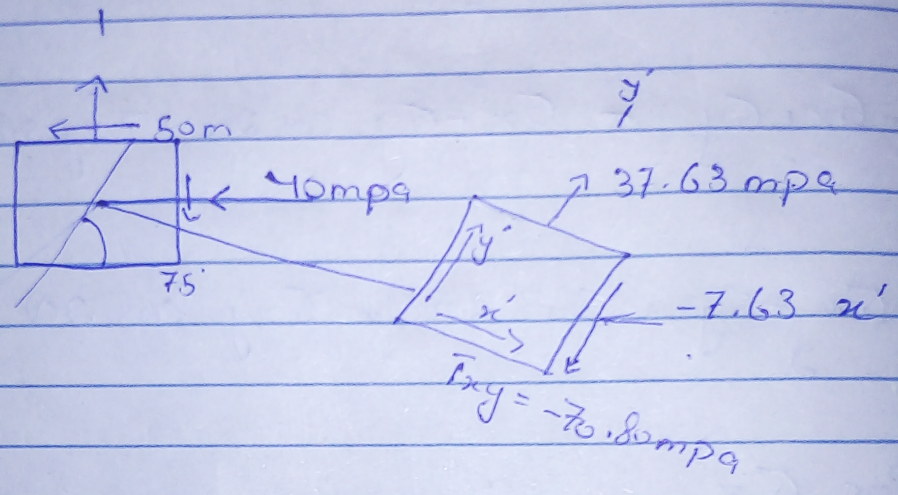
$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

put value

$$\tau_{x'y'} = \frac{(-40 - 70)}{2} \sin 2(15^\circ) + (-50) \cos(-15^\circ)$$

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$$\tau_{xy}' = -70.80 \text{ MPa}$$



Ques

Sho

(a)

(b)

Sol:-

Question :- 2

for the element of a beam shown in Q1, find the following

(a) Principal plane and principal stress

(b) Maximum in plane shear stress and shear plane.

Sol:-

First we find principal plane

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\alpha_p = \frac{2(-50)}{-40 - 30}$$

$$\tan 2\alpha_p = 0.9091$$

$$2\alpha_{p1} = \tan^{-1}(0.9091) = 42.27$$

$$\alpha_{p2} = 21.14^\circ \text{ (for } x\text{-axis)}$$

Also

$$2\alpha_{p2} = 42.27 + 180$$

$$\alpha_{p2} = \frac{222.27}{2} = \alpha_{p1} = 111.135$$

(for y-axis)

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Now we find principal stress as

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-40 + 70}{2} + \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$\sigma_1 = 89.33 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_2 = \frac{-40 + 70}{2} - \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (50)^2}$$

$$\sigma_2 = -59.33 \text{ MPa}$$

We must check which angle goes with principal stress

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$$

$$\sigma_{x1} = \frac{-40 + 70}{2} + \frac{(-40 - 70)}{2} \cos 2(21.14) + (50) \sin 2(21.14)$$

$$\sigma_{x1} = 59.33 \text{ MPa}$$

$$\sin(21.14)$$

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Hence

$$\sigma_1 = 89.33 \text{ mpa with } \theta_{p1} = 111.135^\circ$$

$$\sigma_2 = -59.33 \text{ mpa with } \theta_{p2} = 21.14^\circ$$

Part b)

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Given data

The state of stress which are given

$$\sigma_x = 40 \text{ mpa}$$

$$\sigma_y = 70 \text{ mpa}$$

$$\tau_{xy} = -50 \text{ mpa}$$

Required

Maximum In-plane shear stress = ,
Maximum shear plane ?

Solution

first we find max in plane shear stress

$$\tau_{\text{max in plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (50)^2}$$

$$\tau_{\text{max in plane}} = 74.3 \text{ mpa}$$

$$\tau_{xy} = \frac{\sigma_x + \sigma_y}{2} = \frac{40 + 70}{2}$$

$$\tau_{xy} = 15$$

Now have to find Max shear plane

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$$\tan 2\theta_s = \frac{(3x - 6y)/2}{T_{xy}}$$

$$= \frac{-(40 - 70)/2}{-50}$$

$$\tan 2\theta_s = -1.1$$

$$\theta_s = \frac{1}{2} \tan^{-1}(-1.1)$$

$$\theta_s = -2.3.90 \text{ Ans.}$$