

1

NAME → MUHAMMAD USAMA

ID NO: → 16747

DEPT → BS(CS)

SUBJECT → DISCRETE MATHS

TEACHER → SAIF ULLAH JAN SIR  
NAME

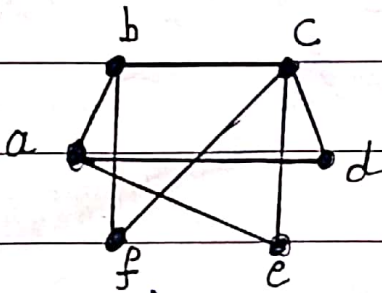
DATE → 24/06/2020

(FINAL TERM EXAM)

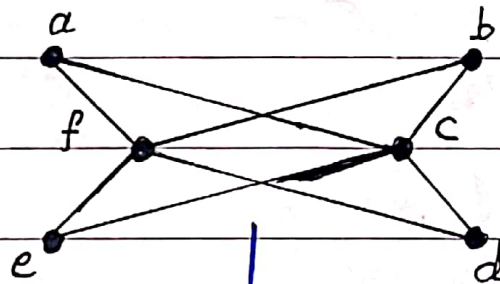
IQRA NATIONAL UNIVERSITY  
(PESHAWAR)

QNO: 1-----

Determine whether the Graph is bipartite.



①



②

ANS::

Bipartite Graph::

A Bipartite Graph  $G$  is a Simple

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Graph whose vertex set can be partitioned into two mutually disjoint non empty subsets  $A$  and  $B$  such that the vertices in  $A$  may be connected to vertices in  $B$ ,

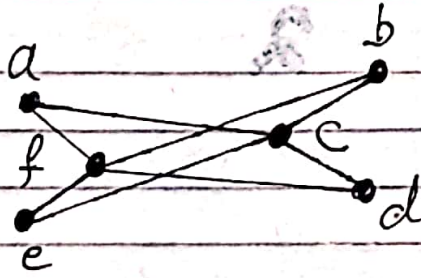
But no vertices in  $A$  are connected to vertices in  $A$  and no vertices in  $B$  are connected to vertices in  $B$ .

→ Determining bipartite Graphs:  $\cup$

The following labeling procedure determines whether a Graph is bipartite or not.

- 1) Label any vertex  $a$ .
  - 2) Label all vertices adjacent to  $a$  with the label  $b$ .
  - 3) Label all vertices that are adjacent to a vertex just labeled  $b$  with label  $a$ .
  - 4) Repeated steps 2 and 3 until all vertices got a distinct label (a bipartite graph) or there is a conflict i.e., a vertex is labeled with  $a$  and  $b$  (Not a bipartite graph)
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(i) -

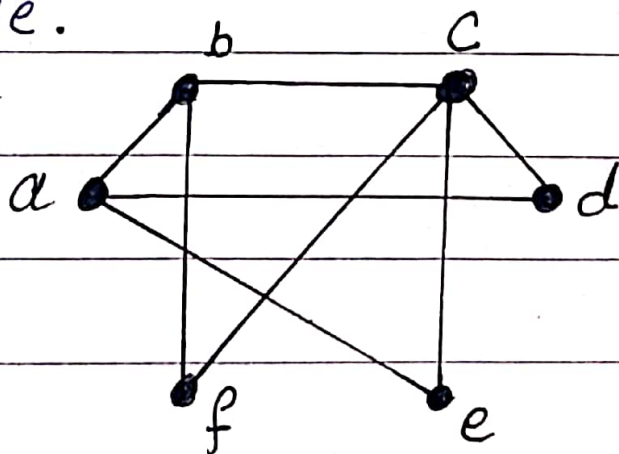
Solution:

- a). First assign red color to vertex a. Next assign blue color to vertices b, c, and d which are adjacent to vertex a. Finally, assign red color to vertex e. As the graph shows on the left-hand side, there are no two adjacent vertices having the same color, hence, (the graph is bipartite) in fact, it is the complete bipartite  $K_{2,3}$  as the graph shown on the right-hand side.

Sol: for the <sup>below</sup> Above Graph,  
The Graph is bipartite  
Showing they are 2-Colorable.

