S M T W T F S Date:

Iqra national university peshawar.

Department of Electrical Engin.

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Module: 2nd

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(1)

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Q1:(a): Estimate $\int \theta \sqrt{1-\theta^2}\, d\theta$

Solution:

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

Let $1-\theta^2 = u$

Then $1-\theta^2 = u$

$$\frac{d}{d\theta}(1-\theta^2 = du/d\theta$$

$$0 - 2\theta = du/d\theta$$

$$-2\theta = du/d\theta$$

~~$\theta / d\theta = du/d$~~

$$\theta\, d\theta = -\frac{1}{2} du$$

put $= -\frac{1}{2} du = \theta du$

$$\theta(1-\theta^2) = u$$

as $\int -\sqrt[4]{\frac{u}{2}}\, du$

$$-\frac{1}{2}\int \sqrt[4]{u}\, du$$

$$-\frac{1}{2}\int u^{1/4}\,du$$

$$= -\frac{1}{2}\,\frac{u^{\frac{1}{4}+1}}{\frac{1}{4}+1}+c$$

$$= -\frac{1}{2}\,\frac{(u)^{5/4}}{5/4}+c$$

put $u = 1-\theta^2$

$$= -\frac{1}{2}\,\frac{(1-\theta^2)^{5/4}}{5/4}+c$$

$$= -\frac{1}{\cancel{2}}\,\frac{\cancel{4}^2(1-\theta^2)^{5/4}}{5}+c$$

$$-\frac{2}{5}\,\frac{(1-\theta^2)^{5/4}}{5}+c$$

$$\boxed{= -\frac{2}{5}\,\frac{(-\theta^2+1)^{5/4}}{5}+c}$$

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Q1(b) Estimate $\int_0^1 x^3(1+x^4)^3\,dx$

using substitution method.

Solution:

$$\int_0^1 x^3(1+x^4)^3\,dx \quad \text{--- (1)}$$

by substitution

let $1+x^4 = u$

$$= \frac{d}{dx}(1+x^4) = du/dx$$

$$0 + 4x^3 = du/dx$$

$$x^3 dx = 1/4 \cdot du \quad \text{put in (1)}$$

$$= \int_0^1 (u)^3 \cdot \frac{1}{4}\,du$$

$$= \frac{1}{4}\left(\int_0^1 u^3\,du\right)$$

$$= \frac{1}{4} \int_0^1 \frac{u^{3+1}}{3+1}$$

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$= \frac{1}{4}\int_0^1 \frac{u^4}{4}$ ——— (ii)

Put $u = 1+x^4$

$= \frac{1}{4}\int_0^1 \frac{1+x^4}{4}$

Apply limits:

$= \frac{1}{4}\left(\frac{1+(1)^4}{4} - \frac{1+0}{4}\right]_0^1$

$= \frac{1}{4}\left(\frac{2}{4} - \frac{1}{4}\right)$

$= \frac{1}{4}\left(\frac{2+1}{4}\right)$

$= \frac{1}{4}\cdot\frac{1}{4} = 1/16$

$\boxed{\int_0^1 x^3(1+x^4)^3 dx = \frac{1}{16}}$ An

(5)

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Q2:(a): Illustrate the center and radius of the shere $x^2+y^2+z^2+3x-4z+1$

Solution:: $x^2+y^2+z^2+3x-4z+1=0$

$$(x^2+3x)+y^2+z^2-4z+1=0$$

$$(x^2+3x+(3/2)^2)+(y+0)^2+(z^2-4z+(-4/2)^2)$$

$$=-1+(3/2)^2+(-4/2)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y+0)^2+(z-2)^2=\frac{21}{4}$$

So $(x_0, y_0, z_0) \Rightarrow$ center

$$=\left(-3/2, 0, 2\right)$$

find Radius $a=\sqrt{\frac{21}{4}}$

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Q2(b) The region b/t the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Apply the integration find the volume of solid.

