IQRA NATIONAL UNIVERSITY
Structural Analysis 1

Final Assignment

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Section = (A)

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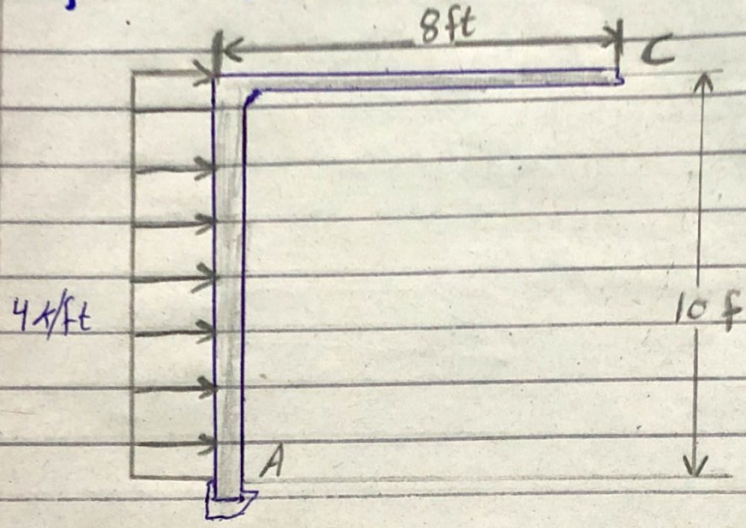
Q 1: Determine the vertical displacement of free end point C on the frame shown in Figure. Take $E = 29(10^3)$ ksi and $I = 600\ in^4$ for both members. Use method of Virtual work.



Sol:-

Given data:

Uniform load = 4k/ft

$E = 29 \times 10^3$ ksi

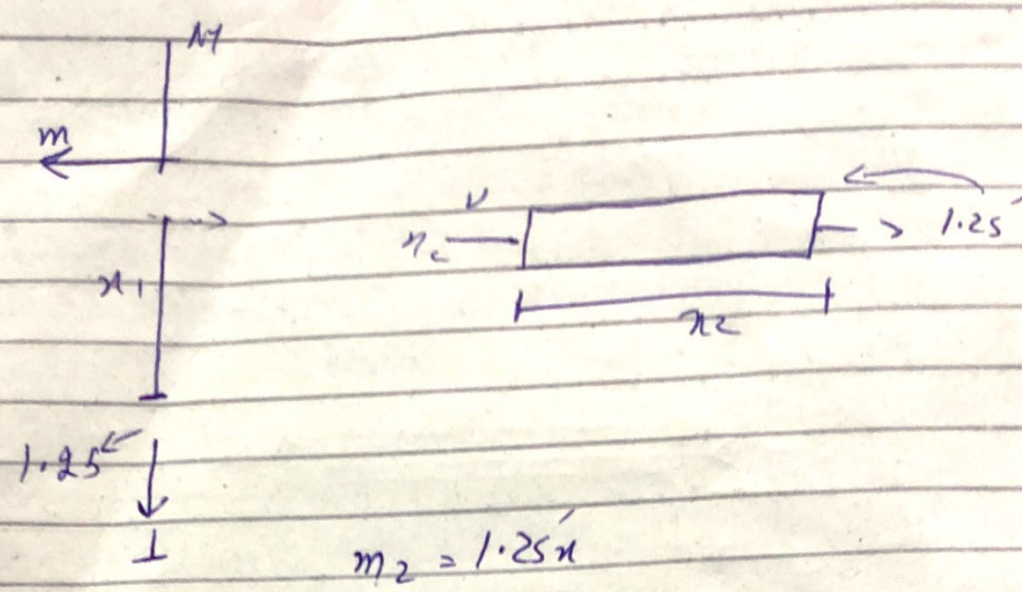
$I = 600\ in^4$

Required:

