Name : ⟶ M. Amir Hamza

ID : ⟶ 7986

Subject: ⟶ Calculus

Dept : ⟶ Civil

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Instructor: ⟶ Shonila Mazhar

Submit ⟶ ✓

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Question # 1 :→ Find PQ where P is the point in three-dimensional space with coordinates (4, 1, 3) and the point Q with coordinates (1, 2, 4). Find the distance b/w P & Q Futher. Find the position vector of the point dividing PQ in the ratio 1:3.

Solution :→

Coordinate of P = (4, 1, 3)

$$OP = 4i + 1j + 3K$$

or

$$OQ = \vec{OQ} - \vec{OP}$$

$$= (i + 2j + 4K) - (4i + 1j + 3K)$$

$$= -3i + 1j + 1K \longrightarrow (1)$$

Now Distance b/w P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \longrightarrow (2)$$

Let M be the point which divided PQ in ratio 1:3 Then by the ratio theorm positive vector of $M = \vec{OM}$

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$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \longrightarrow (3)$$

Hence eq 1, 2 & 3 are required solution

So Answer: ⟶

Q No # 2:→ Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Solution:→

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

By long Division

$$\frac{4x^3+10+4}{2x^2+x} = 2x-1+\frac{11x+4}{2x^2+x}$$

$$= 2x-1+\frac{11x+4}{x(2x+1)}$$

suppose that

$$\frac{11x+4}{x(2x+1)} = A/x + B/{2x+1}$$

xplying both sides by $x(2x+1)$

$$11x+4 = A(2x+1)+Bx \longrightarrow (1)$$

Put $x=0$ & $x=-1/2$ in (1)

$$\boxed{A=4} \quad -11/2+4 = -1/2\, B$$

$$\Rightarrow \frac{-11+8}{2} = -1/2\, B$$

(P-T-O) ⟹

$$\Rightarrow \quad \frac{-11+8}{2} = -\tfrac{1}{2}B$$

$$\therefore \quad \Rightarrow \quad B = 3$$

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

$\therefore$

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int 2x\,dx - \int dx + \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$I = 2\cdot\frac{x^2}{2} - x + 4\ln|x| + \tfrac{3}{2}\ln|2x+1| + c$$

$$\Rightarrow \quad I = x^2 - x + \tfrac{3}{2}\ln|2x+1| + 4\ln|x| + c$$

Answer:→

$\therefore$ ══════ $\therefore$

Q No # 3 :→ Evaluate

(a) $\int_0^2 x^2 e^x \, dx$

Solution :→ $\int_0^2 x^2 e^x \, dx$

Now first find Integration.

$$= \int x^2 e^x \, dx$$

$$= x^2 \int e^x \, dx - \int \left(\left(\int e^x \, dx \right) \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) \, dx.$$

$$= x^2 e^x - 2 \int x e^x \, dx$$

$$= x^2 e^x - 2 \left[x \int e^x \, dx - \int \left(\left(\int e^x \, dx \right) \frac{d}{dx} x \right) \right.$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x \, dx \right.$$

$$= x^2 e^x - 2x e^x + 2 e^x$$

Now put limits

$$\left| x^2 e^x - 2x e^x + 2 e^x \right|_0^2$$

(P-T-O) ⇒

$$= \left(2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$$

$$= (\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2)$$

$$= 2e^2 - 2$$

Answer:→ $2e^2 - 2$

(b) $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

suppose

$$I = \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx \quad \text{——— (1)}$$

Put $\sqrt{x} = u$

$$\Rightarrow \frac{1}{2\sqrt{x}}\,dx = du$$

$$\Rightarrow \frac{dx}{\sqrt{x}} = 2\,du$$

(P-T-O) ⇒

When $x \to 1$, $u \to 1$

When $x \to 2$, $u \to \sqrt{2}$

$\therefore$ Equation (1) becomes

$$I = \int_{1}^{\sqrt{2}} \sin u \cdot 2\,du$$

$$= -2 \left| \cos u \right|_{1}^{\sqrt{2}}$$

$$= -2(\cos\sqrt{2} - \cos 1)$$

Answer: $\to = -2\cos\sqrt{2} + 2\cos 1$

Q No # 4: →

Solution: → Given that

$$u(x,y,z) = \frac{1}{\sqrt{x^2, y^2, z^2}} \longrightarrow (1)$$

we have to verify

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Now,

⇒ Partially diff (1) w.r.t (x)

$$\frac{\partial u}{\partial x} = -\tfrac{1}{2}\,(x^2+y^2+z^2)^{-3/2}$$

Again diff w.r.t 'x', we have

$$\frac{\partial^2 u}{\partial x} = -\left[(x^2+y^2+z^2)^{-3/2} + x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x)\right]$$

$$= -\left[(x^2+y^2+z^2)^{-3/2} - 3x^2(x^2+y^2+z^2)^{-5/2}\right]$$

In the same manner

$$\frac{\partial^2 u}{\partial y^2} = -\left[(x^2+y^2+z^2)^{-3/2} - 3y^2(x^2+y^2+z^2)^{-5/2}\right]$$

and

(P-T-O) ⇒

$$\frac{\partial^2 u}{\partial z^2} = -\left[(x^2+y^2+z^2)^{-3/2} - 3z^2(x^2+y^2+z^2)^{-5/2}\right]$$

Adding above all second order partial derivatives, we have

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2}$$

$$= -(x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}$$

$$= -(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2}$$

$$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-5/2}(x^2+y^2+z^2)$$

$$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-3/2} = 0$$

Hence

$$\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + \partial^2 u/\partial z^2 = 0$$

This is laploce equation in 3 dimension.