Date:____________

Differential Equation

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Section: A

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(Q1)

Solve:

$$\frac{dy}{dx} = e^{y-t}\sec(y)(1+t^2)$$

For $y(0)=0$

Solution:

$$\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2)$$

$y(0)=0$ so $x=0$ $y=0$

$$dy = e^{y}\cdot e^{-t}\sec(y)(1+t^2)\,dt$$

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$$\frac{1}{e^{y}\cdot \sec(y)}\,dy = (1+t^2)e^{-t}\cdot dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\Rightarrow \int e^{-y}\cos y\,dy = \int (1+t^2)\,e^{-t}\cdot dt$$

Using Integration by parts

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy}e^{-y}\right) =$$

$$(1+t^2)\int e^{-t} - \int\left(e^{-t}\cdot \frac{d}{dt}(1+t^2)\right) \longrightarrow eq(1)$$

L.H.S

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy}e^{-y}\right)$$

$$\Rightarrow e^{-y}\sin y - \int\left(\sin y \cdot e^{-y}(-1)\right)$$

$$\Rightarrow e^{-y}\sin y + \int\left(\sin y \cdot e^{-y}\right)$$

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Again using integration by parts

$e^{-y}\sin y + e^{-y}(-\cos y) - \int \left(\int \sin y \frac{d}{dy} \cdot e^{-y} \right)$

$e^{-y} \cdot \sin y + e^{-y}(-\cos y) - \int \left(-\cos y \frac{e^{-y}}{-1} \right)$

$\Rightarrow e^{-y}\sin y + e^{-y}\cos y - \int (\cos y \cdot e^{-y})$

$\Rightarrow$ Since $\int (\cos y e^{-y}) = L \cdot H \cdot S$

Since it is again same to the first one so L·H·S will become

$L \cdot H \cdot S = e^{-y}(\sin y - \cos y) - L \cdot H \cdot S$

$\Rightarrow 2 L \cdot H \cdot S = e^{-y}(\sin y - \cos y)$

$L \cdot H \cdot S = \frac{e^{-y}(\sin y - \cos y)}{2}$

Stationery

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Now taking R.H.S

$$\Rightarrow \int (1+t^2)e^{-t}\,dt$$

$$\Rightarrow -(1+t^2)e^{-t} - \int (-e^{-t}(2t))$$

$$-(1+t^2)e^{-t} + \int (2t)e^{-t}$$

Again using integration by parts

$$\Rightarrow -(1+t^2)e^{-t} + (2t\int e^{-t} - \int(\int e^{-t} \frac{d}{dt}2t)$$

$$\Rightarrow -(1+t^2)e^{-t} + (-2te^{-t} - \int(-e^{-t}.2)$$

$$\Rightarrow -(1+t^2)e^{-t} + (-2te^{-t} + \int(2e^{-t})$$

$$\Rightarrow -(1+t^2)e^{-t} + (-2te^{-t} - 2e^{-t}) + C$$

$$\Rightarrow -(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + C$$

Stationery

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$\Rightarrow -(t^2+2t+3)e^{-t}+c = R.H.S$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t}+c$$

we know that

$x=0 \quad , \quad y=0$

put it above

$\Rightarrow (0-1) = -3+c$

$c = 5/2$

put value of c

$$\boxed{\Rightarrow \frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2+2t+3)e^{-t} + 5/2}$$

Answer

(Q2)

$$(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{x+y}-\sqrt{x-y})dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$$

This is Homogeneous Differential equation in x and y to solve this put

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq ① becomes

$$v+x.\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x.\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x.\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

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$$v + x\frac{dv}{dx} = \frac{1 + \cancel{v} + \cancel{1} - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1 + \sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$v + x \cdot \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v}$$

$$x \cdot \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v} - v$$

$$x \cdot \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2} - v^2}{v}$$

$$x \cdot \frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1 + \sqrt{1-v^2}\right)}{v}$$

$$\frac{\frac{dv}{dx}}{\sqrt{1-v^2}\left(1 + \sqrt{1-v^2}\right)} = \frac{dx}{x}$$

taking integral on b/s

$$\int \frac{x \cdot dv}{\sqrt{1-v^2}\left(1 + \sqrt{1-v^2}\right)} = \int \frac{dx}{d}$$

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$-\ln t = \ln x + \ln c$

$-\ln t \; (1+\sqrt{1-v^2} = \ln cx$

$\ln(1+\sqrt{1-v^2} = -\ln cx$

$\cancel{\ln}(1+\sqrt{1-v^2} = \cancel{\ln}(cx)^{-1}$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$\boxed{x+\sqrt{x^2-y^2} = C, \quad \because \frac{1}{c} = C}$$

Hence, which is our solution

Q No 3 (a)

Sol:-

$(D^4+D^2)\, y = 3x^2+4\sin x-2\cos x$

$\Rightarrow f(D)\, y = f(x)$

As it is non homogenous linear equation so solution will be

$y = y_c + y_p$ — (i)

$D^4-D^2=0 \Rightarrow D^2(D^2+1)=0$

either $D^2=0 \Rightarrow \boxed{D=0}$

$D^2+1=0 \Rightarrow D^2=-1$

$D=\sqrt{-1} \Rightarrow D=i$ or $D=\boxed{0+i}$

Root are real and complex

$y_c = c_1 e^{-0x} + e^{0x}(c_2\cos x + c_3\sin x)$

$y_c = c_1 + c_2\cos x + c_3\sin x$

$y_p = \frac{1}{f(D)} f(x)$

$$y_p = \frac{1}{D^4+D^2}(3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$$f(D) = D^4 + D^2$$

at $D = 0 \Rightarrow f(D) = 0$.

So $f(D) = 4D^3 + 2D$

Now also for $D = 0 \Rightarrow f'(D) = 0$

again Differentiating

$$F''(D) = 12D + 2$$

so for $D = 0$

$$f''(0) = 12(0) + 2 = 2$$

so replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$y_P = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} 4\sin x - \frac{x^2 2\cos}{12D+2}$$

putting $D = 0$ in all

P.T.O

(3) So

$$yp = \frac{x^2 \, 3x^2}{12(0)+2} + \frac{x^2 \, 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$yp = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

So putting in equation (i)

$$y = c_1 + c_2 \cos x + c_3 \cos x + \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$

Ans