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SUBMITTED TO: ENGR. ADEED KHAN

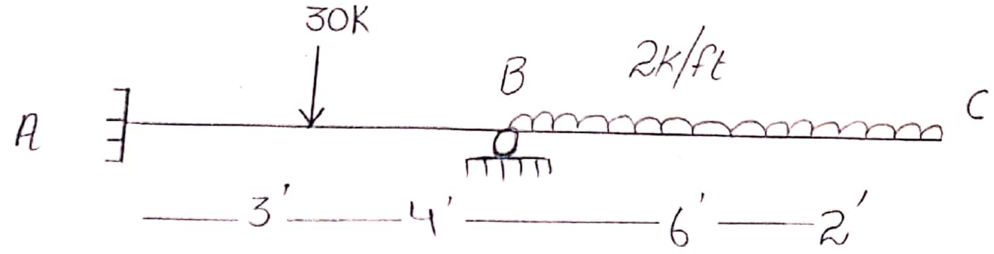
SECTION: B

MODULE: 6^{th}

SUBJECT:- STRUCTURAL ANALYSIS 2

DATE :- 25-9-2020

Question # 01:-



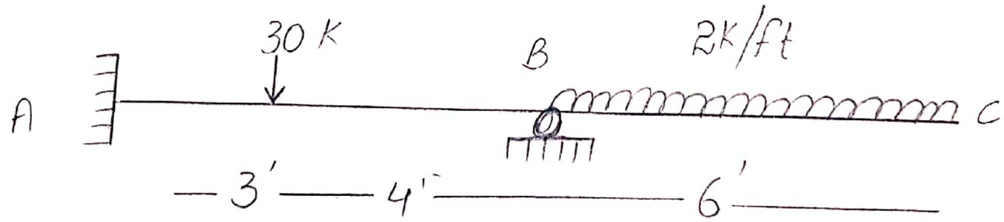
Solution:-

Step # 01:-

Determining Kinamatic innerterminancy.

$K.I = 5^\circ$

So we have to reduce the extended portion.



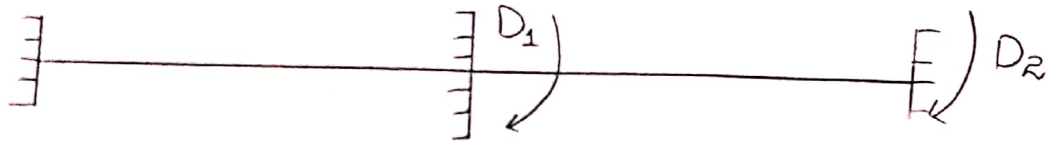
$\Rightarrow \frac{2(2)}{1} = 4K/ft$

Now,

$K.I = 2^\circ$

Step # 02:-

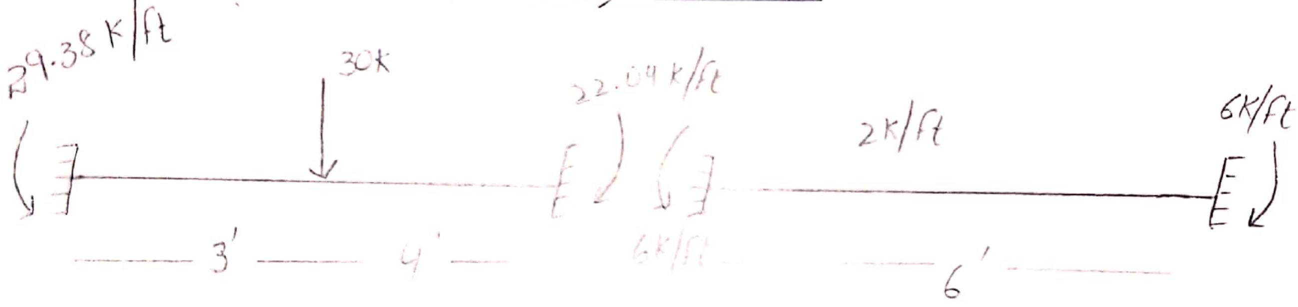
Determine unknown Joint Displacement.



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{pmatrix} AD_1 \\ AD_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step # 03:-

Compute (ADL) Matrix.



