Name :- Muhammad Saad

Id # 16097

Subjet :- Applied Calculus

Teacher :- MAM Shomaila Mazhar

Q1 The function $g(t)$ is defined

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t + 3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

(a) State any point of discontinuity.

To check the possibility of discontinuity of the function at $x = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$ — ①

L.H.L

$$\lim_{t \to 4} g(t) = \lim_{t \to 4} (2t+3)$$

$$= 2(4)+3$$

$$= 11$$

R.H.L

$$\lim_{t \to 4} g(t) = 12$$

$$L.H.L \neq R.H.L$$

so the function is discontinous at $x=4$

(b) Find, if they exist

(i) $\lim_{t \to 3} g$

L.H.L

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} t^2 \quad 0 \leq t \leq 3$$

$$= (3)^2 = 9$$

R.H.L

③

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} (2t + 3)$$

$$= 2(3) + 3$$

$$= 9$$

$$L \cdot H \cdot L = R H \cdot L$$

So $\lim_{t \to 3}$ exist.

Q2 Find the Maclaurin's series for

$$y(x) = x^2 + \sin s$$

Diff w.r.t x

$$y'(x) = \frac{d}{dx}(x)^2 + \frac{d}{dx}\sin x$$

$$y'(x) = 2x + \cos x.$$

Diff again w.r.t x

$$\frac{d}{dx}y'(x) = \frac{d}{dx}(2x + \cos x)$$

$$y''(x) = 2 + (-\sin x)$$

$$y''(x) = 2 - \sin x$$

Diff again w.r.t x

$$\frac{d}{dx}y''(x) = \frac{d}{dx}(2 - \sin x)$$

$$y'''(x) = 0 - \cos x$$

Diff again w.r.t x

$$y''''(x) = -(-\sin x)$$

$$y''''(x) = \sin x$$

Now For maclaurins
Put $x = 0$ in $y'(x), y''(x), y'''(x), y''''(x)$

$y(0) = 0$

$y'(0) = 2(0) + \cos(0)$

$y'(0) = 1$

$y''(0) = 2 - \sin(0)$

$y''(0) = 2$

$y'''(0) = -\cos(0)$

$y'''(0) = -1$

* Maclurins series

$f(x) = f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \text{----}$

$= 0 + \frac{x^1}{1}(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1) + \text{-----}$

$= x + x^2 - \frac{x^3}{3!} + \text{-------}$ Ans.

Q3 Find y'' given

(i) $1 + xy = x^2 + y^2$

Diff w.r.t x

$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$

$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}y^2$

$0 + \left[x\, dy/dx + y \cdot d/dx\, x\right] = 2(x)^{2-1} \cdot 1 + 2y^{2-1}\, dy/dx$

$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$

$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$

Taking dy/dx common

$\frac{dy}{dx}(x - 2y) = 2x - y$

$y' = dy/dx = \frac{2x - y}{x - 2y}$

Diff again

$\frac{d}{dx}y' = \frac{d}{dx}\frac{2x-y}{x-2y}$

$$y'' = \frac{1}{(x-2y)^2}\left[(x-2y)\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x-2y)\right]$$

$$y''(x) = \frac{1}{(x-2y)^2}\left[(x-2y)(2-dy/dx)-(2x-y)(1-2\,dy/dx)\right]$$

Put $dy/dx = \dfrac{2x-y}{x-2y}$

$$y''(x) = \frac{1}{(x-2y)^2}\left[(x-2y)\left(2-\frac{2x-y}{x-2y}\right)-(2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)\right]$$

$$= \frac{1}{(x-2y)^2}\left[\cancel{(x-2y)}\left[\frac{2x-4y-2x+y}{\cancel{x-2y}}\right]-(2x-y)\left[\frac{x-2y-4x+2y}{x-2y}\right]\right]$$

$$= \frac{1}{(x-2y)^2}\left[-3y-(2x-y)\left(\frac{-3x}{x-2y}\right)\right]$$

$$= \frac{1}{(x-2y)^2}\left[\frac{-3y(x-2y)+3x(2x-y)}{(x-2y)}\right]$$

$$= \frac{1}{(x-2y)^2}\left[-3xy + 6y^2 + 6x^2 - 3xy\right]$$

$$= \frac{1}{(x-2y)^3}\left[(6x^2 + 6y^2 - 6xy)\right]$$

$$= \frac{1}{(x-2y)^3}\left[6(x^2 + y^2 - xy)\right]$$

But $x^2 + y^2 = 1 + xy$

$$\frac{1}{(x-2y)^3}\left[6(1 + \cancel{xy} - \cancel{xy})\right]$$

$$y''(x) = \frac{6}{(x-2y)^3} \quad \text{Ans.}$$

3(ii) Find y' by using logarithmic differentiation

$$y = x^3 (1+x)^9 e^{6x}$$

Taking ln on b/s

$$\ln y = \ln[x^3 \cdot (1+x)^9 e^{6x}]$$

$$\ln y = \ln (x)^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + \ln(e^{6x})$$

Diff w.r.t x

$$\frac{d}{dx} \ln y = 3 \frac{d}{dx} \ln x + 9 \frac{d}{dx} \ln (1+x) + \frac{d}{dx} \ln(e^{6x})$$

$\ln(m \times n) = \ln(m) + \ln(n)$

$\ln(m)^n = n \ln(m)$

$$\frac{1}{y} \frac{dy}{dx} = 3 \cdot \frac{1}{x} \cdot 1 + 9 \cdot \frac{1}{1+x} \cdot 1 + \frac{1}{e^{6x}} \frac{d}{dx} e^{6x}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{\cancel{e^{6x}}} \cdot \cancel{e^{6x}} \cdot 6$$

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xing b.s y

$$y\frac{1}{y}\frac{dy}{dx} = y\left[\frac{3}{x} + \frac{9}{1+x} + 6\right]$$

Ans.