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Semester :- 2nd (spring)
Subject :- Linear Algebra
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Mid-Term Assignment # 1
Date :- 16/04/2020

Q 1: Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementry row operation that should be performed in order to solve this system. where **ID_3** is the 3rd digit in your ID and **ID-last** is the last digit of your ID in inverse e.g if your ID is 12345 then ID last = -5.

$$\left[\begin{array}{cccc|c} 1 & ID_3 & 3 & 0 & 5 \\ 0 & 1 & -ID\text{-}last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{array}\right]$$

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My ID = 16694
$ID_3 = 6$
$-ID\text{-}last = -4$

Given Matrix.

$$\left[\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

In elementry Row operation

(1) we can add any two rows together.

(2) we can subtract any two rows from eachother.

(3) We can multiply any rows with a non-zero constant.

(4) we can interchange any two rows ~~two~~ together.

To solve linear system.
The diagonal elements of the matrix must be one from the lift ~~sig~~ side.
Except the diagonal all the elements of the matrix is zero.

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$$\left[\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$ $R_2 + 4R_3 \to R_2$ we put the answer in row 2.

$$\begin{array}{r} -4 \\ +4 \\ \hline 0 \end{array} \quad \begin{array}{r} -24 \\ +7 \\ \hline -17 \end{array}$$

$$\stackrel{R}{=} \left[\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$ $R_1 + (-3R_3) \to R_1$ we put the answer in row$_1$

$$\begin{array}{r} -6 \\ -3 \\ \hline +18 \\ +5 \\ \hline 23 \end{array}$$

$$\stackrel{R}{=} \left[\begin{array}{cccc|c} 1 & 6 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$ Again $R_1 + (-6R_2) \to R_1$ we put the answer in row 1

$$\begin{array}{r} -6 \\ +6 \\ \hline 0 \end{array} \quad \begin{array}{r} -17 \\ \times -6 \\ \hline 102 \end{array}$$

$$\stackrel{R}{=} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 102 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$ Answer-

These are the steps which should be performed in elementry row operation to solve this system.

Q3: The row echelon form is used to solve the system of linear equations

(a) what is the difference between the row echelon and reduced row echelon form? what is the practical uses of reduced row echelon form? Give one example.

Difference b/w Row echelon and R.R. echelon form:

Ans: i Row echelon form:- A matrix is is said to be in row echelon form if it satisfy the following properties.

(1) All zero rows, if there are any appear at the bottom of matrix.

(2) The first nonzero entry from the left of a nonezero row is 1. This entry is called a leading one of its row

(3) For each nonzero row, the leading one appears to the right and below any leading one's in preceding rows.

$$A=\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & -1 \end{bmatrix}$$

ii Reduced Row echelon form:-

A matrix is said to be in reduced row echelon form if it satisfy the following properties.

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R.R. Echelon form.

(1) All zero rows, if there are any appear, at the bottom of matrix.

(2) The first nonzero entry from the left of a nonzero ~~entry~~ ~~from~~ ~~the~~ row is 1. This entry is called a leading one of its row.

(3) For each nonzero row, the leading one appears to the right and below any leading one's in preceding rows.

(4) If a column contains a leading one, then all other entries in that column are zero.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Main difference:

The first three properties of both row echelon and reduced row echelon form is same but the fourth (4th) property of reduced row echelon form does not exist in Row echelon form. That is the main difference.

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Practical use of reduced row echelon form:

Practically we use the reduced row echelon form to solve the linear system in the form of matrix.
we use the reduced row echelon form to find the exact or accurate value of the variables (x, y, z).

Example:

$$w + 6x + 3y = 5$$
$$x - 4y = 7$$
$$y = -6$$
$$z = 6$$

$$\left[\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

when we apply reduced row echelon on this matrix. It becomes in this form as under.

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$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 102 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

This is the answer of the given linear system. which means that.

$$\begin{aligned} w &= 102 \\ x &= -17 \\ y &= -6 \\ z &= 6 \end{aligned}$$

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Q3 (b) Find an echelon form for the below matrix using row operations. where is the ID_2 is 2nd digit in your ID. eg if your ID is 12345 $ID_2 = 2$, $ID_3 = 3$ and first, last is the first and last digit of your ID i.e 15.

$$\begin{bmatrix} 1 & ID_2 & 8 \\ 2 & 8 & -1 \\ -ID_3 & 0 & 0 \\ 1 & -4 & ID-First+last \end{bmatrix}$$

Sol:

ID = 16694   Solve by echelon form

$$= \begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -6 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix}$$ $R_2 - 2R_1$ we put result in R_2.

$$= R \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & -17 \\ -6 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix}$$ $R_3 + 6R_1 \to R_3$ we put result in Row_3.

$$= R \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & -17 \\ 0 & 36 & 48 \\ 1 & -4 & 14 \end{bmatrix}$$ $R_1 - R_4 \to R_4$ we put result in R_4.

$$= R \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & -17 \\ 0 & 36 & 48 \\ 0 & 10 & -6 \end{bmatrix}$$ $6+R_2 - R_4 \to 4$
we put result
in Row 4

$$= R \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & -17 \\ 0 & 36 & 48 \\ 6 & 0 & -17 \end{bmatrix}$$ $6R_1 - R_4 \to R_4$
we put result
in Row 4

$$= R \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & -17 \\ 0 & 36 & 48 \\ 0 & 36 & 31 \end{bmatrix}$$ $R_4 - R_3 \to R_4$
we put result in
Row 4.

$$= R \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & -17 \\ 0 & 36 & 48 \\ 0 & 0 & 17 \end{bmatrix}$$

This the echelon form.

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Q 2:- (b) Below given are the some matrices. Find which one is the row echelon form and which is reduced row echelon form. Explain in your own words for each of the selection in detail.

(a)
$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$ is in echelon form.

Ans It is in row echelon form because the first elements is nonzero and 'e' is the mathematica constant which value is 2.71. and π is also have the constant value which is $22/7$ (3.14).
The bottom starting ~~ending~~ value is zero which is echelon form.

(b)
$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is in echelon form.

Ans: It is also in row echelon form because the first condition of row echelon form is true.
2nd condition is also true.
The 3rd condition of echelon form which is left bottom element are zero which means that it is in echelon form.

(c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Ans:- Yes it is in reduced row echelon form because the All entries of leading one's 1's column must be zero.

(d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Ans:- It is in reduced row echelon form ~~f~~ because the properties of echelon form is exist there and the leading one (1) entry of it row is nonzero from the left. The column contains the leading one, then all other entries in that column are zero. For each nonzero row the leading one appears to the right and below any leading one's in preceding rows.

Q2:: (a) Find the elementry row operation that transform the first matrix into second and reverse row operation that transforms the second matrix into first.

first Matrix $A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$

2nd Matrix $B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

Sol: firstly we use elementry row operation on Matrix 'A', which transform the first matrix into 2nd matrix which is Matrix 'B'.

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first Matrix $A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} -2R_2 + R_3$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

we transform the matrix 'A' into Matrix 'B' By apply elementry Row operation.
which means that matrix 'A' is now equal to matrix 'B'.

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Now 2nd Matrix $B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \; 2R_2 + R_3$$

No we use the reverse Row operation on matrix B.

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

This is the reverse Row operation which we apply on Matrix 'B'.
which transform the Matrix 'B' 2nd Matrix into Matrix 'A' first Matrix.

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