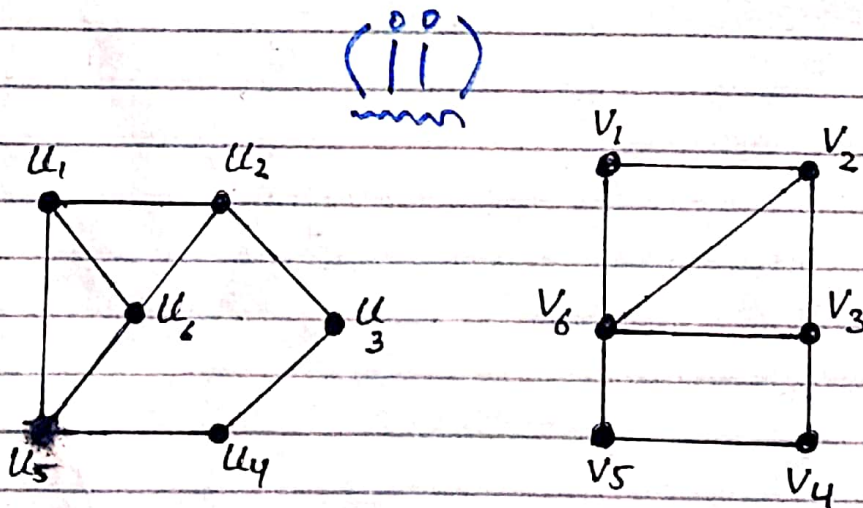
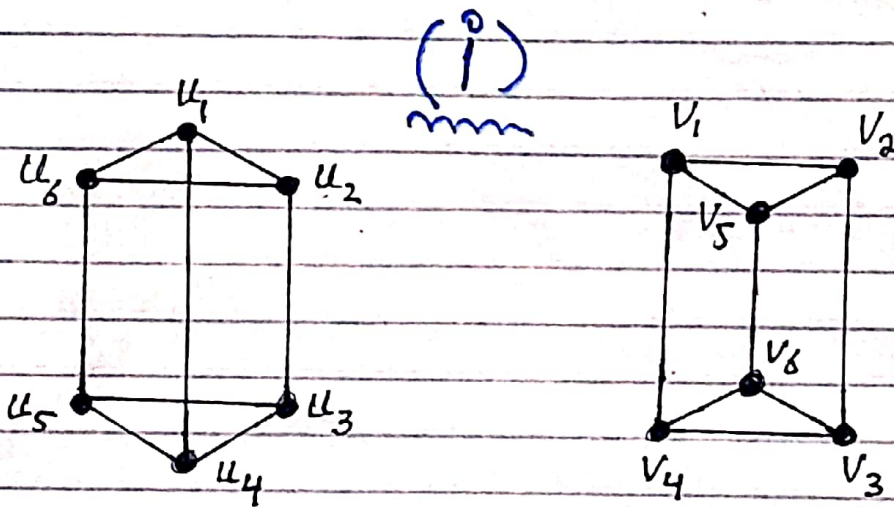
The easy one are  
the one that they are  
2-Colorable you just  
find ~~of the vertices~~  
As 2-colorable i.e.  
a coloring of the  
vertices of the same color  
are never adjacent along  
an edge.

(11)  
m



QNO: 2

Determine whether the Graph is isomorphic.



ANS:

## Isomorphism of a Graph:

The Simple Graph

$G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are

isomorphism  $f$  from  $V_1$

to  $V_2$  with the property

that  $a$  and  $b$  are adjacent

in  $G_1$  if and only if

$f(a)$  and  $f(b)$  are

adjacent in  $G_2$ , for

all  $a$  and  $b$  in  $V_1$ .

Such a function  $f$  is

called isomorphism.

“two Simple Graph

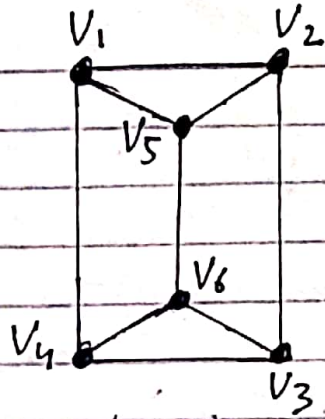
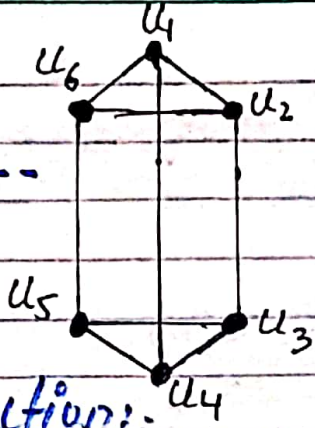
that are not isomorphic

are called nonisomorphic.”



