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i-D	5778
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Q1 (a) Express the eq of Plane Passing through The Point $A(2,-2,1), B(-1,0,3), C(5,-3,4)$.

Sol:- The non Parallel vectors

$\overrightarrow{P_1P_2} = (-3,2,2) \longrightarrow (-1,0,3)-(2,-2,1)$

$\overrightarrow{P_1P_3} = (3,-1,3) \qquad (-1,0,3)-2+2-1$

$(-3,2,2)$

the Perpendicular Vector is :-

$\overrightarrow{P_1P_2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$n = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$n = i(6+2) - j(-9-6) + k(3-6)$

$n = 8i + 15j - 3k$

$n = (8,15,-3)$

Now $P_1(x_0,y_0,z_0) = (2,-3,1)$

$n(a,b,c) = (8,15,-3)$

So eq of plane is

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$8(x-2)+15(y+2)-3(z-1)=0$

$8x+15y-3z-16+30+3=0$

$$\boxed{8x+15y-3z+17=0}$$

②

Q=1 ⓑ Express a pair of ~~two~~ planes whose intersection is the given line

$x=2-3t$, $y=3+t$, $z=2-4t$

Sol:-

$x-2=-3t \Rightarrow t=\frac{x-2}{-3}$

$y=3+t \Rightarrow y-3=t \Rightarrow t=\frac{y-3}{1}$

$z-2=-4t \Rightarrow t=\frac{z-2}{-4}$

So $\frac{x-2}{-3}=\frac{y-3}{1}=\frac{z-2}{-4}$

For 1st plane takes 1st & 2nd

$\frac{x-2}{-3} \times \frac{y-3}{1}$

$x-2=-3y+9$

$\boxed{x+3y-11=0}$

For 2nd plane take 1st & 3rd

$\frac{x-2}{-3}=\frac{z-2}{-4}$

$-4x+8=-3z+6$

$\boxed{-4x+3z+2=0}$

or $\boxed{4x-3z-2=0}$

③

=2 $L(x,y) = (x+1, y, x+y)$ illustrate that L is Linear transformation.

Sol:-

$L(x,y) = (x+1, y, x+y)$

let $U = (x_1, y_1)$ $\quad V = (x_2, y_2)$

$U+V = (x_1, y_1) + (x_2, y_2)$

$U+V = (x_1 + x_2, y_1 + y_2)$

$L(U+V) = L(x_1 + x_2, y_1 + y_2)$

$L(U+V) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

given that $U = (x+2y)$

$L(U) = L(x_1, y_1) = (x+1+y+, x_1+y-1)$

$L(U) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$

$L(U)+L(V) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_2+y_2)$

Since $1 \neq 2$

So not Linear Transformation

Q=3 Using the matrix $A=\begin{bmatrix}1&2&3\\1&1&2\\0&1&2\end{bmatrix}$ then Interpret to decode the message 77 54 38 71 49 29 68 51 33 76 48 40 86 53 52

Sol:-
$$A=\begin{bmatrix}1&2&3\\1&1&2\\0&1&2\end{bmatrix}$$

For the Coding first find A^{-1}

$$A^{-1}=\frac{\text{adj of } A}{|A|}$$

$$A^{-1}=\begin{bmatrix}0&1&-1\\2&-2&-1\\-1&1&1\end{bmatrix}$$

$$x_1=\begin{bmatrix}77\\54\\38\end{bmatrix},\ x_2=\begin{bmatrix}71\\49\\29\end{bmatrix},\ x_3=\begin{bmatrix}68\\51\\33\end{bmatrix},\ x_4=\begin{bmatrix}76\\48\\40\end{bmatrix},\ x_5=\begin{bmatrix}86\\53\\52\end{bmatrix}$$

$$A^{-1}x_1=\begin{bmatrix}0&1&-1\\2&-2&-1\\-1&1&1\end{bmatrix}\begin{bmatrix}77\\54\\38\end{bmatrix}=\begin{bmatrix}16\\8\\5\end{bmatrix}$$

$$A^{-1}x_2=\begin{bmatrix}0&1&-1\\2&-2&-1\\-1&1&1\end{bmatrix}\begin{bmatrix}71\\49\\29\end{bmatrix}=\begin{bmatrix}20\\15\\7\end{bmatrix}$$

$$A^{-1}x_3=\begin{bmatrix}0&1&-1\\2&-2&-1\\-1&1&1\end{bmatrix}\begin{bmatrix}68\\51\\33\end{bmatrix}=\begin{bmatrix}18\\1\\15\end{bmatrix}$$

$$A^{-1}x_4=\begin{bmatrix}0&1&-1\\2&-2&-1\\-1&1&1\end{bmatrix}\begin{bmatrix}76\\48\\40\end{bmatrix}=\begin{bmatrix}8\\16\\12\end{bmatrix}$$

