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Question # 1 : \rightarrow Find PQ where P is the point in three-dimensional space with coordinates $(4, 1, 3)$ and the point Q with coordinates $(1, 2, 4)$. Find the distance b/w P & Q Further. Find the position vector of the point dividing PQ in the ratio $1:3$.

Solution : \rightarrow

Coordinate of $P = (4, 1, 3)$

$$OP = 4i + 1j + 3k$$

$$\begin{aligned} \text{OR } OQ &= \vec{OQ} - \vec{OP} \\ &= (i + 2j + 4k) - (4i + 1j + 3k) \\ &= -3i + 1j + 1k \longrightarrow \textcircled{1} \end{aligned}$$

Now Distance b/w P & $Q = |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \longrightarrow \textcircled{2}$$

Let M be the point which divided PQ in ratio $1:3$ Then by the ratio theorem position vector of $M = \vec{OM}$

$$(\vec{P} - \vec{T} - \vec{O}) \Rightarrow$$

Q No # 2: → Evaluate

3

$$\left(\frac{4x^3 + 10x + 4}{2x^2 + x} \right)$$

Solution: →

$$\left(\frac{4x^3 + 10x + 4}{2x^2 + x} \right)$$

By long Division

$$\frac{4x^3 + 10x + 4}{2x^2 + x} = 2x - 1 + \frac{11x + 4}{2x^2 + x}$$

$$= 2x - 1 + \frac{11x + 4}{x(2x + 1)}$$

Suppose that

$$\frac{11x + 4}{x(2x + 1)} = \frac{A}{x} + \frac{B}{2x + 1}$$

Multiplying both sides by $x(2x + 1)$

$$11x + 4 = A(2x + 1) + Bx \longrightarrow \textcircled{1}$$

~~Put~~ Put $x = 0$ & $x = -\frac{1}{2}$ in $\textcircled{1}$

$$\boxed{A = 4} \quad -\frac{11}{2} + 4 = -\frac{1}{2}B$$

$$\Rightarrow \frac{-11 + 8}{2} = -\frac{1}{2}B$$

$$(P - T - 0) \Rightarrow$$

④

$$\Rightarrow \frac{-11+8}{2} = -\frac{1}{2} B$$

$$\therefore \Rightarrow B = 3$$

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

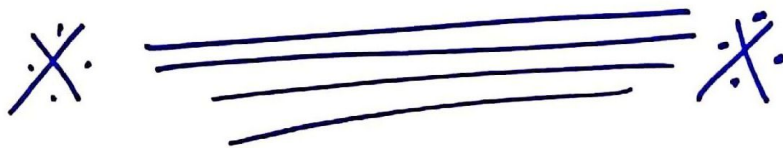
\therefore

$$\int \frac{4x^3+10x+4}{2x^2+x} dx = \int 2x dx - \int dx + \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$$

$$I = 2 \cdot \frac{x^2}{2} - x + 4 \ln|x| + \frac{3}{2} \ln|2x+1| + C$$

$$\Rightarrow I = x^2 - x + \frac{3}{2} \ln|2x+1| + 4 \ln|x| + C$$

Answer: \rightarrow _____ \uparrow



Q No # 3 :-> Evaluate

(5)

(a) $\int_0^2 x^2 e^x dx$

Solution :-> $\int_0^2 x^2 e^x dx$

Now first find Integration.

$$= \int x^2 e^x dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) dx.$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) \right]$$

~~$$= x^2 e^x - 2 [x e^x - \int e^x dx]$$~~

$$= x^2 e^x - 2 [x e^x - \int e^x dx]$$

$$= x^2 e^x - 2 x e^x + 2 e^x$$

Now put limits

$$\left| x^2 e^x - 2 x e^x + 2 e^x \right|_0^2$$

$$(P - T - 0) \Rightarrow$$

$$\begin{aligned}
 &= (2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)) \quad \textcircled{6} \\
 &= (\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2) \\
 &= 2e^2 - 2
 \end{aligned}$$

Answer: $\rightarrow 2e^2 - 2$

(b) $\int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

suppose

$$I = \int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx \quad \text{--- (1)}$$

$$\text{Put } \sqrt{x} = u$$

$$\Rightarrow \frac{1}{2\sqrt{u}} du = du$$

$$\Rightarrow \frac{du}{\sqrt{u}} = 2 du$$

$$(p - r - o) \Rightarrow$$

When $x \rightarrow 1$, $u \rightarrow 1$

When $x \rightarrow 2$, $u \rightarrow \sqrt{2}$

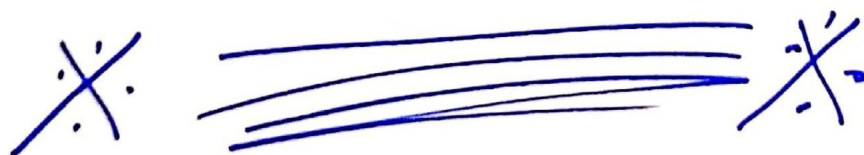
\therefore Equation (1) becomes

$$I = \int_1^{\sqrt{2}} \sin u \cdot 2 du$$

$$= -2 \left| \cos u \right|_1^{\sqrt{2}}$$

$$= -2 (\cos \sqrt{2} - \cos 1)$$

Answer: $\rightarrow = -2 \cos \sqrt{2} + 2 \cos 1$



Q No # 4: →

8

Solution: → Given that

$$u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \longrightarrow (1)$$

we have to verify

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Now,

⇒ Partially diff (1) w.r.t (x)

$$\frac{\partial u}{\partial x} = -\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2}$$

Again diff w.r.t 'x', we have

$$\frac{\partial^2 u}{\partial x^2} = - \left[(x^2 + y^2 + z^2)^{-3/2} + x(-3/2)(x^2 + y^2 + z^2)^{-5/2} (2x) \right]$$

$$= - \left[(x^2 + y^2 + z^2)^{-3/2} - 3x^2 (x^2 + y^2 + z^2)^{-5/2} \right]$$

In the same manner

$$\frac{\partial^2 u}{\partial y^2} = - \left[(x^2 + y^2 + z^2)^{-3/2} - 3y^2 (x^2 + y^2 + z^2)^{-5/2} \right]$$

and

$$(P-T-O) \Rightarrow$$

$$\frac{\partial^2 u}{\partial z^2} = - \left[(x^2 + y^2 + z^2)^{-3/2} - 3z^2 (x^2 + y^2 + z^2)^{-5/2} \right] \quad (9)$$

Adding above all second order partial derivatives, we have

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} &= - (x^2 + y^2 + z^2)^{-3/2} + 3x^2 (x^2 + y^2 + z^2)^{-5/2} \\ &= - (x^2 + y^2 + z^2)^{-3/2} + 3y^2 (x^2 + y^2 + z^2)^{-5/2} \\ &= - (x^2 + y^2 + z^2)^{-3/2} + 3z^2 (x^2 + y^2 + z^2)^{-5/2} \\ &= -3 (x^2 + y^2 + z^2)^{-3/2} + 3 (x^2 + y^2 + z^2)^{5/2} (x^2 + y^2 + z^2)^{-5/2} \\ &= -3 (x^2 + y^2 + z^2)^{-3/2} + 3 (x^2 + y^2 + z^2)^{-3/2} = 0 \end{aligned}$$

Hence

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

This is Laplace equation in 3 dimension.