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(i) ---



Solution:

The graph is isomorphic.

One isomorphism is  $f(u_1) = v_5$ ,  $f(u_2) = v_2$ ,  $f(u_3) = v_3$ ,  $f(u_4) = v_6$ ,  $f(u_5) = v_4$ , and  $f(u_6) = v_1$ .

Sol :-

first we see the solution Graphs.

they are non-isomorphic

because when we compare

vertices  $v$  and  $u$ . that

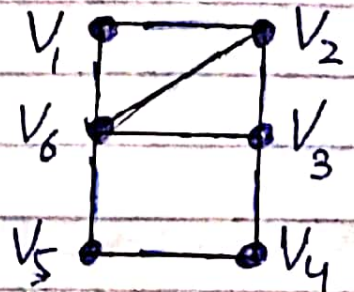
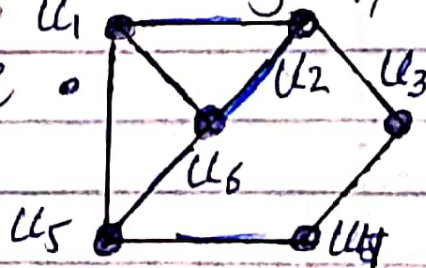
are not same to

one another.

These two

the above graphs is

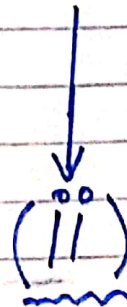
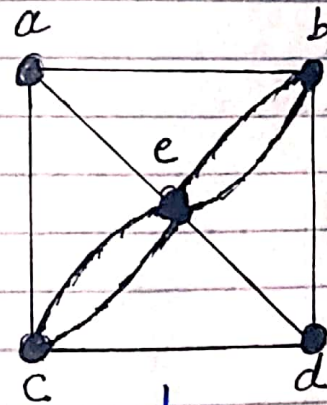
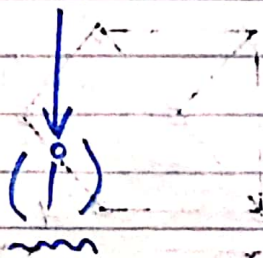
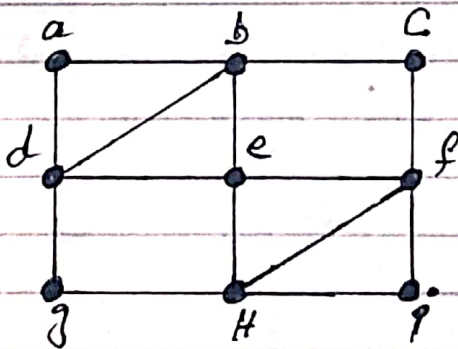
isomorphic.



(ii)  
↑

QNO: 3-----

Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



ANS: ---

EULER PATH AND CIRCUIT:

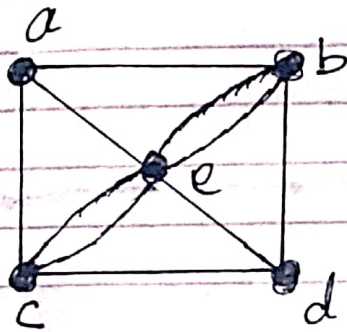
“ An Euler circuit in a Graph  $G$  is simple circuit containing every edge of  $G$ .”

“ An Euler path in  $G$  is a simple path containing every edge of  $G$ .”

Theorem:

A Connected multi-graph with at least two vertices has an Euler circuit if and only if each of its vertices has an even degree.

(i) ---

Solution:

By theorem, we know

this graph does not have  
an Euler circuit. b/c we

have ~~four~~ <sup>two</sup> vertices of odd  
degree. (a and d). This

graph does have an Euler  
path path.