Vertical displacement

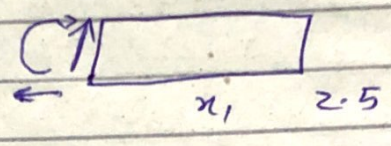
Sol:-

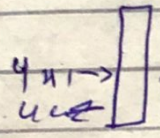
Now vertical moment



$m_2 = 1.25x$

real moment





$$m'' = \frac{40x_1 - \frac{1}{2}x_1(u_2)}{40x_1 - 2u^2}$$

Now By vertical work equation

$$\Delta DC = \int_0^L \frac{m M dx}{E}$$

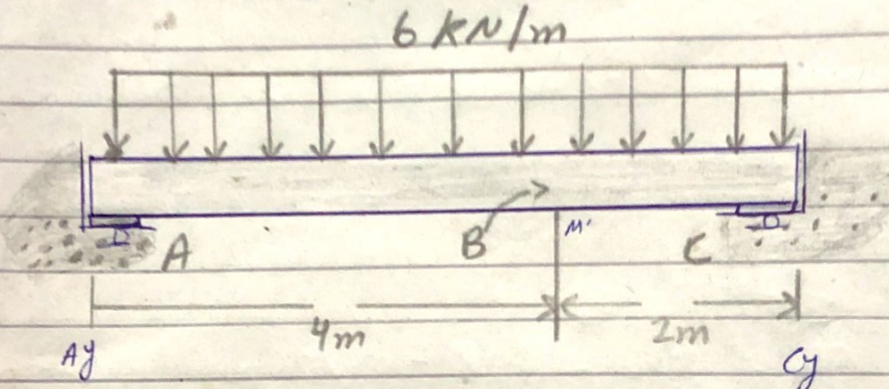
$$\Delta L = \int_0^{10} \frac{(1x_1)(40x_2 - 2u^2)}{E} dx + \int_0^8 \frac{(1.25u_2)(25)}{EI}$$

$$\Delta L = \frac{1}{EI}\left| \frac{4x^3}{3} - \frac{2u^3}{4} \right|_0^{10} + \left| \frac{(31.25u^3}{3} \right|_0^8$$

~~$\Delta L = 1649$~~

$$\Delta L = 10649.60184.$$

Q2:- Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take E = 200 GPa, $I = 60(10)^6\ mm^4$. Use Castigliano's theorem.



Sol:

First we find reactions using equilibrium method;

$\Sigma M_A = 0$ (+)

$(36 \times 3) - M' - Cy \times 6 = 0$

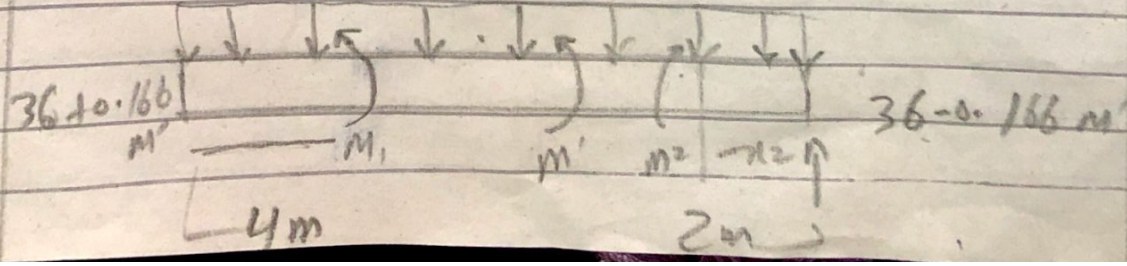
$Cy = \dfrac{(36 \times 3) - M'}{6}$

$Cy = 18 - 0.1667M'$

$\Sigma F_y\ (+) = 0$

$Ay - 36 + 18 - 0.1667M' = 0$

$Ay = 36 + 0.1667M'$



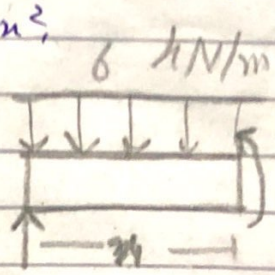
Now we take sections and find moment at that section as;

Section 1 : (x_1)

$\Sigma M = 0 \; +\circlearrowleft$

$-M_1 + (36 + 0.166\,M') x_1 - 3x_1^2 - (6x_1)(x_1/2) = 0$

$M_1 = (36 + 0.1667\,M') x_1 - 3x_1^2$



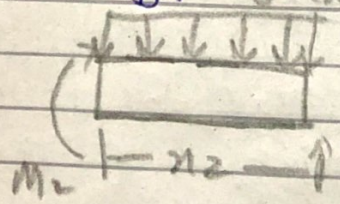
Section 2 : (x_2)

$\Sigma M = 0 \; +\circlearrowleft$

$M_2 + 3x_2 - (36 - 0.1667\,M')\, x_2 = 0$

$M_2 = (36 - 0.1667\,M')\, x_2 - 3x_2$

Now we find reactions and moment at sections by apply unit load (P) at point B.