⇒ For point load (Not at mid):-

for Left End:

$$\frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ K/ft}.$$

for Right End:-

$$\frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ K/ft}.$$

for UDL:-

$$\frac{WL^2}{12} \rightarrow \frac{(2)(6)^2}{12} = 6 \text{ K/ft}$$

$$ADL_1 = +22.04 - 6 = 16.04\ k/ft$$

$$ADL_2 = 6\ k/ft.$$

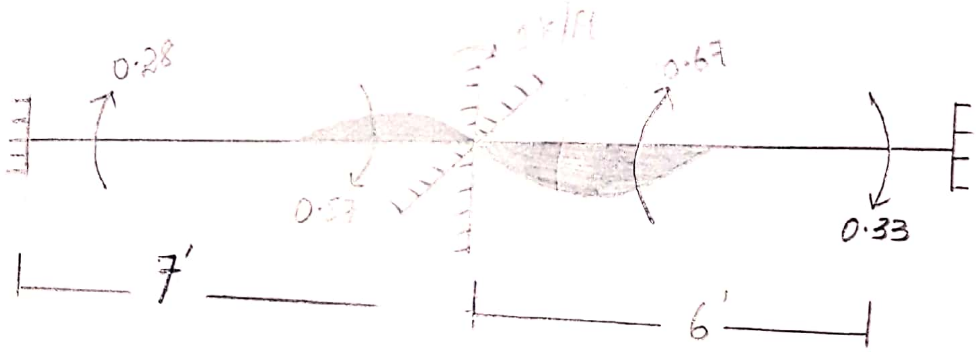
Step # 04:-

Compute [S] Matrix:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a)

$$D_1 = 1k \quad , \quad D_2 = 0$$



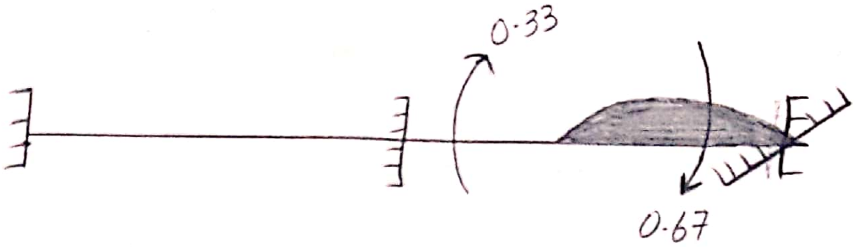
$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$$S_{11} = 0.57 + 0.67$$
$$= 1.24\ EA$$

$$S_{21} = 0.33\ EA$$

b) $D_1 = 0$, $D_2 = 1K$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2E_1}{6} = 0.33$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step # 05:

Compute [D] matrix :-

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times Adj\, A \times \begin{bmatrix} 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

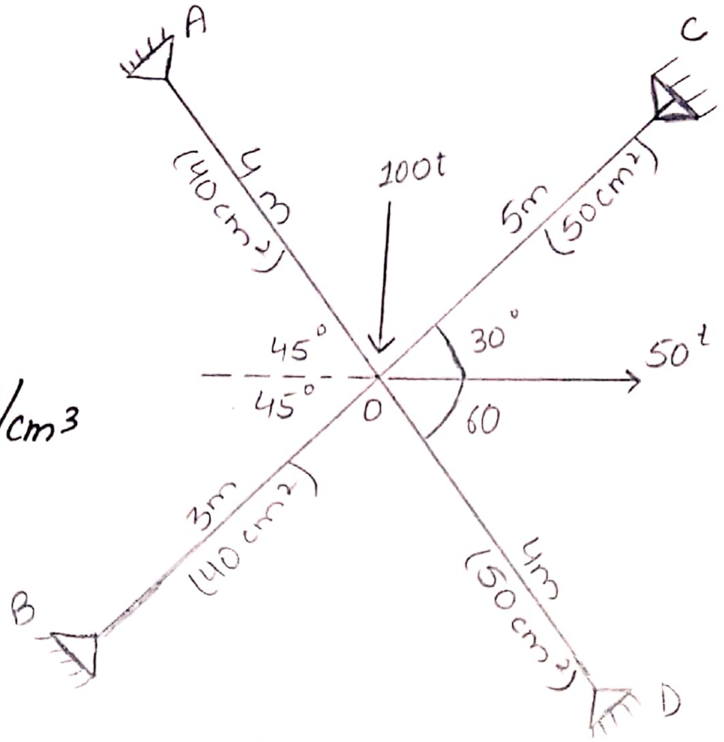
$$Adj\, A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now:

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} 16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.915 \\ 3.894 \end{bmatrix}$$

Question # 02 :-



$E = 2000\ t/cm^3$

Solution:-

For A:

$\sin 45° = P/h = P/4$

$\Rightarrow P = 2.828\ m$

$\cos 45° = b/4$

$\Rightarrow b = 2.828\ m$

For B:

$\sin 45° = P/3$

$\cos 45° = b/h$

$\Rightarrow b = 2.12\ m$

For C:

$\sin 30° = P/h{=}5 \quad \Rightarrow b \Rightarrow 2.5m$

$\cos 30^\circ = b/5$

$\Rightarrow b = 4.33\ m$

Now:

$EA_{(A)} = 2000 \times 40 = 80{,}000\ t$

$EA_{(B)} = 2000 \times 40 = 80{,}000\ t$

$EA_{(C)} = 2000 \times 50 = 100{,}000\ t$

$EA_{(D)} = 2000 \times 50 = 100{,}000\ t$

Step # 01:

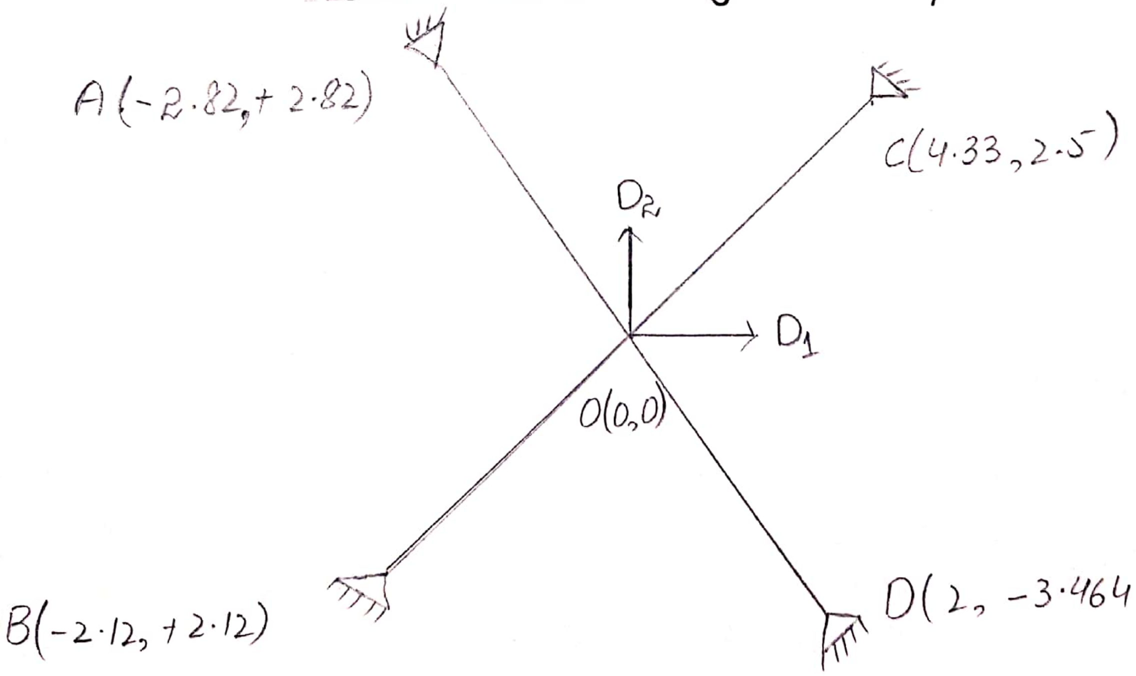
$K \cdot I$

$K \cdot I = 2j - r$

$= 2(5) - 8 = 2^\circ$

Step # 02:

Select unknown Joint Displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} , \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 03:

$$[AMD]_{4\times2} \quad \& \quad [S]_{2\times2}$$

(i) $D_1 = 1 \quad , \quad D_2 = 0$

$$AMD = \frac{EA}{L^2}(\cancel{x}k - xj)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0 - 433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0 \cdot 200) = -125$$

Now

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(xk - xj)^2$$

$$= \frac{80,000}{(400)^3}(282)^2 + \frac{80,000}{(300)^3}(212)^2 + \frac{100,000}{(500)^3}(-433)^2 + \frac{100,000}{(400)^3}(-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3}(x_k - x_j)(y_k - y_j)$$

$$= \frac{80{,}000}{(400)^3}(282)(-282) + \frac{80{,}000}{(300)^3}(212)(212) + \frac{100{,}000}{(500)^3}(-433)(0-250)$$

$$+ \frac{100{,}000}{(400)^3}(-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^3}(y_k - y_j)$$

$$AMD_{12} = \frac{80000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{(400)^2}(346) = 216.25$$

Now,

$$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(y_k - y_j)^2$$

$$\frac{80{,}000}{(400)^3}(-282)^2 + \frac{80{,}000}{(300)^3}(212)^2 + \frac{100{,}000}{(500)^3}(-250)^2 + \frac{100{,}000}{(400)^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step # 04:

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step #05:-

$[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$\begin{bmatrix} 141 \times 0.1183 + (-141) \times (0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 \quad -46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$

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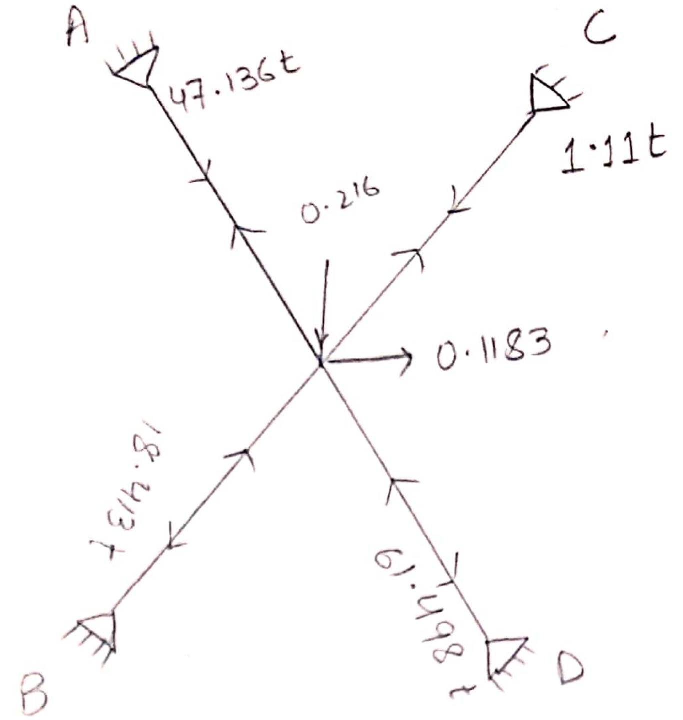
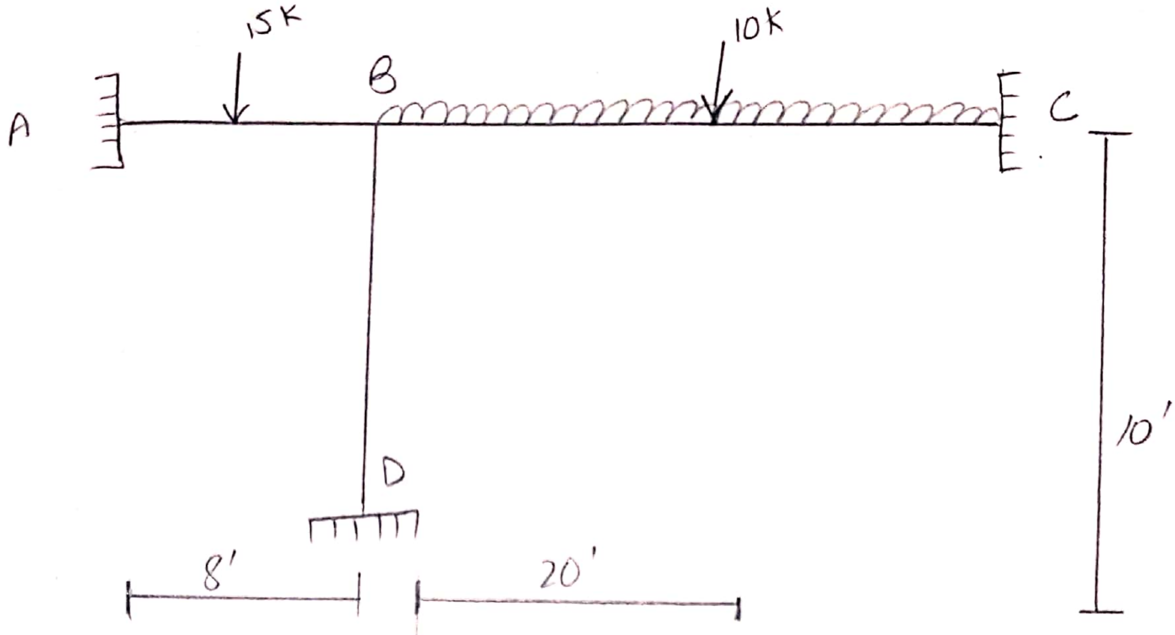