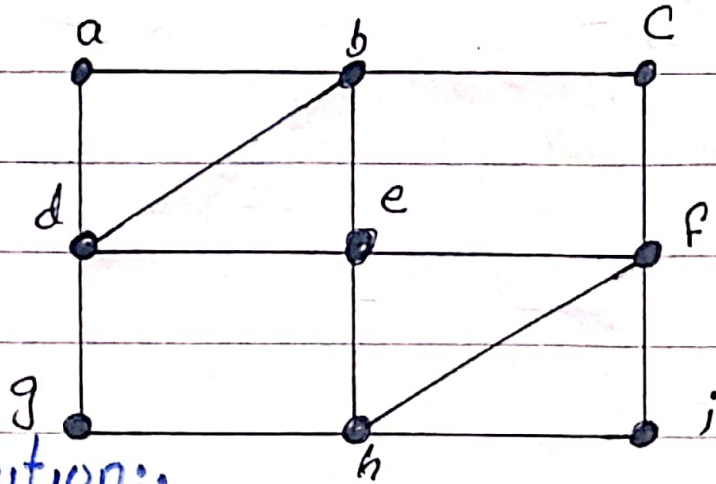
The path is as  
follows: (a, e, c, e, b, e, d, b, a, c, d.)

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Solution:

It is an Euler circuit because its initial and terminal points is one.

$(a, b, c, i, h, g, d, e, f, h, a)$ .

Start and end with  
a:

QNO: 11-----

Are the Simple Graphs

With the following adjacency  
Matrices isomorphic?

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \longrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

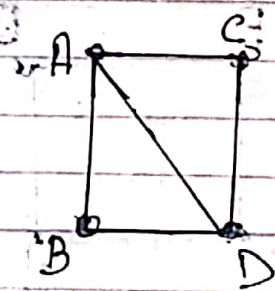
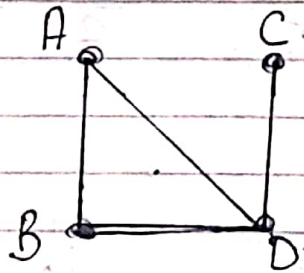
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

ANS:-----

(1)

$$\begin{array}{c}
 A \\
 B \\
 C \\
 D
 \end{array}
 \begin{array}{cccc}
 A & B & C & D \\
 \left[ \begin{array}{cccc}
 0 & 1 & 0 & 1 \\
 1 & 0 & 0 & 1 \\
 0 & 0 & 0 & 1 \\
 1 & 1 & 1 & 0
 \end{array} \right]
 \end{array}$$

$$\begin{array}{c}
 A \\
 B \\
 C \\
 D
 \end{array}
 \begin{array}{cccc}
 A & B & C & D \\
 \left[ \begin{array}{cccc}
 0 & 1 & 1 & 1 \\
 1 & 0 & 0 & 1 \\
 1 & 0 & 0 & 1 \\
 1 & 1 & 1 & 0
 \end{array} \right]
 \end{array}$$

Solution:Adjacency lists:

$$\begin{array}{l}
 a = B, d \\
 b = A, d \\
 c = d \\
 d = A, B, C
 \end{array}$$

$$\begin{array}{l}
 a = C, B, A \\
 b = A, d \\
 c = A, d \\
 d = A, B, C
 \end{array}$$

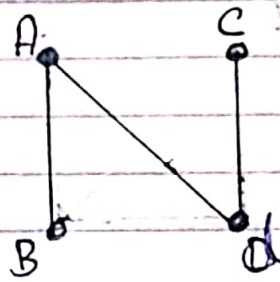
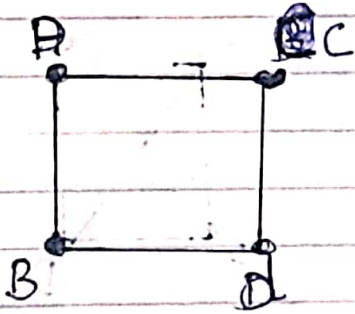
So the given simple graphs with with above Adjacency is “Not isomorphic”



(ii)

$$\begin{array}{c}
 A \\
 B \\
 C \\
 D
 \end{array}
 \begin{bmatrix}
 0 & 1 & 1 & 0 \\
 1 & 0 & 0 & 1 \\
 1 & 0 & 0 & 1 \\
 0 & 1 & 1 & 0
 \end{bmatrix}$$

$$\begin{array}{c}
 A \\
 B \\
 C \\
 D
 \end{array}
 \begin{bmatrix}
 0 & 1 & 0 & 1 \\
 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 \\
 1 & 0 & 1 & 0
 \end{bmatrix}$$

Solution:

Adjacency lists :-

A : C, B

B : A, d

C : A, d

d : B, C

A : B, d

B : A

C : d

d : A, C

So the Above simple  
graphs is "Not isomorphic"