Reaction :

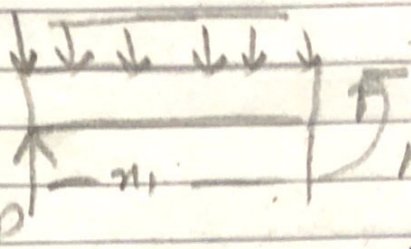
$A_y = 36\left(\frac{7}{6}\right) + P\left(\frac{2}{6}\right)$

$A_y = 18 + 0.33P$

$C_y = 36\left(\frac{2}{6}\right) + P(4/6)$

$C_y = 18 + 0.67P$

Now we find moment at sections as;



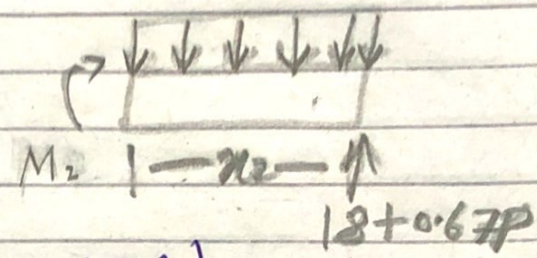
(Section 1) (x_1)

$\Sigma M = 0$ (+↺)

$(18+0.33P)x_1 - M_1 - (6x_1)(x_1/2) = 0$

$M_1 = (18+0.33P)x_1 - 3x_1^2$

(Section 2) (x_2)



$\Sigma M = 0$ (+↺)

$M_2 + 3x_2^2 - (18+0.67P)x_2 = 0$

$M_2 = (18+0.67P)x_2 - 3x_2^2$

For slope:

By taking derivatives of eq(1) and eq(2) w.r.t M' we get

$$\frac{\partial M_1}{\partial M'} = 0.1667 x_1$$

$$\frac{\partial M_2}{\partial M'} = 0.1667 x_2$$

Also put $M' = 0$ in eq(1) and eq(2)

$M_1 = (36x_1 - 3x_1^2)$ kN·m

$M_2 = (36x_2 - 3x_2^2)$ kN·m

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^4 \frac{(36x_1 - 3x_1^2)(0.1667x_1\,dx_1}{EI} + \int_0^2 \frac{(36x_2 - 3x_2^2)(0.1667x_2^2)dx_2}{EI}$$

$$= 1.98 \times 10^{-5}$$

For displacement

Taking derivative of eq(3) and (4) w.r.t P.

$$\frac{\partial M_1}{\partial P} = 0.33x_1$$

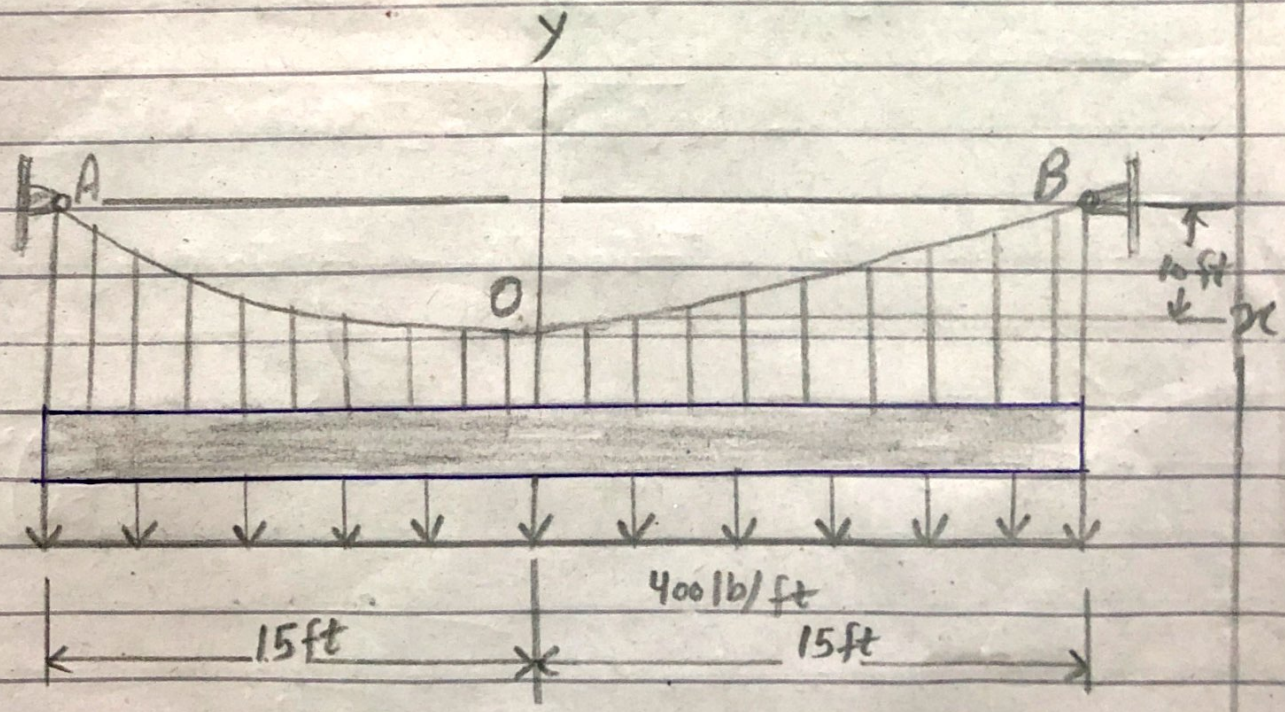
$$\frac{\partial M_2}{\partial P} = 0.67\ x_2$$

