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Question No # 1.

$4y'' - 20y' + 25y = 0$

Sol.

$ay'' + by' + cy = 0$

and the solution for this is $y = e^{\lambda} \to ①$

General solution

$y = C_1 e^{\lambda} + C_2 t e^{\lambda}$

Now

$4\frac{d^2}{dx^2}(y) - 20\frac{d(y)}{dx} + 25(y) = 0 \to Ⓐ$

put eq ① in eq Ⓐ

$= 4\frac{d^2}{dx^2}(e^{\lambda x}) - 20\frac{d}{dx}(e^{\lambda x}) + 25e^{\lambda x} = 0.$

$\Rightarrow \frac{d^2}{dx^2} e^{\lambda x} = \lambda^2 e^{\lambda x} \to Ⓑ$

Now eq Ⓑ and eq ① in Ⓐ

$= 4\lambda^2 e^{\lambda x} - 20e^{\lambda x} + 25e^{\lambda x} = 0$

$\Rightarrow e^{\lambda x}(4\lambda^2 - 20\lambda + 25) = 0$

$e^{\lambda x} \neq 0$

$\Rightarrow 4x^2 - 20\lambda + 25 = 0$

$(2\lambda - 5)^2 = 0$

$\lambda = 5/2$ or $\lambda = 5/2$

$y(x) = y_1(x) + y_2(x)$

$\Rightarrow y(x) = C_1 e^{5/2x} + C_2 x e^{5/2x}$ Ans.

Question No 2A.

sol:)

$$y'' + 2y' + y = 0$$

Now

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \lambda + 1 = 0$$

$$\lambda(\lambda+1) + 1(\lambda+1) = 0$$

$$\lambda = -1, \quad \lambda = 1$$

Roots are real & equal

The multiply of root $\lambda = -1$ is 2 which gives

$$y_1(x) = C_1 e^{-x}, \quad y_2(x) = C_2 e^{-x}$$

where C_1 are Constant The general solution is the sum of above solution

$$y(x) = y_1(x) + y_2(x) \Rightarrow C_1 e^{-x} + C_2 e^{-x} x$$

$$y' = C_1 e^{-x} + C_2 e^{-x}$$

$$y' = C_1 e^{-x} + C_2 e^{-x} - x e^{-x}$$

for unknown Constants using initial conditions.

Now compute.

$$\frac{dy(x)}{dx}$$

$$\frac{dy(x)}{dx} = \frac{d}{dx}\left(C_1 e^{-x} + C_2 e^{-x} x\right)$$

$$= -C_1 e^{-x} + C_2 e^{-x} - C_2 e^{-x} x$$

Now

$y = 4, \; x = 0$

substitute $y = 4$ into $y(x) = e^{-x} C_1 + e^{-1} x C_2$

$$\boxed{4 = C_1} \rightarrow ①$$

Now

$$x=0 \quad , \quad y=-6$$

Subsitute $y'(0)=-6$

$$-6 = c_1 e^{x} + c_2 e^{-x} - c_2 e^{-x} x$$

$$\Rightarrow -6 = c_1 + c_2 \rightarrow (2)$$

Adding eq (1) & (2)

~~$4 = c_1$~~

$$4 = \cancel{c_1}$$
$$-6 = \cancel{c_1} + c_2$$

$$-2 = c_2 \quad \text{or} \quad c_2 = -2$$

Now we get

$$c_1 = 4 \quad \& \quad c_2 = -2.$$

Subsitute $c_1 = 4$ and $c_2 = -2$ into

$$y = c_1 e^{-x} + c_2 e^{-x} x$$

$$\boxed{y = -2e^{-x}(x-2)}$$ Ans.

Question 26

$$x^2y'' + 3xy' + y = 0$$

Sol:>

$a = 3$, $b = 1$

$m^2 + (a-1)\ m + b = 0$

$m^2 + (3-1)\ m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)\ (m+1) = 0$

$m = -1$; , $m = -1$

roots are real and equal

So,

$y = (c_1 + c_2 \ln x)x^m$

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Question No #3

Sol:→

$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$

$y'' + y' - 6y = 0$

$d^2 + ad + by = 0$

$d^2 + d - 6 = 0$

$d^2 + d - 6 = 0$

$d^2 + 3d - 2d - 6 = 0$

$d(d-3) - 2(d+3) = 0$

$d+3 = 0 \quad , (d-2) = 0$

$d = -3 \quad , d = 2$

port are Real and distone.

