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Subject :: linear ALGEBRA
Section :: "A"

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Q 1

Consider the given below Matrix as the augmented Matrix of linear system. Explain in your words the next elementary Row Operation that should be performed in order to solve the system where ID_3 is the 3rd Digit in your ID in reverse. e.g. if your ID is 12345 then $-ID_{last} = -5$?

$$\left[\begin{array}{ccccc} 1 & ID_3 & 3 & 0 & 5 \\ 0 & 1 & -ID_{last} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{array} \right]$$

ANSWER ::

$$ID = 16027$$

given that $ID_3 = 3rd$

Digit of $ID = 0$

$$ID = \text{last Digit reverse} = 7$$

Putting Value ::

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

p.t.0

New Row Operation ::

$$R_2 + 7R_3 \quad \begin{pmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & -35 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{l} R_2 : 0 \ 1 \ -7 \ 0 \ 7 \\ 7R_3 : \underline{0 \ 0 \ 7 \ 0 \ -42} \\ \quad \quad 0 \ 1 \ 0 \ 0 \ -35 \end{array}$$

$$R_1 - 3R_2 \quad \begin{pmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & -35 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{l} R_1 : 1 \ 0 \ 3 \ 0 \ 5 \\ -3R_3 : \underline{0 \ 0 \ -3 \ 0 \ 18} \\ \quad \quad 1 \ 0 \ 0 \ 0 \ 23 \end{array}$$

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Now augmented Matrix convert to system of linear form

i.e.

$$1x_1 + 0x_2 + 0x_3 + 0x_4 = 23$$

$$\Rightarrow x_1 = 23$$

$$x_2 = -35$$

$$x_3 = -6$$

$$x_4 = 0$$

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Q2 ::

(A)

Find the elementary Row Operation that transforms the first Matrix into Second and reverse row Operations that transforms the Second Matrix into first ?

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution ::

In order to transform the first Matrix into the Second, we will Multiply -2 with row 2 ~~and~~ and then add the row 2 to row 3

$$-2R_2 + R_3 ::$$

$$\begin{array}{l} -2 \times \text{row 2} : \quad 0 \quad 1 \quad -4 \quad 2 \\ \quad \quad \quad \quad \quad 0 \quad -2 \quad 8 \quad -4 \end{array}$$

Now adding it to row 3

P.T.O

$$\begin{array}{r}
 0 \quad -2 \quad 8 \quad -4 \\
 + 0 \quad 2 \quad -5 \quad -1 \\
 \hline
 0 \quad 0 \quad 3 \quad -5
 \end{array}$$

Putting the newly formed row 3 into the first Matrix:

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Hence we transformed the first Matrix into the Second Matrix

Now applying reverse row operation and turning the Second Matrix into the first:

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Multiplying 3 with row 2 and adding it to row 3

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$$2R_2 + R_3$$

$$\begin{array}{cccc} 0 & 2 & -8 & 4 \\ 0 & 0 & 3 & -5 \\ \hline 0 & 2 & -5 & -1 \end{array}$$

Putting back the new row 3
obtained above, into the second
matrix and transforming it to
matrix no first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

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(B)

Given below are some Matrices. Find where these are in the forms written in front of them or not. Explain in your own words for each of the solution in detail.

Solution :-

(A)

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

This matrix is not echelon form because the most below row have "e" element is not zero and for echelon form there must be all zero element in the last line.

P.T.O

(B)

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In this one is echelon form because the pivot element having zero in their below row and last row have also zero element

(C)

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

This is not reduced echelon form because having non zero row at the bottom.

"D"

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

this is reduced echelon form
if when we transpose $R_2 \leftrightarrow R_3$
then the bottom line will
be have zero element.

Q3 :
part "A"

The Row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced Row echelon form? What is the practical use of reduced Row echelon form Give one example.?

Row echelon form :-

A Matrix is said to be in ^{row} echelon form when it satisfies the following conditions :-

- (1) The first non-zero element in each Row called leading entry is 1
- (2) Each leading entry is in a row to the right of the leading entry in the previous Row

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(3)

Row with all zero elements, if any are below (after), the row having non-zero element

Example :

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduced Row echelon form:

* A matrix is said to be in reduced row echelon form when it satisfies the following condition:-

(1)

The matrix satisfies condition for a Row echelon form.

(2)

The leading entry in each row is the only non-zero entry in its Row

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Example :

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Therefore we can say that each reduced row echelon form is also a Row echelon form but vice versa is not always true.

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Q.3: Part (B)

Find in echelon form for the below Matrix using row operation when ID_2 is 2nd digit in your ID eg if your ID is 12345 $ID_2 = 2$, $ID_3 = 3$ = ID-first-last and last digit of your ID = 5

$$\begin{bmatrix} 1 & ID_2 & 8 \\ 2 & 8 & -1 \\ -ID_3 & 0 & 0 \\ 1 & -4 & ID_{\text{first}} - \text{last} \end{bmatrix}$$

Solution:

Requirement

$$ID = 16027$$

- (1) $ID_2 = 6$
- (2) $ID_3 = 0$
- (3) First last = 17

P.T.O

$$10R_4 + R_3 \quad \left| \begin{array}{ccc|c} 1 & 10 & 8 & \\ 2 & 8 & -1 & \\ 0 & 0 & 1 & \\ 1 & -4 & 1 & 10 \end{array} \right|$$

$$\frac{1}{8} R_2 + R_1 \quad \left| \begin{array}{ccc|c} 1 & 10 & 8 & \\ \frac{3}{8} & 1 & -\frac{1}{8} & \\ 0 & 0 & 1 & \\ 1 & -4 & 1 & 10 \end{array} \right|$$

$$17R_3 \quad \left| \begin{array}{ccc|c} 1 & 6 & 10 & \\ \frac{3}{8} & 6 & -\frac{1}{8} & \\ 0 & 0 & 17 & \end{array} \right|$$

$$\begin{array}{l} 10R_4 \\ 10R_2 \\ 16R_3 + R_4 \end{array} \quad \left| \begin{array}{ccc|c} 1 & 6 & 8 & \\ \frac{3}{8} & 1 & -\frac{1}{8} & \\ 0 & 0 & 16 & \\ 1 & -4 & 1 & \end{array} \right|$$

$$R_4 + R_3 \quad \left| \begin{array}{ccc|c} 1 & 6 & 8 & \\ \frac{3}{8} & 1 & -\frac{1}{8} & \\ 0 & 0 & 16 & \\ 1 & -4 & 17 & \end{array} \right|$$

P.T.O

$$\frac{2}{8} R_2 \quad \left| \begin{array}{ccc} 1 & 6 & 8 \\ 1 & 6 & -1/4 \\ 0 & 0 & 16 \\ 1 & -4 & 17 \end{array} \right|$$

$$R_3 - R_2 \quad \left| \begin{array}{ccc} 1 & 6 & 10 \\ 1 & 0 & 4 \\ 0 & 0 & 16 \\ 1 & -4 & 17 \end{array} \right|$$

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