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SubJect Differential Eq.

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Q1 (a) Estimate the General Solution of
$y' = (x+2)\, y^2$

Sol:- $y' = (x+2)\, y^2$

$$\frac{dy}{dx} = (x+2)\, y^2$$

$$\int \frac{1}{y^2}\, dy = \int (x+2)\, dx$$

$$\int y^{-2}\, dy = \int (x+2)\, dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + C$$

multiPlying both sides by -1

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + c \right)$$

$$y = -\left(\frac{1}{x^2/2 + 2x + c} \right)$$

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Q1 (b) Estimate the general solution of $y' = (y+9x)^2$

Sol:- let $(y+9x) = u$

$$\frac{dy}{dx} y + 9 = \frac{du}{dx}$$

$$\frac{d}{dx} y = \frac{d}{dx} u - 9$$

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9$$

$$\int \frac{1}{u^2+9} du = \int dx$$

$$\int \frac{1}{(3)^2 + (u)^2} du = \int dx$$

$$\frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) = x + c_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + 3c_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + c$$

$$\frac{u}{3} = \tan(3x + c)$$

$$u = 3\tan(3x + c)$$

$$y + 9x = 3\tan(3x + c)$$

$$\boxed{y = -9x + 3\tan(x + c)}$$

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Q2: Estimate the general solution of $x^3 dx + y^3 dy = 0$

Sol:- $x^3 dx + y^3 dy = 0$

$$M dx + N dy = 0$$

$$M = x^3, \quad N = y^3$$

$$\frac{\partial M}{\partial y} = 0, \quad \frac{\partial N}{\partial x} = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{so} \quad \text{exact}$$

$$U = \int M dU + K(y)$$

$$U = \int x^3 d\upsilon + K(y)$$

$$U = \frac{x^4}{4} + (Ky) \quad \text{—— (1)}$$

$$\frac{\partial U}{\partial y} = 0 + \frac{d}{dy} K(y)$$

$$\frac{\partial U}{\partial y} = \frac{d}{dy} K(y)$$

Since we know that

$$\frac{\partial U}{\partial y} = N = y^3$$

$$y^3 = \frac{d}{dy} K(y) \Rightarrow \int y^3 = \int d\,(Ky)$$

$$\Rightarrow K(y) = \frac{y^4}{4} + C_1 \quad \text{Put in} \text{ --- } (i)$$

$$U = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 - C_1 = \frac{x^4}{4} + \frac{y^4}{4}$$

$$\Rightarrow \boxed{C = \frac{x^4}{4} + \frac{y^4}{4} = U} \text{ Ans}$$

Q3 ⓐ Find the general solution

$$4y'' - 20y' + 25y = 0$$

Sol: Assuming ~~the~~ a solution of the form

$y = e^{rx}$ we get the auxiliary equation

$$4r^2 - 20r + 25 = 0$$

that is

$$r_1 = r_2 = \frac{5}{2}$$

$$\therefore y = c_1 y_1 + c_2 y_2 = c_1 e^{\frac{5}{2}t} + c_2 t e^{\frac{5}{2}t}$$

$$\Rightarrow y' = \frac{5}{2} c_1 e^{5/2 t} + c_2 \left(e^{5/2 t} + \frac{5}{2} t e^{5/2 t}\right)$$

Solve For c_1 and c_2

Given $y(0) = 2 \wedge y'(0) = -3$

$$c_1 + 0 = 2$$

$$\therefore \frac{5}{2} c_1 + c_2 = -3 \Rightarrow c_1 = 2 \wedge c_2 = -8$$

$$\boxed{\Rightarrow y(t) = 2e^{5/2 t} - 8te^{5/2 t}}$$

Q 3 (b) Estimate general solution of

$$4y'' - 6y' - 7y = 0$$

Sol:- $4y'' - 6y' - 7y = 0$;

$$y = c_1 e^{\frac{(3+\sqrt{37})t}{4}} + c_2 e^{\frac{(3-\sqrt{37})t}{4}}$$

Second order homogenous differential equation with constants.

it has form of $ay'' + by' + cy = 0$

For an equation $ay'' + by' + cy = 0$ assume solution of the form

e^{rt} Rewrite equation with

$$y = e^{rt}$$

$$4((e^{rt}))'' - 6((e^{rt}))' - 7e^{rt} = 0$$

$$e^{rt}(4r^2 - 6r - 7) = 0$$

$$e^{rt}(4r^2 - 6r - 7) = 0$$

Solve $e^{rt}(4r^2 - 6r - 7) = 0$;

$$r = \frac{3+\sqrt{37}}{4}, \quad r = \frac{3-\sqrt{37}}{4}$$

Rewrite the equation with

$y = e^{rt}$

$$4((e^{rt}))'' - 6((e^{rt}))' - 7e^{rt} = 0$$

Simplify above

$$e^{rt}(4y^2 - 6y - 7) = 0$$

Solve $e^{rt}(4y^2 - 6y - 7) = 0$; $r = \frac{3+\sqrt{37}}{4}$, $r = \frac{3-\sqrt{37}}{4}$

$$r = \frac{3+\sqrt{37}}{4}, \quad r = \frac{3-\sqrt{37}}{4}$$

For two real roots

$r_1 \neq r_2$ the general solution takes ~~from~~ form

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$$c_1 e^{\frac{3+\sqrt{37}}{4}t} + c_2 e^{\frac{3-\sqrt{37}}{4}t}$$

$$\boxed{= c_1 e^{\frac{3+\sqrt{37}}{4}t} + c_2 e^{\frac{3-\sqrt{37}}{4}t}}$$