Solution:

Given That

$$y = \sqrt{x}$$

$$0 \le x \le 4 \Rightarrow a \le x < b.$$

as $V = \int_a^b \pi y^2 \, dx$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx = \pi \left. \frac{x^2}{2} \right|_0^4$$

$$V = \frac{\pi}{2}\left((4)^2 - 0\right)$$

$$= 8\pi$$

Q3: if $A = 2i - 4j + \sqrt{5}k$, and $B = -2i + 4j - \sqrt{5}k$ then illustrate the vector $proj_A B$

Solution: By pof projet

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = (-2j + 4j + \sqrt{5}k) \cdot (2j - 4j + 5k)$$

$$B \cdot A = (-2\cancel{j} + 2\cancel{j} + 4\cancel{j} - 4\cancel{j} - \sqrt{5}\cancel{k} + \sqrt{5}\cancel{k})$$

$$B \cdot A = -4 - 16 - 5$$

$$\boxed{B \cdot A = -25}$$

Now

$$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2j - 4j + \sqrt{5}k)$$

$$A \cdot A = (2\cancel{i} + 2\cancel{i}) + (-4\cancel{j} - 4\cancel{j}) + (\sqrt{5}\cancel{k} + \sqrt{5}\cancel{k})$$

$$\boxed{A \cdot A = 4 + 16 + 5}$$

So $$proj_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A.$$

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As $proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

By putting values

$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$

$= (-1)(2i + -4j + \sqrt{5}k)$

$\boxed{= -2i + 4j - \sqrt{5}k}$ Ans

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Q4: Find the area of region between the graph and the x-axis,
where $y = -x^2 + 5x - 4$, $[0, 2]$

Solution:

given that

$$y = fx = x^2 + 5x - 4$$

and

$$[a, b] = [0, 2]$$

As

$$a = 0$$
$$b = 2$$

So area under graph will

$$A = \int_a^b f(x)\,dx$$ ~~By putting value~~

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

$$\left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right)\Big|_0^2$$

$$= \left(\frac{(-2)^3}{3} + \frac{5(2)^2}{2} - 4(2)\right) = \frac{(0)^3}{3} + \frac{5(0)^2}{2} - 4(0)$$

$$= \left(\frac{-4}{3} + \frac{20}{2} - 8\right) - 0 + 0 - 0$$

$$= \frac{-4}{3} + 10 - 8$$

$$= \frac{-4}{3} + 2 \Rightarrow \frac{-4+6}{3}$$

$$\boxed{A = \frac{2}{3}} \Rightarrow 0 \cdot 6$$

Result $= 0 \cdot 6$

Q.5: (a) Estimate the angle between
$A = i - 2j - 2K$ and
$B = 6i + 3j + 2K$

Solution:

$$A = i - 2j - 2K$$

$$|A| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2K$$

$$|B| = \sqrt{36+9+4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A| \cdot |B|}\right)$$

$$\theta = \cos^{-1}\left(\frac{(i-2j-2K)\cdot(6i+3j+2K)}{3 \times 7}\right)$$

$$\theta = \cos^{-1}\left(\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right)$$

$$\theta = \cos^{-1}\left(-\frac{4}{21}\right)$$

Q5(b):: Change into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$.

Solution::

$$x^2+y^2+(z-1)^2=1$$

$$=(\rho\sin\phi\cos\theta)^2+(\rho\sin\theta\sin\theta)^2$$
$$+(\rho\cos\theta-1)^2=1$$

$$=\rho^2\sin^2\theta\cos^2\theta+\rho^2\sin^2\theta\sin^2\theta$$
$$+\rho^2\cos^2\theta+1-2\rho\cos\theta=1$$

$$=\rho^2\sin^2\theta(\cos^2\theta+\sin^2\theta)$$
$$+\rho^2(\sin^2\theta)+\rho^2\cos^2\theta+2\cdot\rho\cos\theta=1-1$$

$$=\rho^2\sin^2\theta+\cos^2\theta)-2\rho\cos\theta=0$$

$$=\rho^2=2\rho\cos\theta$$

$$=\rho=2\cos\theta$$