(5)

$$A^{-1}x_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

A B C D E F - - - - - - W, X, Y, Z

1 2 3 4 5 6 - - - - - - 23 24 25 26

Talha $\longrightarrow$ Umar $\quad A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$m = 15$, $e = 5$, $e = 5$, $t = 20$, $t = 20$

$o = 15$, $m = 13$, $o = 15$, $r = 18$, $r = 18$, $o = 15$, $w = 23$

$$x_1 = \begin{bmatrix} 15 \\ 5 \\ 5 \end{bmatrix}, x_2 = \begin{bmatrix} 20 \\ 20 \\ 15 \end{bmatrix}, x_3 = \begin{bmatrix} 13 \\ 15 \\ 18 \end{bmatrix}, x_4 = \begin{bmatrix} 18 \\ 15 \\ 23 \end{bmatrix}$$

$$Ax_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 15 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 38 \\ 28 \\ 15 \end{bmatrix}$$

$$Ax_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \\ 15 \end{bmatrix} = \begin{bmatrix} 105 \\ 70 \\ 50 \end{bmatrix}$$

$$Ax_3 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 15 \\ 18 \end{bmatrix} = \begin{bmatrix} 97 \\ 64 \\ 51 \end{bmatrix}$$

$$Ax_4 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 18 \\ 15 \\ 23 \end{bmatrix} = \begin{bmatrix} 19 \\ 79 \\ 61 \end{bmatrix}$$

So Talha Send the message

38 28 15 105 70 50 97 64 51 19 79 61

Q-4 Find an equation of the (b) plane passing through the point $(-1,3,2)$ & perpendicular to the vector $n = (0,1,-3)$.

Sol:- Eq of Plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

Given that

$$P_0\ (x_0, y_0, z_0) = (-1,3,2)$$
$$n = (a,b,c) = (0,1,-3)$$

So
$$0(x-(-1)) + 1(y-3) - 3(z-2)$$

$$0(x+1) + 1(y-3) - 3(z-2)$$

$$0 + y - 3 - 3z + 6$$

$$\Rightarrow y - 3z - 3 + 6$$

$$\Rightarrow \boxed{y - 3z + 3} \text{ ANS}$$

(7)

Q-5 Find an Eigen Values & Eigen vector of matrix $A=\begin{bmatrix}1 & 1\\ -2 & 4\end{bmatrix}$

Sol:- Given $A=\begin{bmatrix}1 & 1\\ -2 & 4\end{bmatrix}$, $x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

$x_1+x_2=\lambda x_1$ — ①

$-2x_1+4x_2=\lambda x_2$ — ②

$\lambda I=\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$

$A-\lambda I=\begin{bmatrix}1 & 1\\ -2 & 4\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$

$=\begin{bmatrix}1-\lambda & 1\\ -2 & 4-\lambda\end{bmatrix}$

$=\begin{bmatrix}1-\lambda & 1\\ -2 & 4-\lambda\end{bmatrix}=0$

$(1-\lambda)(4-\lambda)+2=0$

$4-\lambda-4\lambda+\lambda^2+2=0$

$\lambda^2-5\lambda+6=0$

$\lambda^2-3\lambda-2\lambda+6=0$

$\lambda(\lambda-3)-2(\lambda-3)=0$

$(\lambda-3)(\lambda-2)=0$

$\boxed{\lambda_1=3}$
$\boxed{\lambda_2=2}$ are two eigen values

for eigen vector:- $\lambda_1=3$ Put in eq (1) & (2)

So $x_1+x_2=3x_1$

$-2x_1+4x_2=3x_2$

$-2x_1+x_2=0$

$-2x_1+x_2=0$

$\Rightarrow$ $\boxed{x_1=\frac{1}{2}x_2 \quad x_1=\frac{1}{2}x_2}$

for $\lambda_2 = 2$ Put in eq (1) & (2)

$x_1 + x_2 = 2x_1 \Rightarrow x - 2x_1 + x_2 = 0$

$-x_1 + x_2 = 0 \quad \boxed{x_1 = x_2}$

(2) $-2x_1 + 4x_2 = 2x_2 \Rightarrow -2x_1 + 4x_2 - 2x_2 = 0$

$-2x_1 + 2x_2 = 0 \quad \boxed{x_1 = x_2}$
