

FAREEHA JEHANGIRI

ID 16051

SEMESTER 2<sup>ND</sup>

SUBJECT LINEAR ALGEBRA

INSTRUCTOR SIR SHAKEEL

SECTION "A"

PROGRAM BS(SE)



# QUESTION NO. 1

Consider the given below matrix as the augmented matrix of linear system. Explain should be performed in your words the next elementary row operation that should be performed in order to solve this system. Where  $ID_3$  is the 3<sup>rd</sup> digit in your ID and  $ID_{Last}$  is the last digit of your ID in inverse e.g. if your ID is 12345 then  $-ID_{Last} = -5$ .

$$\begin{bmatrix} 1 & ID_3 & 3 & 0 & 5 \\ 0 & 1 & -ID_{Last} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{bmatrix}$$

$R_1 - 3R_3$

$$\begin{bmatrix} 1-0 & 0-0 & 3-3 & 0-0 & 5-0 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{bmatrix}$$



$$R_2 + R_3 \begin{bmatrix} 1 & 0 & 0 & 0 & 5+15 \\ 0 & 1+0 & -1+1 & 0+0 & 7-6 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Answer.

## Question No. 2

Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$



In order to transform first matrix into the second we will multiply  $-2$  with row 2 and then add the row 2 to row 3

$$-2R_2 + R_3 \therefore$$

$$\begin{array}{r} -2 \times \text{row 2} \\ = \end{array} \begin{array}{cccc} 0 & 1 & -4 & 2 \\ 0 & -2 & 8 & -4 \end{array}$$

Now adding it to row 3

$$\begin{array}{cccc} + & 0 & -2 & 8 & -4 \\ & 0 & 2 & -5 & -4 \\ \hline & 0 & 0 & 3 & -5 \end{array}$$

Putting the newly formed row 3 into the first matrix.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Hence we transformed the first matrix into the second matrix.

Now adding reverse row operation and turning the second matrix into the first



$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Multiplying 2 with row 2 and adding it to row 3

$$2R_2 + R_3:$$

$$\begin{array}{cccc} 0 & 2 & -8 & 4 \\ 0 & 0 & 3 & -5 \\ \hline 0 & 2 & -5 & -1 \end{array}$$

Putting back the new row 3 obtained above, into the second matrix and transforming it to matrix as first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}.$$

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# QUESTION NO. 2

## PART B

Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words of each of the selection in detail.

a

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & \pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

This matrix is not in echelon form because the most below row have "e" element which is not zero and for echelon form there must be all zero element in the last line.

b

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



(7)

b) This one is the echelon form because the pivot element having zero in their below row and the last row have also zero element.

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

c) This is not reduced echelon form because having non-zero rows at the bottom.

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

This is reduced echelon form if when we transpose  $R_2 \leftrightarrow R_3$  then the bottom line will be have all zero element.

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# QUESTION 3

## PART "A"

(a)

The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form?

What is the practical use of reduced row echelon form? Give an example?

Row Echelon Form :: A Matrix is said to be in row (column) echelon form when it satisfies the following conditions.

1. The first non-zero element in each row, called leading entry is 1.
2. Each leading entry is in a row to the right of the leading entry in the previous row.
3. Rows with all zero elements, if any are below (after) the row having



a non-zero element

For Example:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduced Row Echelon Form:

\* A Matrix is said to be in reduced row echelon form when it satisfies the following conditions

- 1- The Matrix satisfies conditions for a row echelon form
- 2- The leading entry in each row is the only non-zero entry in its row.

For example

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore we can say that each reduced row echelon form is also a row echelon form, but vice versa is not always true



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# QUESTION No. 3

## PART "B"

$$\begin{bmatrix} 1 & 1D_2 & 8 \\ 2 & 8 & -1 \\ 1D_3 & 0 & 0 \\ 1 & -4 & 1D\text{-first-last} \end{bmatrix}$$

Solution

I put my ID at the matrix.

Now this matrix solution is that  
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$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2-2 & 8-12 & -1-16 \\ 0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix} \quad 2R_1 - R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & +17 \\ 0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix} \quad R_3 \leftrightarrow R_2$$



(1)

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 1 & -4 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 11 & -4-6 & 11-8 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & -10 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0-0 & -10(-4) & 3-(-17) \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & -6 & 20 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & -6 & 20 \\ 0 & 0 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -1 \\ 0 & -6 & 20 \\ 0 & 0 & 0 \end{bmatrix}$$

(12) (8)

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0+1 & -6+6 & 20+8 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 + R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 1 & 0 & 28 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0 \times 1 & 0 \times -4 & 28 \times -7 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \times R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0 & 0 & 196 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Answer!}$$

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