A
C
47.136t
1·11t
0·216
0·1183
18.413t
61.498t
B
D

Question # 03:-



Solution:-

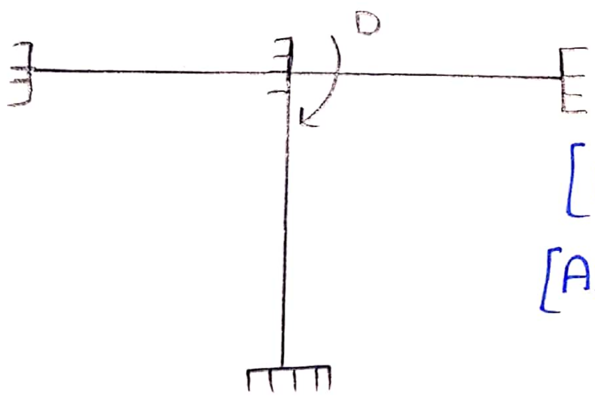
Step # 01:-

Determine kinematic interminancy

$K \cdot I = 1°$

Step # 02:-

Determine unknown Joint Displacement

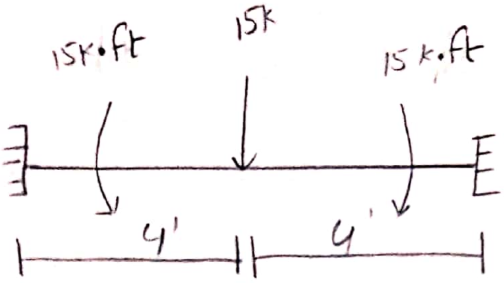


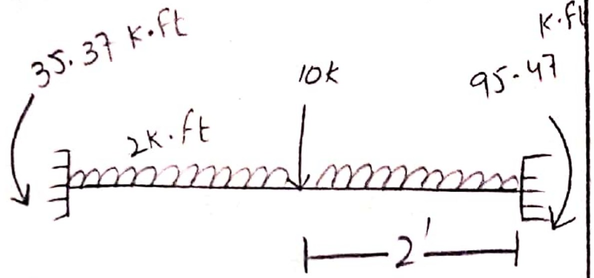
$[D] = [?]$

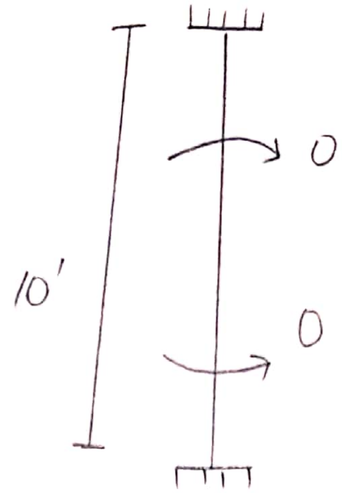
$[AD] = [0]$

Step # 03:

Compute [ADL] Matrix.







