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1971 national university
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Department of Electrical Engin.

Final Assignment Spring 2020

Date: 26/6/2020

Subject: calculus

Module: 2nd

Total marks: 50

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Q1(a): Estimate $\int_0^1 \sqrt{1-\theta^2} d\theta$

Solution:

$$\int_0^1 \sqrt{1-\theta^2} d\theta$$

$$\text{Let } 1-\theta^2 = u$$

$$\text{Then } 1-\theta^2 = u$$

$$\frac{d}{d\theta} (1-\theta^2) = \frac{du}{d\theta}$$

$$0-2\theta = \frac{du}{d\theta}$$

$$-2\theta = \frac{du}{d\theta}$$

~~$$\frac{d\theta}{d\theta} = \frac{d\theta}{d\theta}$$~~

$$\theta d\theta = -\frac{1}{2} du$$

$$\text{put } = -\frac{1}{2} du = \theta du$$

$$\theta (1-\theta^2) = u$$

$$\text{at } \int -4 \sqrt{\frac{u}{2}} du$$

$$-\frac{1}{2} \int 4 \sqrt{u} du$$

$$-\frac{1}{2} \int u^{1/4} du$$

$$= -\frac{1}{2} \frac{u^{1/4+1}}{1/4+1} + C$$

$$= -\frac{1}{2} \frac{(u)^{5/4}}{5/4} + C$$

put $u = 1 - \theta^2$

$$= -\frac{1}{2} \frac{(1 - \theta^2)^{5/4}}{5/4} + C$$

$$= -\frac{1}{2} \frac{4^2 (1 - \theta^2)^{5/4}}{5} + C$$

$$= \frac{-2}{5} \frac{(1 - \theta^2)^{5/4}}{5} + C$$

$$= \frac{-2}{5} \frac{(-\theta^2 + 1)^{5/4}}{5} + C$$

Q1(b) Estimate $\int_0^1 x^3 (1+x^4)^3 dx$

using substitution method.

Solution: $\int_0^1 x^3 (1+x^4)^3 dx$ — (1)

by substitution

$$\text{let } 1+x^4 = u$$

$$= \frac{d}{dx} (1+x^4) = du/dx$$

$$0 + 4x^3 = du/dx$$

$$x^3 dx = \frac{1}{4} \cdot du \quad \text{put in (1)}$$

$$= \int_0^1 (u)^3 \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \left(\int_0^1 u^3 du \right)$$

$$= \frac{1}{4} \int_0^1 \frac{u^{3+1}}{3+1}$$

$$= \frac{1}{4} \int_0^1 \frac{u^4}{4} \quad \text{--- (ii)}$$

put $u = 1+x^4$

$$= \frac{1}{4} \int_0^1 \frac{1+x^4}{4}$$

Apply limits:

$$= \frac{1}{4} \left(\frac{1+(1)^4}{4} - \frac{1+0}{4} \right) \int_0^1$$

$$= \frac{1}{4} \left(\frac{2}{4} - \frac{1}{4} \right)$$

$$= \frac{1}{4} \left(\frac{2-1}{4} \right)$$

$$= \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$\boxed{\int_0^1 x^3 (1+x^4)^3 dx = \frac{1}{16} \text{ An}}$$

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Q2: (a): Illustrate the center and radius of the sphere

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

Solution: $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

$$(x^2 + 3x) + y^2 + z^2 - 4z + 1 = 0$$

$$\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) + (y+0)^2 + \left(z^2 - 4z + \left(-\frac{4}{2}\right)^2\right)$$

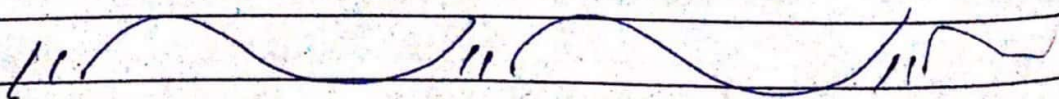
$$= -1 + \left(\frac{3}{2}\right)^2 + \left(-\frac{4}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 + (y+0)^2 + (z-2)^2 = \frac{21}{4}$$

So $(x_0, y_0, z_0) \Rightarrow$ center

$$= \left(-\frac{3}{2}, 0, 2\right)$$

find Radius $a = \sqrt{\frac{21}{4}}$



Q2(b) The region b/t the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, and the x -axis is revolved about the x -axis to generate a solid. Apply the integration find the volume of solid.

