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Question Number 1

Part (a)

Ans:- Given $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$

Solution:

let

$$1-\theta^2 = u$$

$$\frac{d}{d\theta} 1-\theta^2 = \frac{d}{d\theta} u$$

$$-2\theta = \frac{du}{d\theta}$$

$$d\theta = -\frac{1}{2} du$$

Now:

$$= \int (u)^{\frac{1}{4}} \cdot \left(-\frac{1}{2}\right) du$$

$$= -\frac{1}{2} \int u^{\frac{1}{4}} du \qquad \because \frac{1}{4}+\frac{1}{1} = \frac{1+4}{4} = 5/4$$

$$= -\frac{1}{\cancel{2}} \cdot \frac{4^{2}}{5} u^{5/4} + C$$

$$= -\frac{2}{5} u^{5/4} + C$$

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By back Substitution:

$= -\frac{2}{5}(1-\phi^2)^{5/4} + c.$

Q1 Part (b)

Ans: Given:

$$\int_0^1 x^3(1+x^4)^3 \, dx.$$

Solution:

Let

$1+x^4 = u$ ———— ①

$\frac{d}{dx} = (1+x^4) = \frac{d}{dx} u$

$4x^3 = \frac{du}{dx}.$

$x^3 \, dx = \frac{1}{4} dx$

Now put $x=0$ in eq ①

$1+0^4 = 1$

$u = 1$

& put $x=1$ in eq ①

$1+(1)^4 = 2$

(3)

$u = 2$

$$= \int_1^2 (u)^3 \frac{1}{4} \, dx$$

$$= \frac{1}{4} \int_1^2 u^3 \, dx$$

$$\frac{1}{4} \frac{u^4}{4} \Big|_1^2$$

$$= \frac{1}{4} \left(\frac{(2)^4}{4} - \frac{(1)^4}{4} \right)$$

$$= 3/8$$

Result:

$$= 3/8 \quad \text{Ans.}$$

Question Number 2

Part (a)

illustrate the centre and radius of the sphere.

Ans:-

Given.

$$x^2+y^2+z^2+3x-4z+1.$$

$$(x^2+3x)y^2+z^2-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+((y-0)^2+z^2-4z+\left(\frac{-4}{2}\right)^2)$$

$$=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y+0)+(z-2)^2=\frac{21}{4}$$

So,

$$(x_0, y_0, z_0) \quad \text{center} = \left(\frac{-3}{2}\right), 0, 2)$$

Radius:

$$r_e = \sqrt{\frac{21}{4}}$$

Q2 Part-(b)-

Ans: Given:

$$y = \sqrt{x}$$

$$0 \le x \le 4 \Rightarrow a \le x \le b.$$

Solution:

we know that:

$$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \frac{x^2}{2} \Big|_0^4$$

$$V = \frac{\pi}{2} \left((4)^2 - 0 \right).$$

$$V = \frac{\pi}{\cancel{2}} \times \cancel{16}^{8}$$

$$\boxed{V = 8\pi}$$ Ans,

Question Number 3

If $A = 2i - 4j + \sqrt{5}K$ & $B = -2i + 4j - \sqrt{5}K$ then illustrate the vector $Proj_A B$

Sol :-

we know projection formula.

$$Proj_A B = \frac{B \cdot A}{|A|^2} A \quad \text{——— } (1)$$

$$B \cdot A = (-2i + 4j - \sqrt{5}K) \cdot (2i - 4j + \sqrt{5}K)$$

$$B \cdot A = (-4 - 16 - (\sqrt{5})^2)$$

$$B \cdot A = -4 - 16 - 5$$

$$B \cdot A = -25 \quad \text{——— } (a)$$

$$A \cdot A = (2i - 4j + \sqrt{5}K) \cdot (2i + 4j + \sqrt{5}K)$$

$$A \cdot A = 4 + 16 + 5$$

$$A \cdot A = 25 \quad \text{——— } (b)$$

Now by putting values in eq (1)

So,

$$Proj_A B = \frac{-\cancel{25}}{\cancel{25}} (2i - 4j + \sqrt{5}K)$$

$$'' = -(2i - 4j + \sqrt{5}K)$$

$$\boxed{Proj_A B = -2i + 4j - \sqrt{5}K}$$

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Question Number 4

Q: Find the area of the region b/w the graph and the x-axis. where

$$y = -x^2 + 5x - 4 \quad , \quad [0, 2].$$

Solution:-

AS $a = 0$, $b = 2$.

$$A = \int_a^b f(x)\,dx.$$

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx.$$

$$A = \left(-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x\right)\Big|_0^2$$

$$A = -\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2) - (0).$$

$$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right).$$

$$A = -\frac{8}{3} + \frac{\cancel{20}^{10}}{\cancel{2}} - 8$$

$$A = \frac{-2}{3} = 0.6$$

AS area is never in negative so take the value in positive

$\boxed{A = 0.6}$ Ans/—

Question Number 5

Part -(a)-

Estimate the angle between $A = i - 2j - 2K$ and $B = 6i + 3j + 2K$

Solution:

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$$

$$|A| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$$

$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$$

$$|B| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7.$$

As we know the formula.

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right).$$

$$\theta = \cos^{-1}\left(\frac{(i - 2j - 2K).(6i + 3j + 2K)}{3 \times 7}\right)$$

$$\theta = \cos^{-1}\left(\frac{(1)(6) - (2)(3) + (-2)(2)}{21}\right)$$

$$\Phi = \cos^{-1}\left(-\frac{4}{21}\right)$$

$$\Phi = 100.98^\circ$$

Q5 Part (b)

Ans: Given:-

$$x^2+y^2+(z-1)^2 = 1.$$

Solution:

$$x^2+y^2+(z-1)^2 = 1$$

$$(\rho\sin\phi\cos\Phi)^2+(\rho\sin\phi\sin\Phi)^2+(\rho\cos\phi-1)^2 = 1$$

$$= \rho^2\sin^2\phi\cos^2\Phi+\rho^2\sin^2\phi\sin^2\Phi+\rho^2\cos^2\phi+1-2\rho\cos\phi = 1$$

$$= \rho^2\sin^2\phi(\cos^2\Phi+\sin^2\Phi)+\rho^2\cos^2\phi+1-2\rho\cos\phi = 1$$

$$= \rho^2(\sin^2\phi)+\rho^2\cos^2\phi-2\rho\cos\phi = 1-1$$

$$= \rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi = 0$$

$$= \rho^2 = 2\rho\cos\phi$$

$$\boxed{\rho = 2\cos\phi}$$ Ans