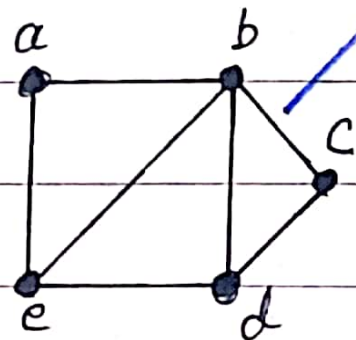
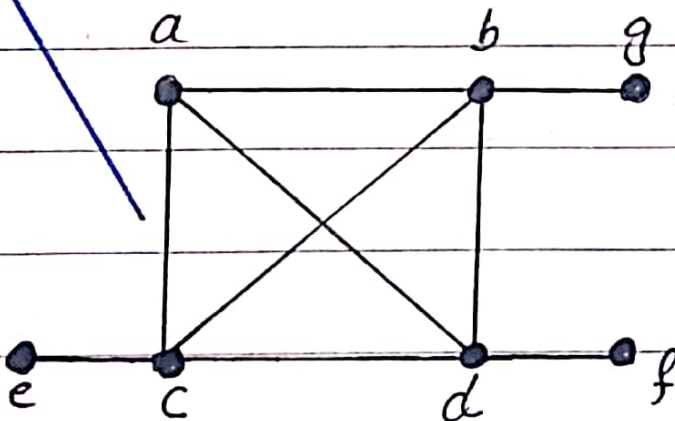
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Q NO: 5-----

Determine whether the given graph has a Hamiltonian circuit. if it does, find such a circuit. if it does, give an argument to show why no such circuit exists.

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ANS:.....HAMILTON PATH AND CIRCUITS

A path  $x_0, x_1, \dots, x_{n-1}, x_n$  in the  $G$  graph  $G = (V, E)$  is called a HAMILTON ~~CIRCUIT~~ PATH if  $V = \{x_0, x_1, \dots, x_{n-1}, x_n\}$  and  $x_i \neq x_j$  for  $0 \leq i < j \leq n$ .

A Circuit  $x_0, x_1, \dots, x_{n-1}, x_n, x_0$  (with  ~~$n$~~   $n > 1$ ) in a  $G$  graph  $G = (V, E)$

is called a Hamilton Circuit.

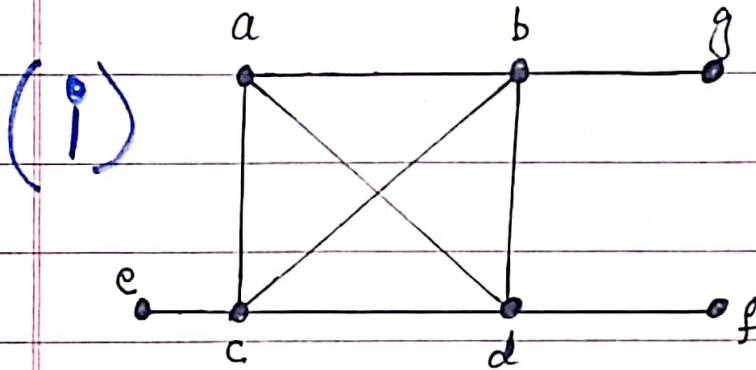
if  $x_0, x_1, \dots, x_{n-1}, x_n$  is a Hamilton path.

↳ Dirac's Theorem:

if  $G$  is a simple graph with  $n$  vertices

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With  $n \geq 3$  Such that  
the degree of every vertex  
in  $G$  is at least  $n/2$ ,  
then  $G$  has a Hamilton  
Circuit.

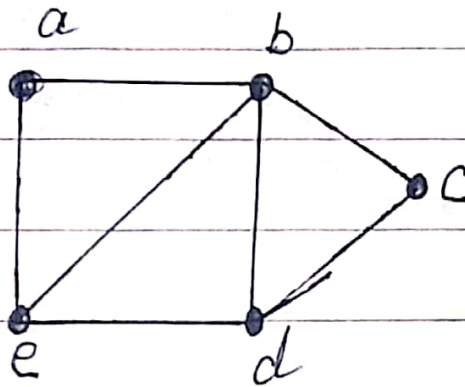


Solution:

There is no  
Hamilton circuit because  
they are vertices of degree 1  
(pendants) in the graph.

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(ii) -

Solution:

This Graph Has  
A Hamilton Circuit.

a, b, c, d, e, a is a Circuit.

The End\*