Also put $P=0$ in eq (3) and (4)

$M_1 = (18x_1 - 3x_1^2)$

$M_2 = (18x_2 - 3x_2^2$

Q 3:- The cable is subjected to the uniform loading. If the slope of the cable at point O is zero, determine the equation of the curve and the force in the cable at O and B.



Sol:

Step (1) Parabola equation

$$y = \frac{h}{L^2} x^2 = \frac{10}{15^2} \times x^2$$

$$y = 0.0444 x^2$$

Step (2)

$$FH = T_o = \frac{w_o L^2}{2h} = \frac{400 \times 15^2}{2 \times 10} = 4500 \text{ lb}$$

$$= 4.5 k$$

Step (3)

$$T_B = T_{max} = \sqrt{(FH)^2 + (w_o L)^2}$$

$$= \sqrt{(4500)^2 + (400 \times 15)^2}$$

$$= 7500 \text{ lb} = 7.5 k$$

Step (4)

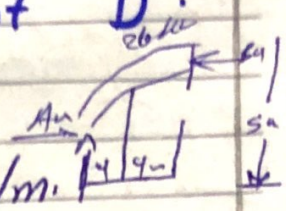
$$T_B = T_{max} = w_o L \sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

$$= 400 \times 15 \sqrt{1 + \left(\frac{15}{2 \times 10}\right)^2}$$

$$= 7500 \text{ lb}$$

$$= 7.5 k$$

Q4:- The three-hinged spandrel arch is subjected to the uniform load of 30 kN/m. Determine the internal moment in the arch at Point D.



Given data:

Uniform load = 30 kN/m.

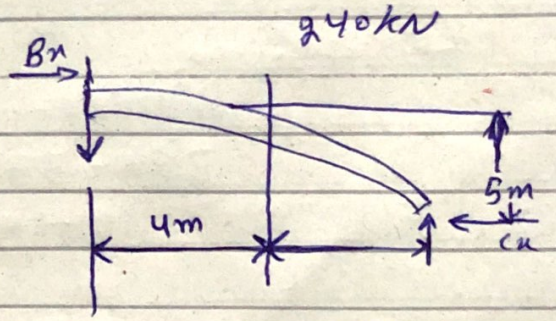
Required:

Internal moment at D = ?

Member AB

Sol:-

Dividing into two member AB and BC



Member BC

AB.

$\zeta + \Sigma M_A = 0 \quad B_x(5) + B_y(8) - 240(4) = 0 \rightarrow (a)$

BC

$\zeta + \Sigma M_C = 0 \quad -B_x(5) + B_y(8) + 240(4) = 0 \rightarrow (b)$

Adding eq (a) and (b)

$B_x(5) + B_y(8) - 240(4) = 0$

$-B_x(5) + B_y(8) + 240(4) = 0$

$0 + 2B_y(8) + 0 = 0$

$$2By(8) = 0$$

$$\Rightarrow By = 0 \cdot kN.$$

Putting the value of "By" in eq (b)

$$eq(b) \Rightarrow -Bx(5) + 0(8) + 960 = 0.$$

$$Bx(5) = 960$$

$$\frac{Bx(5)}{(5)} = \frac{960}{5}$$

$$\boxed{Bx = 192\ kN}$$

"Now at segment DB"

$$\zeta + \Sigma M_D = 0$$

$$192(2) - 150(2.5) - M_D = 0$$

$$384 - 375 - M_D = 0$$

$$9 - M_D = 0$$

$$\Rightarrow M_D = 9\ kN \cdot m$$