$y = c_1 e^{-3x} + c_2 e^{2x}$

chaice for y.

$y_1 = k_3 x^2 + k_2 x^2 + k_1 x + k_0$

$y' = 3k_3 x^2 + 2k_2 x + k_1$

$y_1 = 6k_{3x}^{2} - 2k_2$

putin ①.

$6k_3 x - 2k_2 + 3k_3 x^2 + 2k_3 x + k_1 - 6k_3 x^3 - 6k_x^3$

$-6k_2 x^3 - 6k_1 x - 6kx - 6x^3 - 3x^3 + 12x.$

comparing

$-6k_3 = 6 \quad , -6k_2 + 3k_1 = -3$

$, -6k_2 + 3(-1) = -3$

$, -6k_2 - 3 = -3$

$, -6k_2 = -3 + 3$

$, 6k_2 = 0$

$\boxed{k_2 = 0}$

$6k_3 + 2k_2 + k_2 = 12x$

$6(-1) + 2(0) + k = 12.$

$-6 + k_1 = 12.$

$\boxed{k_1 = 2}$

$-2k_2 + k_1 + k_0 = 0$

$-2(0) - 2 + k_0 = 0$

$\boxed{k_0 = 2}$

Question No # 4

Sol

$$y'' - 4y' + 4y' = x^2 e^{2x}.$$

For equation.

$$y'' - 4y' + 4y = 0.$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda^2(\lambda - 2) - 2(\lambda - 2) = 0$$

$$\lambda = 2, \lambda = 2.$$

Roots are Real & equal

$$y = (c_1 + c_2 x) e^{2x}$$

$$y_1 = c_1 e^{2x} + c_2 x e^{2x}.$$

$$y_1 = e^{2x}, \quad y_2 = x e^{2x}.$$

$$y_1' = 2e^{2x}, \quad y_2' = e^{2x} + 2ex^{2x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2ex^{2}x \end{vmatrix}$$

$$W = e^4 + 2xe^4 - 2xe^{4x}$$

$$W = e^{4x}$$

$$y_p = -y_1 \int \frac{y_2\, y(x)}{W} + y_2 \int \frac{y_1\, y(x)}{W}$$

$$y_p = e^{-2x} \int \frac{x e^{3x} \cdot x^2 e^{2x}\, dx}{e^{2x}} \int \frac{e^{2x} x^2 e^{2x}\, dx}{e^{2x}}$$

$$y_p = -e^{2x} \int \frac{x^3 \cancel{e^{4x}}}{\cancel{e^{4x}}} + x e^{3x} \int x^2 \frac{\cancel{e^{4x}}}{\cancel{e^{4x}}}\, dx$$

$$y_p = -e^{2x}\int x^2 dx + xe^{2x}\int x^2 dx.$$

$$y_p = -e^{2x} \cdot \frac{x^4}{4} + xe^{3x} \cdot \frac{x^3}{3}$$

So

$$y = y_x + y_p.$$

$$y = C_1 e^{3x} + C_2 x e^{-2x} - e^{3x}\frac{x^4}{4} + xe^{2x}\frac{x^3}{3}$$

an

Question No #5

Sol:

e^{-3x} in hyperbolic equation it will be e^{+3x} then it will be solved.

$y' = e^{0x}$, $y^2 = e^{+3x}$

$y = C_1e^{0x} + C_2e^{+3x}$

So roots are real and distinct

$y = (C_1e^{\lambda_1 x} + C_2e^{\lambda_2 x})$

So $\lambda_1 = 0$, $\lambda_2 = 3$

$\lambda_1 = 0$, $\lambda_2 - 3 = 0$

$(\lambda)(\lambda - 3) = 0$

$\lambda^2 - 3\lambda = 0$

So

$\lambda^2 - a\lambda + b = 0$

As

$a = -3$, $b = 0$

So

$y'' + ay' + by = 0$

$y'' - 3y = 0$

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