Solution:

Given that

$$y = \sqrt{x}$$

$$0 \leq x \leq 4 \Rightarrow a \leq x < b$$

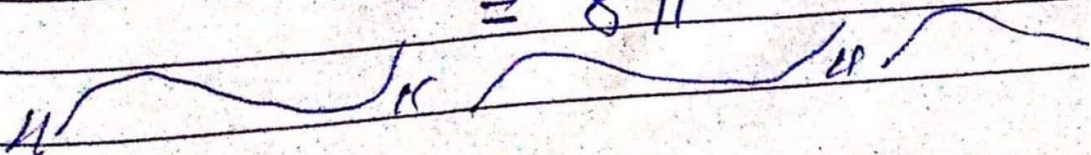
$$\text{as } V = \int_a^b \pi y^2 dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 x dx = \pi \frac{x^2}{2} \Big|_0^4$$

$$V = \frac{\pi}{2} (4^2 - 0)$$

$$= 8\pi$$



Q3: If $A = 2i - 4j + \sqrt{5}k$, and
 $B = -2i + 4j - \sqrt{5}k$ then
 illustrate the vector $\text{proj}_A B$

Solution: By dot product

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot$$

$$(2i - 4j + \sqrt{5}k)$$

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = (-2i + 2i + 4j - 4j - \sqrt{5}k + \sqrt{5}k)$$

$$B \cdot A = -4 - 16 - 5$$

$$\boxed{B \cdot A = -25}$$

Now

$$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$A \cdot A = (2i + 2i) + (-4j - 4j) + (\sqrt{5}k + \sqrt{5}k)$$

$$\boxed{A \cdot A = 4 + 16 + 5}$$

$$\therefore \text{proj}_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A$$

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$$\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A$$

By putting values

$$= \left(\frac{-25}{9.5} \right) (2j - 4j + \sqrt{5}k)$$

$$= (-1) (2j + 4j + \sqrt{5}k)$$

$$= -2j + 4j - \sqrt{5}k \text{ Ans}$$

Q4: Find the area of region between the graph and the x -axis, where $y = -x^2 + 5x - 4$, $[0, 2]$

Solution:

$$A = \frac{A}{B} + \frac{A}{B}$$

given that

$$y = f(x) = x^2 + 5x - 4$$

and

$$[a, b] = [0, 2]$$

As

$$a = 0$$

$$b = 2$$

So area under graph will

$$A = \int_a^b f(x) dx \quad \text{By putting the value}$$

$$A = \int_0^2 (-x^2 + 5x - 4) dx$$

$$\left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x \right) \Big|_0^2$$

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$$= \left(\frac{(-2)^2}{3} + \frac{5(2)^2}{2} - 4(2) \right) = \frac{(0)^3}{3} + \frac{5(0)^2}{2} - 4(0)$$

$$= \left(\frac{-4}{3} + \frac{20}{2} - 8 \right) = 0 + 0 - 0$$

$$= \frac{-4}{3} + 10 - 8$$

$$= \frac{-4}{3} + 2 \Rightarrow \frac{-4 + 6}{3}$$

$$\boxed{A = \frac{2}{3}} \Rightarrow 0.6$$

Result = 0.6

Q.5: (a) Estimate the angle between
 $A = i - 2j - 2k$ and
 $B = 6i + 3j + 2k$

Solution:

$$A = i - 2j - 2k$$

$$|A| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1} \left(\frac{A \cdot B}{|A| \cdot |B|} \right)$$

$$\theta = \cos^{-1} \left\{ \frac{(i - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7} \right\}$$

$$\theta = \cos^{-1} \left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21} \right)$$

$$\theta = \cos^{-1} \left(-\frac{4}{21} \right)$$

Q5(b): Change into a spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

Solution:

$$x^2 + y^2 + (z-1)^2 = 1$$

$$= (f \sin \phi \cos \theta)^2 + (f \sin \phi \sin \theta)^2 + (f \cos \phi - 1)^2 = 1$$

$$= f^2 \sin^2 \phi \cos^2 \theta + f^2 \sin^2 \phi \sin^2 \theta$$

$$+ f^2 \cos^2 \phi + 1 - 2f \cos \phi = 1$$

$$= f^2 \sin^2 \phi (\cos^2 \theta + \sin^2 \theta)$$

$$+ f^2 (\sin^2 \phi) + f^2 \cos^2 \phi + 2 \cdot f \cos \phi = 1 - 1$$

$$= f^2 (\sin^2 \phi + \cos^2 \phi) - 2f \cos \phi = 0$$

$$= f^2 = 2f \cos \phi$$

$$= f = 2 \cos \phi$$