⇒ Point load at center :-

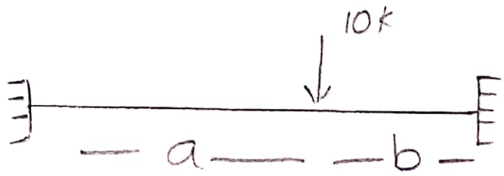
$$\frac{PL}{8} = \frac{(15)(8)}{8} = 15 \text{ k.ft}$$

⇒ Uniformly distributed load :-

$$\frac{wL^2}{12} = \frac{2(20)^2}{12} = 66.67 \text{ K.ft}$$

Point load (Not at mid) :-

Suppose:



For left end:-

$$\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ K.ft}$$

For right end:-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ K.ft}$$

So, total Moment at left end:

$$19.2 + 66.67 = 85.87 \text{ K.ft}$$

Similarly, at right end:-

$$28.8 + 66.67 = 95.47 \text{ K.ft}$$

So: $[ADL] = -85.87 + 15 = -70.87$ K.ft

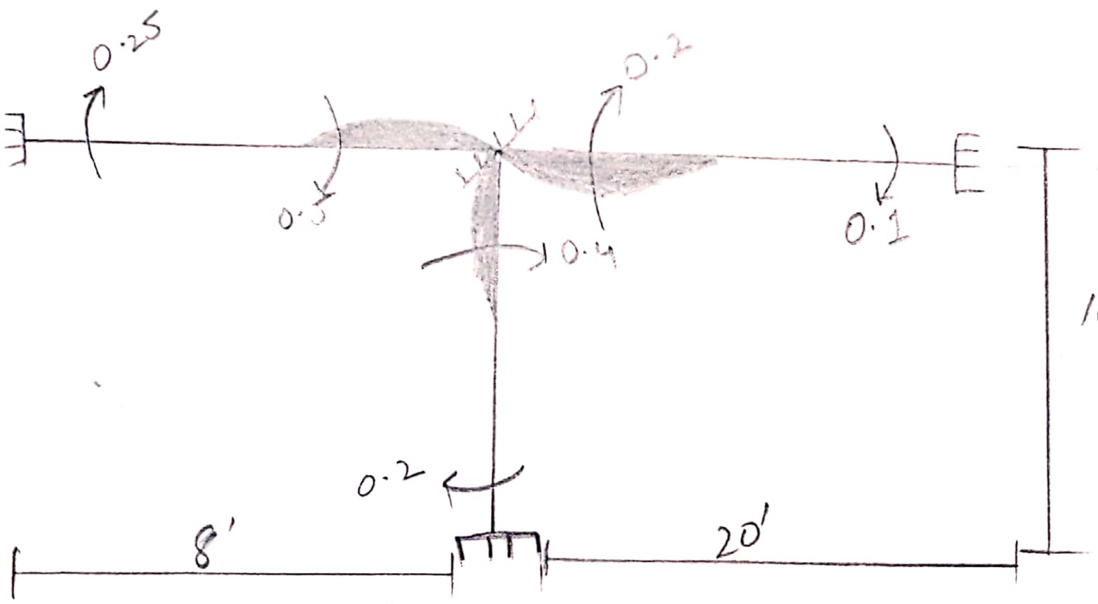
Step#04:-

Determine $[S]$ Matrix :

$$[S] = [S_{11}]$$

Now:

$$D = 1K$$



$\Rightarrow \dfrac{4EI}{8} = 0.5$ $\quad \dfrac{2EI}{8} = 0.25$ $\quad \Rightarrow \dfrac{4EI}{10} = 0.4$

$\Rightarrow \dfrac{4EI}{20} = 0.2$ $\quad \dfrac{2EI}{20} = 0.25$ $\quad \Rightarrow \dfrac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

Step #05:-

Compute $[D]$ Matrix :-

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = 1/1.1 \times [0] - [-70.87]$

$= \dfrac{70.87}{1.1}$

$[D] = [64.42]\ 1/EI$