Name= Muhammad Zubair Khan

ID= 7677

Differential equation

Question no = 01

$f(t) = 1+t = \pi \leq t < \pi$

Here we use the formular

$$f(x) = a_0 + \sum_{n=1}^{n} a_n \cos t + \sum_{n=1} b_n \sin t \quad (1)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right)$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$\boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

2

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)(\cos t)\,dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int \frac{\sin nt}{n}\,d(1+t)\,dt\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1 - (-1)\right)$$

$$\boxed{a_n = 0}$$

(3)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\int (1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\sin nt \frac{d}{dt}(1+t)\,dt\right)$$

$$b_n = \frac{1}{\pi}(1+t)\left(-\frac{\cos nt}{n}\Big|_{-\pi}^{\pi} - \int -\frac{\cos nt}{n}(1)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \cancel{\left(\frac{\sin nt}{n^2}\Big|_{-\pi}^{\pi}\right)}^{0}\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos} + \pi\cos\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos\pi - \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

so eqn become

$$f(x) = \frac{1}{2\pi}\ 2\pi + \pi^2) + 0\sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin t$$

o

Question No: 9

09/ $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

Eigen values = ?

Sol:

$\boxed{\text{step} = 1}$

we have;

$(A - \lambda I)x = 0$ A = Given matrix

$\boxed{\text{step} = 2}$

we have;

The characteristic equation is given by;

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & \lambda \end{vmatrix} = 0$$

$\boxed{\text{step} = 3}$

Scanned with CamScanner

$\lambda^3 - \left[\begin{matrix} \text{Sum of} \\ \text{Diagonal elem} \end{matrix} \right] \lambda^2 + \left[\begin{matrix} \text{Sum of} \\ \text{Diagonal minors} \end{matrix} \right]$

$\lambda - |A| = 0$ — (B)

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By Putting values in eqn (B)

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ — (C)

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By Putting values in eqn (C)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Scanned with CamScanner

Using quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 1 \\ b &= -4 \\ c &= -3 \end{aligned}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2} \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigen values =

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$

Scanned with CamScanner

Page 01

Q:3

$$5x + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z = 1$$
$$x + y + z + m = 0$$

Sol

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & 9 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

02

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & -15 & -1 & -1 \\ 0 & 0 & \frac{-23}{5} & 6/5 & \frac{-1}{5} \end{array}\right] \begin{array}{l} \\ \\ R_3 - 5R_2 \\ R_4 - 2R_2 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & \frac{1}{15} & 1/15 \\ 0 & 0 & \frac{-23}{5} & -1/5 & -1/5 \end{array}\right] \left(\frac{-1}{15}\right) R_3$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & 0 & 113/75 & 8/75 \end{array}\right] R_4 + \frac{23}{5} R_3$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 1 & 1 & 1/15 & 1/15 \\ 0 & 1 & 0 & 8/113 & 8/113 \end{array}\right] \left(75/113\right) R_4$$

~~113~~

Putting value of x in eq (3)

$$2 + \frac{1}{15}\left(\frac{8}{113}\right) = \frac{1}{15}$$

$$2 + \frac{8}{1695} = \frac{1}{15}$$

$$2 + \frac{1}{15} - \frac{8}{1695}$$

$$2 + \frac{113-8}{1695}$$

$$2 = 105/1695$$

Putting values in eqn (2)

$$y + \frac{9}{5}\left(\frac{105}{1695}\right) \frac{-3}{5}\left(\frac{8}{113}\right) = \frac{-2}{5}$$

$$y + \frac{945}{8475} - \frac{24}{565} = \frac{-2}{5}$$

$$y + \frac{945-360}{8475} = \frac{-2}{5}$$

$$y + \frac{585}{8475} = \frac{-2}{5}$$

$$y = \frac{-2}{5} - \frac{585}{8475}$$

$$y = \frac{-3390-585}{8475}$$

$$y = -\frac{3975}{8475}.$$

Putting values eqn in (1)

$$x + \frac{3975}{8475} + 2\left(\frac{105}{1695}\right) + \frac{8}{113} = 1$$

$$x + \frac{3975}{8475} + \frac{210}{1695} + \frac{8}{113} = 1$$

$$x + \frac{3975 + 1050 + 600}{8475} = 1$$

$$x + \frac{5625}{8475} = 1$$

$$x = 1 - \frac{5625}{8475}$$

$$x = \frac{8475 - 5625}{8475}$$

$$x = \frac{2850}{8475}$$

$$x = \frac{570}{1695}$$

$$x = \frac{114}{339}$$

Scanned with CamScanner

Q=4

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}}$$

$u(x,t) = \sin(x+2t)$ is solution

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

it will satisfy the above eqn

$$\frac{\partial u}{\partial t} = \cos(x+2t) \cdot \frac{d}{dt}(x+2t)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

again $\frac{\partial^2 u}{\partial t^2} = -2\sin(x+2t) \frac{dt}{dt}(x+2t)$

$$= \frac{\partial^2 u}{\partial t^2} = \cancel{2}\sin(x+2t)\frac{dt}{dt}(x+2t)$$

$$\boxed{= \frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

A

Now $\frac{du}{dx} = \cos(x+2t)$.

$\frac{d^2u}{dx^2} = -\sin(x+2t)$

$\Rightarrow \boxed{\frac{d^2u}{dx^2} = -\sin(x+2t)}$ — (B)

comparing A & B

$c = 2$

$\Rightarrow -4\sin(x+2t) = -c^2 \sin(x+2t)$

$\Rightarrow -4\sin(x+2t) + c^2 \sin(x+2t) = 0$

Two is Possible if $c = \pm 2$

$-4\sin(x+2t) + (\pm 2)\sin(x+2t) = 0$

$\Rightarrow 0 = 0$

These $u(x,t) = \sin t(x+2t)$

is solution of I-D

were equation